Practice Exercises-1

1. Find the steady state temperature distribution in an infinite slab of width b in which heat generation rate per unit volume is given by \( q'' = ax + bx^2 \) where ‘a’ and ‘b’ are constants. The face \( x=0 \) is cooled by a fluid whose temperature is \( T_\infty \) and the associated heat transfer coefficient is ‘h’ while the face \( x=b \) is perfectly insulated (i.e. no heat flows in or out at this face).

Determine the temperature of the face at \( x=0 \) using the above temperature distribution. Determine the temperature of this face by performing an overall energy balance.

2. You are in a hurry one morning and took out the immersion heater from the bucket and put it on a nearby table without switching the power off. The heater can be modeled as a hollow cylinder with insulation and metal coverings as shown in the figure. The ambient temperature is 300K and the outer and inner wall (innermost radius = 50mm) are exposed to the surrounding atmosphere with heat transfer coeff. of 10 W/m² K. The numbers in the brackets are the thicknesses of the various walls. The heater element is very thin and can be assumed to be isothermal. If the heater can withstand a temperature of 1200 K do you think your heater will get damaged. Assume steady 1-D conditions and that the heater power is 5000 W/m. Ignore any radiative transfer. Assume \( K_{\text{metal}} = 20 \) W/mK; \( K_{\text{insulation}} = 0.1 \) W/mK.

3. A hot surface at 100 °C is to be cooled by attaching aluminum fins to it. Each fin is made by joining two cylinders of length 2cm each and diameters 2cm and 1cm respectively as shown in the figure. If the surrounding air is at 30 °C and the surrounding heat transfer coefficient can be assumed to be 10 W/ m² K determine the heat lost by each fin. \( k_{\text{Al}} = 240 \) W/mK

4. A Nuclear fuel element of thickness ‘2L’ is covered with a steel cladding of thickness ‘b’. Heat generated within the nuclear fuel at a rate \( q''' \) is removed by a fluid at \( T_x \) which adjoins one surface and is characterized by a convective coefficient ‘h’. The other surface is well insulated, and the fuel and steel have thermal conductivities of \( k_f \) and \( k_s \) respectively. (a) Obtain an
equation of the temperature distribution \( T(x) \) in the nuclear fuel. Express your results in terms of \( q''', \ k_f, \ L, \ b, \ k_s, \ h \) and \( T_x \). (b) Sketch the temperature distribution for the entire system.

5. A special computer chip has two thin regions 1 and 2 embedded in a matrix which has a thermal conductivity of 4.0 W/K. Heat is generated in regions 1 and 2 only and equal to 50 kW/m² and 13.33 kW/m² respectively. The chip is convectively cooled at its outer surface, with a fluid stream \( h_1 = 1000 \) W/m² K and \( T_{x1} = 30^\circ \text{C} \). The chip is joined to the circuit board at its inner surface. The thermal contact resistance between the chip and the board surface can be ignored. The board thickness and thermal conductivity are \( L = 1 \) mm and \( k = 2.0 \) W/mK, respectively. The other surface of the board is exposed to cooling air for which \( h_2 = 1000 \) W/m² K and \( T_{x2} = 20^\circ \text{C} \). Evaluate the steady temperatures of the two regions. Assume the heat generation regions to have negligible thickness. The thickness of the surrounding matrix is shown in the figure.

6. A rod of length ‘L’, which acts as a handle, protrudes from a furnace door. Since it is perceived to loose heat to the ambient, you have decided to insulate it. You know some heat transfer and therefore think that the insulation should be thicker at the base and thinner at the tip and therefore provide a linearly varying thickness of insulation as shown below with the diameter of the insulation at the base being four times the diameter of the fin i.e. diameter of fin with insulation is ‘4D_{fin}’ at the door edge. Note that it is reasonable to ignore transverse temperature gradients in the fin but not so for the insulation.

Set up the equations required for obtaining the temperature profile within this rod. The furnace door can be assumed to be at a constant temperature \( T_w \). Assume that the surrounding air is at \( T_{\infty} \) and the heat transfer coefficient is very large to permit you to make a suitable assumption, if necessary.
7. A semicircular stainless steel pipe is used in a miniature furnace for removing heat from the interior of the furnace as shown below. The radius of the pipe is 1cm and the pipe is very long to permit a per unit length calculation. The pipe is mounted on the wall which has a very low thermal conductivity and therefore the flat surface can be considered to be insulated. The furnace gases can be assumed to be at a temperature of such that the entire pipe facing the gases is maintained constant at 300°C while the fluid in the pipe can be assumed to be maintained at 250°C for some process application. The semicircular portion of the pipe is assumed to be of negligible thickness while the plate that it is welded to, is 2mm thick. If the internal heat transfer coefficient for the flowing fluid is 1000W/m²K, determine the heat being transferred to the fluid if the flow rate is 1X10⁻⁶ kg/s. The specific heat of the pipe material and the fluid are 200J/kgK and 2000J/kgK while the thermal conductivity of the pipe and fluid are 200 and 2 W/mK.

