4.9.2 Sequence Networks of a loaded Synchronous Generator:

A three-phase synchronous generator, having a synchronous impedance of $\bar{Z}_s$ per phase, with its neutral grounded through a impedance $\bar{Z}_n$ is shown in Fig.4.44. The generator is supplying a balanced three phase load. The generator voltages $\bar{E}_a$, $\bar{E}_b$ and $\bar{E}_c$ are balanced and hence treated as positive sequence set of voltage phasors and can be expressed as:

$$\begin{bmatrix} \bar{E}_{abc} \\ \end{bmatrix} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} [\bar{E}_a]$$ \hspace{1cm} (4.87)

As the generator is supplying a three-phase balanced load, the following KVL equations can be written for each phase:

$$\bar{V}_a = \bar{E}_a - \bar{Z}_s \bar{I}_a - \bar{Z}_n \bar{I}_n$$

$$\bar{V}_b = \bar{E}_b - \bar{Z}_s \bar{I}_b - \bar{Z}_n \bar{I}_n$$ \hspace{1cm} (4.88)

$$\bar{V}_c = \bar{E}_c - \bar{Z}_s \bar{I}_c - \bar{Z}_n \bar{I}_n$$
Substituting the neutral current $\bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c$ in equation (4.88), and writing the resulting equation in matrix form, we get:

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \end{bmatrix} - \begin{bmatrix} \bar{Z}_s + \bar{Z}_n & \bar{Z}_n & \bar{Z}_n \\ \bar{Z}_n & \bar{Z}_s + \bar{Z}_n & \bar{Z}_n \\ \bar{Z}_n & \bar{Z}_n & \bar{Z}_s + \bar{Z}_n \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

(4.89)

The above matrix equation can be expressed in a compact form as:

$$\begin{bmatrix} \bar{V} \end{bmatrix}_{abc} = \begin{bmatrix} \bar{E} \end{bmatrix}_{abc} - \begin{bmatrix} \bar{Z} \end{bmatrix}_{abc} \begin{bmatrix} \bar{I} \end{bmatrix}_{abc}$$

(4.90)

where,

$\begin{bmatrix} \bar{V} \end{bmatrix}_{abc} = [\bar{V}_a \bar{V}_b \bar{V}_c]^T$ is the vector of terminal phase voltages.

$\begin{bmatrix} \bar{I} \end{bmatrix}_{abc} = [\bar{I}_a \bar{I}_b \bar{I}_c]^T$ is the vector of terminal phase currents.

$\begin{bmatrix} \bar{Z} \end{bmatrix}_{abc}$ is the impedance matrix which can be easily identified from equation (4.89).

Replacing the phase quantities of equation (4.90) by corresponding sequence quantities, using the transformation equation (4.83) and equation (4.85) one can write:

$$\begin{bmatrix} \bar{A} \end{bmatrix}[\bar{V}]_{012} = \begin{bmatrix} \bar{A} \end{bmatrix}[\bar{E}]_{012} - \begin{bmatrix} \bar{Z} \end{bmatrix}_{abc}[\bar{A}] [\bar{I}]_{012}$$

(4.91)

Premultiplying both sides of the equation (4.91) by $[\bar{A}]^{-1}$ and after simplifications one gets:

$$\begin{bmatrix} \bar{V} \end{bmatrix}_{012} = [\bar{E}]_{012} - [\bar{Z}]_{012} \begin{bmatrix} \bar{I} \end{bmatrix}_{012}$$

(4.92)

where, $[\bar{Z}]_{012}$ is Generator Sequence Impedance Matrix and is defined as:

$$[\bar{Z}]_{012} = [\bar{A}]^{-1} [\bar{Z}]_{abc} [\bar{A}] = \begin{bmatrix} \bar{Z}_s + 3\bar{Z}_n & 0 & 0 \\ 0 & \bar{Z}_s & 0 \\ 0 & 0 & \bar{Z}_s \end{bmatrix}$$

$[\bar{E}]_{012}$ is the generated sequence voltage vector and is defined as $[\bar{E}]_{012}$ since the generated voltages are always balanced and contain only the positive sequence component.

