Thus matrix $A$ is a simple yet effective way of summarizing the customer's recency for $r$ time periods. Furthermore depending on the concept of next present value of contribution and spending we may depict the reward matrix of company with respect to this customer as shown below:

Table 11.3: Reward matrix for the company depending on one customer

\[
\begin{array}{c|c}
1 & 1 \\
1 & P - M \\
2 & -M \\
. & . \\
t^{th} & -M \\
. & . \\
j^{th} & -M \\
. & . \\
r & 0 \\
\end{array}
\]

In case the company is interested to find the expected profit for this scenario, then we need to calculate the following which is given by $EV_{t=1}$ and that is shown below:

Table 11.4: Matrix for expected value earned by the company depending on one customer

\[
\begin{array}{c|c}
1 & (P - M) \times A \\
2 & \left\{ \frac{-M}{(1 + \eta)^{2}} \right\} \times A^2 \\
. & . \\
t^{th} & \left\{ \frac{-M}{(1 + \eta)^{t-1}} \right\} \times A^{t-1} \\
. & . \\
j^{th} & \left\{ \frac{-M}{(1 + \eta)^{j-1}} \right\} \times A^{j-1} \\
. & . \\
r & 0 \\
\end{array}
\]

Thus for an infinite time zone consideration we need to find $\lim_{t \to \infty} EV_t$ which will give a lot of information to the company about customer retention and his/her life value.
Let us assume that a marketing firm or a company is trying to acquire a new group of customers (in a particular age group or in a particular income group, etc.). In case the company is successful in doing so it makes a profit (say for example through increase in its market share or increase in number of products sold, etc.,) of say $p$ at the present time instance. This $p$ in marketing terms would be called the **net contribution** at the present time, the company makes to its coffers through its ability by successfully acquiring a particular customer. We also assume that this value of $p$ would be gained by the company in the first or subsequent instance after the customer is acquired/retained. Also consider that there may be many instances or time periods when the customer, does not make any purchase to add value to the company, but that does not stop the company from **spending** a net amount denoted by $M$, which is at present value and denotes how much money the company/firm is willing to spend in order to retain it customer say through television advertisement, direct marketing, etc. Furthermore, assume that the company believes the probability that the customer will purchase at the end of any particular period is a function of customer’s recency only. This denotes the actually number of periods elapsed since the customer's last purchase, i.e., if the customer purchased at the end of last period, then his/her recency value is 1 for the current period. Let us also assume that the probability the customer will purchase at the end of any period is $p_r$, where $r$ is customer's recency. Thus the probability transition matrix $A$ may be written as given below : 

Table 11.2: Transition probability matrix for the brand retention example

```
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>i^{th}</th>
<th>...</th>
<th>j^{th}</th>
<th>...</th>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_1$</td>
<td>$1-p_1$</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$p_2$</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>i^{th}</td>
<td>$p_i$</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>j^{th}</td>
<td>$p_j$</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>$1-p_j$</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>r</td>
<td>$p_r$</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>1</td>
</tr>
</tbody>
</table>
```
**Example 11.2**

As a second example of the use of Stochastic processes in marketing consider the study of customer life time value and its implication for interactive marketing. The concepts of interactive marketing helps us to decide the allocation of marketing budget amongst different media, apart from assisting the marketing manager to make plans of how to retain his/her existing customers in the long run. Customer retention refers to situations in which customers who are not retained are considered lost for good. Thus in a situation where one is interested to study customer retention, nonresponse signals the end of the firm's relationship with the customer. In contrast, customer migration situations are those in which nonresponse does not necessarily signal the end of the relationship.

It has been studied that Markov Chain Model can handle both customer migration and customer retention situations. In addition, the flexibility inherent in the Markov Chain Model which is the fact to do with it being a probabilistic model, helps us to explicitly account for the uncertainty surrounding customer relationships. Thus one would be interested to explore the usefulness of Markov Chain Model to model the relationship between an individual customer and a marketing firm. Moreover we would also be able to find the retention rates using concepts like probability of retention. Or say for example rather than talking about average profits from a segment of customers, we would be more interested to discuss about the expected profit from the firm's relationship with a customer or group of customers. It has been noted that Markov Chain Model works well with popular Recency, Frequency, Monetary value (RFM) framework, which direct marketers use to categorize customers and manage customer relationships as well.
If one wants to analyze brand loyalty using the concept of Markov Chain model, then the number of states very naturally becomes equal to \( J \), as we are only interested in studying the long term transition probabilities of a particular person in our study group switching from one brand say the \( i^{th} \) to another one say the \( j^{th} \) in the group of \( J \), where \( i = j = 1, 2, ..., J \) in any combination, i.e., one can move from \( i = 4 \) to \( j = 9 \), or from \( i = 10 \) to \( j = 10 \), etc. Hence theoretically one may consider that after the survey the transition probability matrix between the \( J \) number of brands may be noted down as given below:

