3 Laminar Internal Flows - l14 to l20

1. Show that the axial velocity profile in a fully developed flow in an annulus is given by

$$\frac{u}{\bar{u}} = \frac{2}{A} \left[ 1 - \left(\frac{r}{r_o}\right)^2 + B \ln \left(\frac{r}{r_o}\right) \right]$$

whrer $B = ((r^*)^2 - 1)/\ln r^*$, $A = 1 + (r^*)^2 - B$ and $r^* = r_i/r_o$. Hence show that the maximum velocity will occur at $(r_m^*)^2 = 0.5B$ and $fRe = 16 (1 - (r^*)^2)/A$.

2. Determine Fully-Developed Pressure drop per unit length ( N / m$^3$ ) of the following ducts. In each case $Re_{Dth} = 500$. Take $\rho = 1000$ kg / $m^3$, $\mu = 8 \times 10^{-4}$ kg/m-s. ( Hint: Wherever possible, use solns given in the class. Else, evaluate from series solutions. )

9
Figure 5: Duct of Rhombic cross-section

(a) Rectangular duct: \(( b = 1 \text{ cm}, \ a = 4 \text{ cm} )\)
(b) Annulus \(( r_i = 1 \text{ cm}, \ r_o = 2.2 \text{ cm} )\)
(c) Annular Sector Duct \(( r_i = 1 \text{ cm}, \ r_o = 2.2 \text{ cm}, \ \theta_0 = 60 \text{ degrees} )\)
(d) Equilateral Triangle: \(( a = 2 \text{ cm} )\)
(e) Equilateral Triangle with Rounded Radius: \(( a = 2 \text{ cm}, \ r / a = 0.1 )\)

3. Using Kantarovich method, derive analytic expression for 
\((f \ Re)_{D_h} = F(x_0, b/a, \Phi)\) for the duct shown in figure 5.

4. Write a general computer program for predicting \((f \ Re)_{D_h}\) and \(N u_{D_h}\) in 
ducts of arbitrary cross section using the method described in lectures 
16 and 18.

5. Using the computer program developed in the previous problem, cal-
culate \((f \ Re)_{D_h}\) and \(N u_{D_h}\) for the duct shown in figure 5.

6. Consider FD flow and heat transfer between two parallel plates. Include 
effect of viscous dissipation. Constant wall heat flux \(q_w\) is applied at 
both surfaces. Show that \((f \ Re)_{D_h} = 24\) and

\[
N u_{D_h} = \frac{140}{17 + 108 \ Br} \quad \text{where} \quad Br \equiv \frac{\mu \ \overline{u}^2}{q_w \ D_h}
\]
7. A heat exchanger is to be designed to cool lubricating oil (\( \rho = 785 \text{kg/m}^3 \), \( k = 0.12 \text{ W/m-K} \), \( \nu = 0.0247 \text{ m}^2/\text{hr} \) and \( \text{Cp} = 2 \text{ kJ/kg-K} \)) from 60°C to 40°C. The oil velocity is 0.75 m/s and the tube surface temperature is 27°C. Calculate the required tube (dia 6 mm) length.

8. Water at 35°C enters a tube (2.5 cm ID) with a velocity 1.25 m/s. The tube wall temperature is constant at 95°C. Calculate the tube-length necessary to raise water temperature by 45°C. Also calculate pumping power. Use a) Dittus-Boelter correlation b) Expression for Nu derived from universal law.

9. Liquid mercury flows through a long tube (2.5 cm dia) with a velocity 1 m/s. Calculate \( h \) for constant wall heat flux (\( \rho = 13264 \text{ kg/m}^3 \), \( \text{Cp} = 0.1365 \text{ kJ/kg-K} \), \( k = 11.5 \text{ W/m-K} \), \( \mu = 9 \times 10^{-4} \text{ N-s/m}^2 \))

10. Consider flow of air (\( Pr = 0.7 \)) between two parallel plates 1 m wide and 2.5 cm apart. The top plate receives heat flux \( q_{\text{top}} = 650 \text{ W/m}^2 \) whereas the bottom plate is insulated. If \( Re_{Pa} = 500 \) and air enters the passage with \( T_{in} = 32^\circ\text{C} \), calculate and plot variations of \( T_{w,\text{top}} \) and \( T_{w,\text{bottom}} \) till \( x^+ = 0.1 \) (Hint: Use results of slide 4, 119).

11. Consider flow of air in a passage formed by two parallel plates \( W = 1 \) m wide, \( L = 1.4 \) m long and separated by distance 3 mm. Heat flux at both the plates varies as

\[
q_w = 950 + 2500 \sin \left( \frac{\pi x}{L} \right) \left( \frac{W}{m^2} \right)
\]

The air inlet temperature is 50°C and system pressure is 7 bar. If the air mass flux is 250 kg/m²-hr, calculate and plot variation of \( T_b, T_w \) and \( Nu_x \) with axial distance \( x \) (m). Evaluate properties at 100°C.