Substituting $[\bar{E}]_{012}$ and $[\bar{Z}]_{012}$ in equation (4.92) we get:

$$\begin{bmatrix} \bar{V}_{a0} \\ \bar{V}_{a1} \\ \bar{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{E}_a \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & \bar{Z}_0 & 0 \\ 0 & \bar{Z}_1 & \bar{Z}_2 \\ 0 & 0 & \bar{Z}_2 \end{bmatrix} \begin{bmatrix} \bar{I}_{a0} \\ \bar{I}_{a1} \\ \bar{I}_{a2} \end{bmatrix}$$

(4.93)

where, $\bar{Z}_1 = \bar{Z}_2$ is the positive sequence generator impedance, $\bar{Z}_2 = \bar{Z}_2$ is the negative sequence gener-
ator impedance and $Z_0 = Z_a + 3Z_n$ is the zero sequence generator impedance. Expanding the above equation, one can write separate equation for each of the sequence components as:

$$
\begin{align*}
\bar{V}_{a0} &= -\bar{Z}_0 \bar{I}_{a0} \\
\bar{V}_{a1} &= \bar{E}_a - \bar{Z}_1 \bar{I}_{a1} \\
\bar{V}_{a2} &= -\bar{Z}_2 \bar{I}_{a2}
\end{align*}
$$

From equation (4.94), it is evident that the three sequence components are independent of each other. The current of a particular sequence produces a voltage drop of that sequence only, hence the three sequences are decoupled from each other. The three sequence networks of a synchronous generator are shown in Fig. 4.45.

![Sequence networks of a synchronous generator](image)

Figure 4.45: The sequence networks of a synchronous generator

### 4.9.3 Sequence networks of a transmission line:

For a static device such as a transmission line, the phase sequence of voltages and currents have no effect on the impedance offered by the line as both positive and negative phase sequences encounter identical line geometry. Hence, the positive and negative sequence impedances offered by a line are identical i.e. $Z_1 = Z_2$.

The zero sequence currents, however, are in phase and flow through the conductors and return through grounded neutral and/or ground wires. As a result, the ground or ground wire are to be included in the path of the zero sequence currents. The zero sequence impedance $Z_0$ is, therefore, different from $Z_1$ and $Z_2$ due to the inclusion of the ground return path. $Z_0$ is usually more than three times of $Z_1$ or $Z_2$. The three sequence networks of the transmission lines are shown in Fig. 4.46.

![Sequence networks of a transmission line](image)

Figure 4.46: The sequence networks of a transmission line

### 4.9.4 Sequence networks of a transformer:

For short circuit studies, the shunt magnetizing branch of transformer is neglected as the current through it is negligible as compared to short circuit current. The transformer is, therefore, modelled with an equivalent series leakage impedance. Since the transformer is also a static device like a
transmission line, the series leakage impedance will not change if the phase sequence of applied voltage is reversed. Therefore, the positive and negative sequence impedances offered by a transformer are equal. The zero sequence current flows through a transformer if paths for it to flow exist on the primary as well the secondary sides. For such transformers the zero sequence impedance is equal to the leakage impedance, as a consequence \( \bar{Z}_0 = \bar{Z}_1 = \bar{Z}_2 \). The positive and negative sequence networks of a transformer are identical to the positive and negative sequence networks of a transmission line as shown in Fig. 4.46 (a) and (b). The sequence networks for zero sequence depends on the winding connections and whether or not the neutrals are grounded. To derive these circuits for different transformer connections, one has to keep in mind that an open circuit will exist on the primary (secondary) side if there is no ground return for primary (secondary) currents or if there is no corresponding path for secondary (primary) zero-sequence currents. The different three-phase transformer connections and their equivalent zero-sequence networks are discussed next. It is assumed that the neutrals, if grounded, are solidly grounded.

