ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-34 REYNOLDS FLOW MODEL
1. Reynold Flow Model - Definitions
2. Reynolds Flux Hypothesis
3. Momentum transfer with suction/blowing
4. Single phase Conv H T with suction/blowing
5. Inert mass transfer without heat transfer
6. Inert mass transfer with heat transfer
7. Mass transfer with heat transfer and simple chemical reaction (SCR)
8. Mass transfer with heat transfer and arbitrary chemical reaction (ACR)
This model postulates ficticious mass flux \( g \) called Reynolds flux in the \( \infty \) - state.

Model develops algebraic relations that are related to real transport phenomena by reference to \( g \).

Model develops \( N_w = g \times B \) relation.

The objective is to relate \( g \) to heat transfer coefficient \( h_{Nw \rightarrow 0} \).

TW - state is just inside the Considered phase.

TL - state is just inside the Neighbouring phase.

In the T - state, temperature and compositions are uniform.
Mass Transfer Flux is sum of all species transferred across the interface. Thus

\[ N_w = \sum_j N_{j,w} = \sum_k N_{k,T} \]

where species j and k need not be the same.

\( q_w \) is positive when flowing towards the interface from the considered phase.

\( q_l \) is positive when flowing away from the interface in the Neighbouring phase.

Therefore, \( (q_w - q_l) = N_w (h_{TW} - h_{TL}) \) where \( h_{TW} \) and \( h_{TL} \) are enthalpies of the transferred substance at the TW and TL states. When \( (q_w - q_l) \neq 0 \), phase-change occurs.

\( q_{rad} \) from Considered phase to the interface is positive. It is accounted in the Neighbouring phase for convenience.
In different applications with or without chemical reaction, complex processes occur in the considered phase.

To avoid solution of any differential equations, it is assumed that the complex processes can be effectively simulated by:

1. A fictitious flux \( g \) through the \( \infty - \infty \) plane towards the interface carrying with it properties of the \( \infty \) - state.
2. A fictitious flux \( N_w + g \) through the \( \infty - \infty \) plane away from the interface carrying with it properties of the \( w \) - state.

The magnitude of \( g \) is not affected by presence of gradients of \( \omega_j \) or \( T \) or turbulence, radiation etc in the considered phase.

The \( g \) - flux is also not affected by direction of \( N_w \).

The \( g \)-flux is supposed to produce same effect at the interface that the real flow is likely to produce.
Consider a solid surface \( T_s \) to be protected from hot gases \( T_\infty > T_s \) and \( \omega_{v,\infty} < \omega_{v,w} \) by a thin water film. The Reynolds Hypothesis states that a g-flux of hot gases towards the interface together with \( N_w + g \) flux of cooler moist air away from the interface will produce the effect: \( N_w > 0, q_w > 0 \) and \( q_l < 0 \).

In this sense, the g-flux is considered capable of responding to the mass-fraction and temperature gradients as well as turbulence effects in the considered phase as shown in the right figure.
Consider control volume between $\infty$- and w- states. Then

\[
\text{Rate of Momentum Flux In} = g \, U_\infty
\]
\[
\text{Rate of Momentum Flux Out} = \tau_w + (N_w + g) \, u_w = \tau_w
\]

Hence,

\[
g \, \frac{U_\infty}{\tau_w} = 1 = \frac{N_w}{\rho \, V_w}
\]
or

\[
N_w = g \times \frac{\rho \, v_w \, U_\infty}{\tau_w} = g \times \frac{v_w / U_\infty}{\tau_w / (\rho \, U_\infty^2)}
\]

Further

\[
g = \frac{\tau_w}{U_\infty} = \rho \, U_\infty \left( C_{f,x} / 2 \right)
\]
Here, CV between T- and ∞-states is considered. Then

\[
\text{Rate of Heat Flux In} = g \ h_\infty + N_w \ h_T \\
\text{Rate of Heat Flux Out} = (N_w + g) \ h_w
\]