8. The furnace shown in the figure is used to cure chemical samples. A solid rod is inserted into the furnace with the sample mounted on it and is kept there, till the sample has attained a predetermined temperature and is then removed. The rod is to be positioned very accurately and therefore manufactured with high precision and therefore overheating of the rod is not permitted. Heat is therefore removed from the base of the rod maintaining it at 100°C using a control mechanism. The conductivity of the rod can be assumed to be 100 W/mK, the diameter 10mm and length 100mm. The furnace temperature is 1000°C and in the computation of heat transfer to the rod, the rod temperature can be assumed to be much smaller than the furnace temperature. Thickness of furnace walls can be ignored and rod tip can be assumed insulated. Determine the heat removal required at the base of the rod. Assume σ=5.67X10⁻⁸ W/K⁴m², ε=1 for all surfaces.

9. You are evaluating a new chapatti ‘making and cooking’ machine. The system is made horizontal, several dough balls are put in between the left plate and the middle plate and the left plate is closed. The system is turned upside down and the same operation is performed with the right plate. The system is made vertical and after a few seconds the two plates are separated and the chapattis fall into a bottom tray and the process repeats. The left and right plates are identical and made of a material with k=1W/mK. The left plate of 20mm thickness has a heater of negligible thickness and power 10kW/m², exactly in the middle as shown. You model the process to be a steady one even though it really is not since the process is happening very fast and the heaters are on during the entire operation. You also assume that all the chapattis together
form a uniform sheet of chapatti of negligible thickness (and therefore no temperature gradients within). The chapatti has to be maintained at 100°C for proper cooking. Using a 1D approximation in the direction from top to middle to bottom plate, determine the heater temperature that will exist.

10. A builder wishes to insulate the wall of a room of a building by embedding wood within concrete as shown below. The total thickness of the concrete and wood are 10cm and 1cm respectively and are fixed from structural constraints. Using a 1D analysis, determine the minimum heat leak into the room if you are permitted to change the location of the wood within the concrete. Assume the inner wall of the room is at 25°C and outer wall is at 40°C. Use $K_{\text{concrete}}=1.0\,\text{W/mK}$, $K_{\text{wood}}=0.1\,\text{W/mK}$.

11. At steady conditions the temperatures at the ends of a system consisting of two alloy steel plates 10cm and 20cm long joined together with a stone plate between them as shown are 100°C and 50°C. There is no contact resistance between the plates. Determine the temperatures of the ends of the stone plate if the temperature difference between the two faces is 20°C.

12. The figure shown is a methodology for measuring the thermal conductivity of sample materials with low thermal conductivity. The unknown material is sandwiched between two rods of known thermal conductivity. A heater is located at one end which supplies a known heat input and the
end is maintained at convenient constant temperature. At steady conditions the temperatures are noted on the rods at specified locations and the thermal conductivity of the sample is calculated. Assuming the contact resistance between the sample and rod on either side is 0.001 m²K/W determine the thermal conductivity of the sample. The temperatures measured at the two locations shown are 125°C (x=5cm), 100°C (x=10cm), and the heater power is 5W. The sample and the rods have a diameter of 5cm and the length of each rod and sample are 15cm and 1cm respectively.
1. Consider the 2D unsteady slab of length=width='L', problem worked out in class. In class all the four sides were exposed to the same heat transfer coefficient and free stream temperature and the solution was obtained as the product of the solutions for two 1D cases. The following case is to be now considered: Free stream temperature on all walls same but heat transfer coeff. different on opposite walls. Can a similar product solution as discussed in class be used in the form shown in the figure below. Now assume a slightly different case: Free stream temperature on opposite walls different but heat transfer coefficient same. Again is the product solution as shown the figure valid here? Assume initial temperature=Ti.

![Diagram](image)

2. Consider a regenerative heat exchanger where the material in the heat exchanger is subjected to alternate hot and cold fluid streams. The hot gas gives heat to the material and the cold gas takes up the heat and becomes hotter. The regenerative material is assumed to be composed of a large number of spherical balls of 5mm diameter each. In a typical cycle the balls are at 50°C when they are exposed to hot fluid at 80°C and after 5min the balls are exposed to cold fluid at 30°C for another 1.5mins. Determine the temperature of the balls at the end of 2.5mins and 6.0mins from the start of the cycle. Each ball can be considered to be separate and not affected by the others for the analysis. The heat transfer coefficient between the balls and fluid can be assumed to be 50W/m²K and for the balls, ρ=3000 kg/m³, Cₚ= 800J/kgK, K=5W/mK. Use the Duhamel’s theorem to obtain the solution.

3. Determine the temperature ‘T’, in the slab of length ‘L’ and width ‘W’ shown below as a function of time. The left and bottom walls are perfectly insulated. The top and bottom walls have wall flux that varies as shown in the figure. The slab is at temperature ‘Tᵢ’ initially. Split the problem into simpler 1D problems for which solutions are already available. The split should be mathematically justifiable. Assume all required properties are known.

![Diagram](image)
4. Obtain the temperature profile in the cross-section shown for heat conduction with no heat generation for the boundary conditions as shown. Do not use separation of variables to obtain the solution but use the solutions already discussed earlier and use superposition to obtain the solution.