**Figure 4.46:** The sequence networks of a transmission line

\[
\bar{Z}_1 \quad \bar{I}_{a1} \quad \bar{Z}_2 \quad \bar{I}_{a2} \quad \bar{Z}_0 \quad \bar{I}_{a0}
\]

**(a) Positive Sequence**  
**(b) Negative sequence**  
**(c) Zero sequence**

**Figure 4.47:** The zero-sequence equivalent circuit of a Star-Star transformer with both neutrals grounded

(a) **Star-Star connections with both neutrals grounded:** Since both the neutrals are grounded, the phasor sum of three unbalanced phase currents is equal to three times the zero sequence current \( \bar{I}_{a0} \) (equation (4.82)). Hence, the zero sequence currents can flow in the primary and secondary
windings and the transhomer, therefore, can be represented by the equivalent zero-sequence leakage impedance. The equivalent circuit is shown in Fig. 4.47.

(b) **Star-Star connections with only one neutral grounded:** When the neutral of only one winding is grounded, the phase currents of the ungrounded winding must add up to zero. This implies that the zero sequence currents can not exist in the ungrounded winding and hence the zero sequence currents can not exist even in the transformer side with neutral grounded. The transformer in this case, is represented as an open circuit between primary and secondary windings and the equivalent circuit is shown in Fig. 4.48.

(c) **Star-Star connections with only no neutral grounded:** In this case also the phasor sum of the phase currents of both the windings is zero and hence the zero sequence currents can not exist on any winding in this case also. The zero sequence equivalent network is represented as an open circuit between the two windings and is shown in Fig. 4.49.
(d) **Star-delta connections with neutral grounded:** A zero sequence current on the grounded star-winding will cause a circulating zero-sequence current in the closed delta-winding. However, the zero-sequence current on the delta-winding can not exist on line side of the winding and is confined only to the closed delta-winding. As a result, an open circuit exists between the star and the delta sides. But, as the zero-sequence currents can exist on the line-side of the grounded star winding, the zero-sequence leakage impedance of the transformer is connected to ground on the star side of the transformer and an open circuit exists between the two windings. The equivalent circuit for this connection is shown in Fig. 4.50.

![Equivalents circuit](image)

**Figure 4.50:** The zero-sequence equivalent circuit of a Star-Delta transformer with Star-side neutral grounded

(e) **Star-delta connections with ungrounded neutral grounded:** Since the neutral is isolated, no zero-sequence current can exist in the star side of the transformer and as a consequence zero-sequence currents can not exist in the delta side. The transformer is, therefore, represented as an open-circuit and the equivalent circuit is shown in Fig. 4.51.

![Equivalents circuit](image)

**Figure 4.51:** The zero-sequence equivalent circuit of a Star-Delta transformer with Star-side neutral ungrounded

(f) **Delta-delta connections with ungrounded neutral grounded:** In this case, the zero-sequence currents can only circulate within the closed delta windings and can not exit on line sides of both
the windings. Hence, an open circuit exists between the two windings as far as zero-sequence currents are concerned. To permit the circulating zero-sequence current to exist, the zero-sequence leakage impedance is represented as a closed path with the ground. The equivalent circuit is shown in Fig. 4.52.

![Diagram](a) Symbol (b) Connection Diagram (c) Equivalent circuit

**Figure 4.52:** The zero-sequence equivalent circuit of a Delta-Delta transformer

![Diagram](a) Symbol (b) Connection Diagram (c) Equivalent circuit

**Figure 4.53:** The zero-sequence equivalent circuit of a Star-Delta transformer with neutral grounded through impedance

**Point to remember:** If the neutral of a transformer is grounded through a grounding impedance $\bar{Z}_n$, as shown in Fig. 4.53, then, the total zero-sequence equivalent impedance to be used in the equivalent circuit is

$$
\bar{Z}_{0\text{total}} = \bar{Z}_0 + 3\bar{Z}_n
$$

This is due to the fact that the neutral current is 3 times the zero-sequence current per phase.

Next, the concepts of unsymmetrical fault analysis are developed with help of Thevenin’s equivalent circuit of sequence networks and symmetrical components in the next lecture.