Equating and rearranging and taking \( T_{ref} = 0 \)

\[
N_w = g \times B_h \ \rightarrow \ B_h = \frac{h_\infty - h_w}{h_w - h_T} = \frac{(c_p T)_\infty - (c_p T)_w}{(c_p T)_w - (c_p T)_T}
\]

where \( N_w = \rho_w \ V_w \)

If specific heats are equal

\[
B_h = \frac{T_\infty - T_w}{T_w - T_T}
\]
Further, CV between $\infty$- and $w$-states is considered. Then

\[
\text{Rate of Heat Flux In} = g \, h_\infty + N_w \, h_w
\]
\[
\text{Rate of Heat Flux Out} = (N_w + g) \, h_w + q_w
\]

Equating and rearranging \( g = \frac{q_w}{(h_\infty - h_w)} \)

If specific heats are equal

\[
g = \frac{q_w}{c_p \, (T_\infty - T_w)} = \frac{h_{\text{cof},v_w}}{c_p}
\]

Thus, \( g \) is related to \( h_{\text{cof},v_w} \).
Inert MT without HT - L34($\frac{8}{14}$)

1. Here, in all states, temperatures are equal ( $T_T = T_w = T_\infty$ )

2. Consider CV between T- and $\infty$ - states. Then

\[
\text{Rate of mass Flux In} = g \omega_{v,\infty} + N_w \omega_{v,T}
\]
\[
\text{Rate of Mass Flux Out} = (N_w + g) \omega_{v,w}
\]

3. Equating and rearranging

\[
N_w = g \times B_m \quad \rightarrow \quad B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}}
\]

where for a pure liquid in the T-state, $\omega_{v,T} = 1$.

$\omega_{v,w}$ is evaluated from equilibrium at $T_w$
Here, let $T_\infty \neq T_w \neq T_T$. Then

Consider, CV between $\infty$- and T-states is considered. Then

Rate of Heat Flux In $= g h_{m,\infty} + N_w h_{m,T}$

Rate of Heat Flux Out $= (N_w + g) h_{m,w}$

Equating: $N_w = g \times B_{mh}$ where $B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T}}$

$h_m = \omega_v h_v + (1 - \omega_v) h_a$

$h_v = c_{p,v} (T - T_{ref}) + \lambda_{ref}$, $h_a = c_{p,a} (T - T_{ref})$

Making $Le = 1$ assumption, $B_{mh} = B_m$ from which

$\omega_w \sim h_w (T_w)$ relationship is iteratively calculated.
Now, consider CV between $\infty$- and $w$-states. Then

$$q_w + (g + N_w) h_{m,w} = g h_{m,\infty} + N_w h_{m,TW}$$

$$N_w = g \times B_{mh} \quad \text{where} \quad B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,TW} + q_w/N_w}$$

But, $(q_w - q_l) = N_w (h_{TW} - h_{TL})$. Hence

$$B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,TL} + q_L/N_w}$$

If $T_w = T_{bp}$, $q_L = 0$ and $h_{m,w} - h_{m,TL} = h_{fg}$.

If radiation is included then $N_w h_{m,T} + q_l + q_{rad} = N_w h_{TL}$. Hence

$$B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,T} - q_{rad}/N_w}$$
Here, we have 3 species: $fu$, $O_2$ and Pr with stoichiometric coefficient $r_{st} = \text{kg of } O_2 / \text{kg of fuel}$.

Consider CV between $\infty$- and T-states. Then

For fuel
\[ g \omega_{fu,\infty} + \omega_{fu,T} N_w - \omega_{fu,w} (g + N_w) = -|R_{fu}| \]

For $O_2$
\[ g \omega_{O_2,\infty} + \omega_{O_2,T} N_w - \omega_{O_2,w} (g + N_w) = -|R_{O_2}| \]

But, $R_{O_2} = r_{st} R_{fu}$. Hence, dividing the 2nd Eqn by $r_{st}$ and subtracting from 1st Eqn, we have

\[ N_w = g \times B_\Phi \rightarrow B_\Phi = \frac{\Phi_{\infty} - \Phi_w}{\Phi_w - \Phi_T} \rightarrow \Phi = \omega_{fu} - \frac{\omega_{O_2}}{r_{st}} \]