Table 11.1 : Transition probability matrix for the brand loyalty example

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>.</th>
<th>( i^{th} )</th>
<th>.</th>
<th>( j^{th} )</th>
<th>.</th>
<th>( J )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( p_{1,1} )</td>
<td>( p_{1,2} )</td>
<td>.</td>
<td>( p_{1,i} )</td>
<td>.</td>
<td>( p_{1,j} )</td>
<td>.</td>
<td>( p_{1,J} )</td>
</tr>
<tr>
<td>2</td>
<td>( p_{2,1} )</td>
<td>( p_{2,2} )</td>
<td>.</td>
<td>( p_{2,i} )</td>
<td>.</td>
<td>( p_{2,j} )</td>
<td>.</td>
<td>( p_{2,J} )</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>( i^{th} )</td>
<td>( p_{i,1} )</td>
<td>( p_{i,2} )</td>
<td>.</td>
<td>( p_{i,i} )</td>
<td>.</td>
<td>( j )</td>
<td>.</td>
<td>( p_{i,J} )</td>
</tr>
<tr>
<td>( j^{th} )</td>
<td>( p_{j,1} )</td>
<td>( p_{j,2} )</td>
<td>.</td>
<td>( p_{j,i} )</td>
<td>.</td>
<td>( p_{j,j} )</td>
<td>.</td>
<td>( p_{j,J} )</td>
</tr>
<tr>
<td>( J )</td>
<td>( p_{J,1} )</td>
<td>( p_{J,2} )</td>
<td>.</td>
<td>( p_{J,i} )</td>
<td>.</td>
<td>( p_{J,j} )</td>
<td>.</td>
<td>( p_{J,J} )</td>
</tr>
</tbody>
</table>

Now if we denote this matrix as \( A \), our aim is to study \( A \times A \times \ldots \) in the long run such that as a brand manager of a particular product in a company can analyze the trend of switching between brands of a group of customer in order to draw some meaningful conclusion in the long run.

In order to find \( A \), things like income effect, age, quality, availability, image, etc., can be brought in to the study which throw some light on the how these variables have a positive/negative impact on brand switching and how one can find the respective probability as given above. The way to analyze that can be through the questionnaire made which would give us a good picture about how to modify and quantify the probabilities of transition between brands in the short term, which can be further analyzed to find the long term probabilities such that brand loyalty as an objective value can be studied more intensely and in detail.
As mentioned above it is true that **Markov Chain Method** may be used quite successful in forecasting/predicting brand switching, yet it has its own drawbacks and limitation, which are:

1. Customers do not always buy products in certain intervals and they do not always buy the same amount of a certain product each time. In practical terms it means that in the future, two or more brands may be bought at the same time.
2. Customers always enter and leave markets, which imply that markets are never stable.
3. The transition probabilities of a customer switching from the \(i^{th}\) brand to the \(j^{th}\) brand are not constant for all customers. Moreover these probabilities themselves may change from customer to customer and from time to time. Thus the transitional probabilities may change according to the average time taken between two purchases.
4. The time between different purchases may be a function of the last brand bought.
5. It is best not to consider other marketing environment such as sales promotions, advertising, competition etc., as it becomes very cumbersome to handle the combined effect of these variables in our **Markov Chain model**.

Consider we want to analyze the brand loyalty for cosmetics used by females in urban India who are in the age group of 20 to 40 years. In order to do this study, assume we conduct a marketing survey with a set of questionnaire which has say \(K\) number of questions. Also consider the number of consumers who are asked to answer these questionnaires is \(N\) in number. Finally the number of brands of different cosmetics is \(J\). Also assume that the questions which are used to give us some information about the behaviour about the consumer is based on a Likert scale of 1 to 5, where the number 1 denotes strongly disagree while 5 implies strongly agree. The questions one can ask in the questionnaire may pertain to income, age, present brand of cosmetics used, preference and liking between the \(J\) number of brands of cosmetics, etc.
Application of renewal theory in marketing

Example 11.1

Consider a consumer buys or utilizes the service of the same manufacturer originated product or service repeatedly over time rather than buying from multiple suppliers. A consumer who shows a penchant of buying the same product or service is said to possess brand loyalty for that particular product or service. This means the consumer consistently purchases the same brand within a product class. Thus brand loyalty, which includes some degree of pre-dispositional commitment towards a brand, may be perceived as being affected by multidimensional/multivariate multi factors. Some of the known factors are customers' perceived value, brand trust, customers’ satisfaction, repeat purchase behavior, commitment level, etc. Hence a marketing or advertising firm, which enhances customer loyalty through brand loyalty, can ensure strong positive effects on the profitability of a brand.

It is quite interesting to study how one can quantify brand loyalty and also understand how long term forecasting of market share can be undertaken utilizing some simple concepts of stochastic processes. Concepts like Markov Chain are used to study brand loyalties in marketing and in this example we discuss such a case, though theoretically which is mainly due to lack of any marketing data.
Module 11: Application of stochastic processes in areas marketing
Lecture 37: Application of renewal theory in Marketing with examples

The Lecture Contains:

- Application of renewal theory in marketing
- Examples