5. Solutions for steady temperature distribution for a few two dimensional problems have been obtained in class for Cartesian coordinates. Now try to attempt the solution in radial coordinates. The geometry is as shown below and is a 90° annulus with inner and outer radii temperature ‘T_i=0’ and ‘T_o=0’ respectively. The temperatures at the θ=0° and θ=90° are ‘0’ and ‘T_90=1’ respectively.

(a) Write down the governing differential equation and boundary conditions that are required to obtain a solution. Now attempt a separation of variables type solution and obtain the resulting two one dimensional ordinary differential equations along with their corresponding boundary equations.

(c) Now attempt a solution. As usual one of the above two equations will give the ‘λ_m’ and the corresponding functions which we will call ‘R_m’ – this is chosen to be the radial direction. You can assume the solution ‘R_m’ and ‘λ_m’ to be known. The other equation you should be able to get a solution. Now, complete the solution for the above problem in the form of a series solution. Obtain this with the constants ‘C_m’ still undetermined.

(d) Now, obtain the constants ‘C_m’ in terms of the ‘R_m’ and ‘λ_m’ given above to complete the solution. Leave the solution in the form of an integral since you will not be able to perform the integration since ‘R_m’ is not obtained.
6. A copper block of density 8000 kg/m$^3$ radius 5mm and specific heat 300J/kgK at ambient temperature 30$^\circ$C is suddenly dipped into a hot water bath with temperature 60$^\circ$C. After time 10secs the hot water suddenly experiences a volumetric heat generating source and the temperature immediately rises to 70$^\circ$C and stays at this level for another 10secs after which the water temperature again rises instantly to 80$^\circ$C and stays at this level for another 10secs. The heat transfer coefficient between the water and the block can be assumed to be constant and equal to 3000 W/m$^2$K. Use the Duhamel’s theorem to determine the temperature of the copper block after 25 secs from the start of the transient.

7. Consider the solution for the temperature distribution for the 2D slab shown below that has been discussed in one of the presentations:

![2D slab diagram]

The following condition is required for the evaluation of the constants in the proposed series solution:

$$\int Sin my Sin ny dy = 0$$ for $m \neq n$ and m,n being integers

Show that this condition can be shown to be true without taking recourse to trigonometric relations using appropriate forms of equations obtainable from the governing equation and the boundary conditions for the problem.

8. A slab with insulated sides is initially at a uniform temperature $T_i$. At time $t>0$, a uniform heat source is activated within the slab while the two sides are maintained exposed to a flow with constant heat transfer coefficient at a temperature $T_\infty$. Determine the variation of the temperature of the slab. Note that you can split this into two problems for which solutions were derived in class.

9. Consider the solution of the temperature in a two dimensional rectangular geometry with constant thermal conductivity. The four walls are at temperatures $T_1$, $T_2$, $T_3$, $T_4$. It is possible to reduce this problem to sub-problems in such a way that the solution obtained for the problem discussed with one non-zero temperature in class can be used without needing to solve additional differential equations. Split the problem such that (a) four subproblems need be solved and (b) only THREE subproblems need to be solved. Explain your steps carefully and do not try to solve the sub-problems.
10. The top portion (i.e. \( y = t \)) of a thin plate of thickness \( t \), and length \( L \) is heated by a uniform radiant flux \( q_r \) W/m\(^2\) as shown in the figure. The plate is very long in the direction normal to the plane of the paper and can be assumed to have constant thermal conductivity \( k \). The bottom portion (i.e. \( y = 0 \)) is insulated. The plate is cooled by maintaining the two ends at \( x = 0 \) and \( x = L \) at a constant temperature \( T_c \). Determine the maximum temperature of the plate and also the heat removed from either end per unit width normal to the plane of the paper.

11. Consider the unsteady conduction in a long cylindrical rod of radius \( r_0 \) and constant thermal conductivity \( k \). The initial temperature is uniform throughout at \( T_i \). The material is a strange composite material whose thermal diffusivity \( \alpha \) is a function of the radius and is given to be \( f(r) \), where \( f(r) \) is a function which can be assumed to be known. Suddenly the temperature of the entire outer surface of the rod is changed to \( T_0 \) and kept at this value and the temperature within the rod is allowed to change. Set up the differential equation for this situation along with the appropriate boundary conditions for obtaining the temperature variation with time. A separation of variables solution will work here. Show that a summation solution will be required. Assume any unknown constants and functions can be numerically obtained and represent them symbolically. Show the methodology to obtain the unknown constants in terms of the summation solution.

12. A slab with insulated sides is initially at a uniform temperature \( T_i \). At time \( t > 0 \), a uniform heat source is activated within the slab while the two sides are maintained exposed to a flow with constant heat transfer coefficient at a temperature \( T_{\infty} \). Determine the variation of the temperature of the slab. Note that you can split this into two problems for which solutions were derived in the handouts.

13. Consider the derivation of the Duhamel’s theorem in a slightly different form that done in the text. Use a general conduction equation and split the initial condition on the lines of the solved example problem.