By the same reasoning, it is also possible to define

\[ \Phi = \omega_{fu} + \frac{\omega_{pr}}{1 + r_{st}} = \frac{\omega_{pr}}{1 + r_{st}} + \frac{\omega_{O_2}}{r_{st}} \]
1. Invoking the energy conservation principle and assuming $c_{p,k} = c_{pm}$, we have shown that

$$h_m = \sum_k \omega_k h_k = c_{pm}(T - T_{ref}) + \omega_{fu} \Delta h_c = c_{pm}(T - T_{ref}) + \frac{\omega_{O_2}}{r_{st}} \Delta h_c$$

2. Consider CV between $\infty$- and w-states. Then

$$g h_{m,\infty} + N_w h_{m,TW} = (g + N_w) h_{m,w} + q_w$$

3. Taking the 2nd definition for $h_m$, we have $N_w = g \times B_{mh}$, where taking $T_{ref} = T_w$

$$B_{mh} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{m,TW} + q_w/N_w}$$

$$B_{mh} = \frac{c_{pm}(T_\infty - T_w) + \Delta h_c (\omega_{O_2,\infty} - \omega_{O_2,w})/r_{st}}{\Delta h_c (\omega_{O_2,w} - \omega_{O_2,TW})/r_{st} + q_w/N_w}$$
For a volatile liquid fuel or transpiration cooling by a combustible gas (e.g. \( H_2 \)), \( \omega_{O_2,w} = 0 \) and since the transferred substance does not contain \( O_2 \), \( \omega_{O_2,TW} = 0 \). Hence,

\[
B_{mh} = c_{pm} \left( T_\infty - T_w \right) + \Delta h_c \frac{\omega_{O_2,\infty} / r_{st}}{q_w / N_w}
\]

Now, for a liquid fuel at its boiling point (\( T_w = T_{bp} \)),

\[
q_w - q_l = N_w (h_{m,TW} - h_{m,TL}) = N_w \lambda_{fu}. \quad \text{Hence,}
\]

\[
B_{mh} = c_{pm} \left( T_\infty - T_{bp} \right) + \Delta h_c \frac{\omega_{O_2,\infty} / r_{st}}{\lambda_{fu} + q_L / N_w}
\]

For an atomised tiny liquid droplet, if \( T_T \approx T_{bp} \), \( q_L = 0 \).
When it is difficult to ascertain mass-fractions of compounds in different states, it is preferred to use conserved property $\eta_{\alpha} = \sum_k \eta_{\alpha,k} \omega_k$ where $\eta_{\alpha,k} = M_\alpha / M_k$.  

Consider CV between $\infty$- and T-states. Element balance gives

$$g \eta_{\alpha,\infty} + N_w \eta_{\alpha,T} = (g + N_w) \eta_{\alpha,w} \quad \text{or}$$

$$N_w = g \times B_m \quad \rightarrow \quad B_m = \frac{\eta_{\alpha,\infty} - \eta_{\alpha,w}}{\eta_{\alpha,w} - \eta_{\alpha,T}}$$

It is usually convenient to combine different $\eta_{\alpha}$ into a new conserved property variable $\Phi$ as was shown in lecture 32 (slide 18)
Thus, in all cases, we have shown that

\[ N_w = g \times B \rightarrow B = \frac{\psi_\infty - \psi_w}{\psi_w - \psi_T} \]

where \( \psi = \omega_v, \eta_\alpha, h_m \) and \( \Phi \).

Any linear combinations of \( \psi \) are also solutions.

The result \( N_w = g \times B \) from the Reynolds flow model is different from the result \( N_w = g^* \ln (1 + B) \) for 1D Stefan flow model (diffusion) and Couette flow model (convection).

The result from the model correctly identifies \( B_f \) in momentum transfer and shows that \( g = h_{cof,vw}/c_p \) in single phase Conv HT.

The Reynolds flow model result will be employed to provide interface boundary conditions to the Boundary Layer flow model in the next lecture.