4.4 Example of $[\bar{Z}_{\text{Bus}}]$ matrix formulation in the presence of mutual impedances

Consider the network shown in Fig. 4.32.

A tree for the network is shown in Fig. 4.32. The system data is given in Table 4.1.

![Figure 4.32: The power system for $[\bar{Z}_{\text{Bus}}]$ example](image)

![Figure 4.33: Tree of the network](image)

Table 4.1: System data

<table>
<thead>
<tr>
<th>Element no.</th>
<th>Self Bus code</th>
<th>Bus code Impedance $\bar{z}_{pq,pq} (p.u.)$</th>
<th>Mutual Bus code</th>
<th>Impedance $\bar{z}_{pq,rs} (p.u.)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 1(1)</td>
<td>0.4</td>
<td>0 - 1(2)</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0 - 1(2)</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0 - 2</td>
<td>0.5</td>
<td>0 - 1(1)</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>2 - 3</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1 - 3</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Step 1: The algorithm starts building the $[\bar{Z}_{Bus}]$ matrix element by element. To initiate the process, start with element 1 connected between nodes $p = 0$ and $q = 1$, shown in Fig. 4.34. The $[\bar{Z}_{Bus}]$ matrix of the partial network is given as,

$$[ar{Z}_{Bus}] = (1) \begin{bmatrix} j0.4 \end{bmatrix}$$

Step 2: Next add element 2 connected between $p = 0$ and $q = 1$ which is mutually coupled to the existing element 1, connected between $\rho = 0$ and $\sigma = 1$. This new element is a link as it does not create a new node, the partial network for this step is shown in Fig. 4.35. The augmented $[\bar{Z}_{Bus}^{(temp)}]$ matrix after the addition of this element, is given by

$$[\bar{Z}_{Bus}^{(temp)}] = (1) \begin{bmatrix} j0.4 & \bar{Z}_{1\ell} \\ \bar{Z}_{\ell1} & \bar{Z}_{\ell\ell} \end{bmatrix}$$

$$\bar{Z}_{\ell1} = \bar{Z}_{01} - \bar{Z}_{11} + \frac{\bar{y}_{0-1(2),0-1(1)}(\bar{Z}_{01} - \bar{Z}_{11})}{\bar{y}_{0-1(2),0-1(2)}}$$

$$\bar{Z}_{\ell\ell} = \bar{Z}_{0\ell} - \bar{Z}_{1\ell} + \frac{1 + \bar{y}_{0-1(2),0-1(1)}(\bar{Z}_{0\ell} - \bar{Z}_{1\ell})}{\bar{y}_{0-1(2),0-1(2)}}$$

where, $\bar{Z}_{01}$ and $\bar{Z}_{0\ell}$ are the elements of $[\bar{Z}_{Bus}]$ matrix associated with the reference node.

The primitive impedance matrix $[\bar{Z}]$ for the partial network is

$$[\bar{Z}] = \begin{bmatrix} 0-1(1) & 0-1(2) \\ 0-1(2) & 0-1(1) \end{bmatrix} \begin{bmatrix} j0.4 & j0.2 \\ j0.2 & j0.5 \end{bmatrix}$$

The primitive admittance matrix $[\bar{Y}]$ for the partial network in nothing but the inverse of primitive
impedance matrix $[\bar{Z}]$ and is given by

$$[\bar{Y}] = [\bar{Z}]^{-1} = \begin{bmatrix} 0-1(1) & 0-1(2) \\ 0-1(2) & \end{bmatrix} = \begin{bmatrix} -j3.125 & j1.25 \\ j1.25 & -j2.5 \end{bmatrix}$$

With $\bar{Z}_{01} = 0$, since 0 is the reference node, $\bar{Z}_{\ell 1}$ is evaluated as

$$\bar{Z}_{\ell 1} = -j0.4 + \frac{j1.25(-j0.4)}{-j2.5} = -j0.2 = \bar{Z}_{1\ell}$$

Also as $\bar{Z}_{0\ell} = 0$, since 0 is the reference node, and hence, $\bar{Z}_{\ell\ell}$ is calculated as

$$\bar{Z}_{\ell\ell} = j0.2 + \frac{1 + j1.25(j0.2)}{-j2.5} = j0.50$$

The augmented $\bar{Z}^{(\text{temp})}_{\text{bus}}$ matrix is given as

$$\bar{Z}^{(\text{temp})}_{\text{bus}} = \begin{bmatrix} (1) & (\ell) \\ (\ell) & \end{bmatrix} = \begin{bmatrix} j0.4 & -j0.2 \\ -j0.2 & j0.5 \end{bmatrix}$$

The row and column corresponding to the $\ell$th row and column corresponding to a link addition, (shown in red in the above matrix), need to be eliminated as the link addition does not create a new node. The $[\bar{Z}_{\text{bus}}]$ matrix, after the addition of second element to the partial network, is calculated using the following expression

$$[\bar{Z}_{\text{bus}}] = [\bar{Z}_{\text{bus}}] - \frac{\bar{Z}_{1\ell} \bar{Z}_{\ell 1}}{\bar{Z}_{\ell\ell}}$$

$$= j0.4 - \frac{(-j0.2)(-j0.2)}{j0.5}$$

$$\bar{Z}_{\text{bus}} = (1) \begin{bmatrix} j0.32 \end{bmatrix}$$

Note that the size of $\bar{Z}_{\text{bus}}$ matrix is still $(1 \times 1)$ as no new node has been added to the partial network as yet.

**Step 3:** Next add element 3, which is connected between the nodes $p = 0$ and $q = 2$. This is a branch addition as a new node, node 2 is created. This element is mutually coupled to the existing element 1. Hence, the primitive $[\bar{Z}]$ matrix of the partial network, shown in Fig. 4.36, is
The primitive $\bar{y}$ matrix is calculated as $\bar{z}^{-1}$ and is equal to

$$
\bar{y} = \begin{bmatrix}
0-1(1) & 0-1(2) & 0-2 \\
0-1(1) & j0.4 & j0.2 & j0.1 \\
0-1(2) & j0.2 & j0.5 & 0 \\
0-2 & j0.1 & 0 & j0.5
\end{bmatrix}
$$

The modified $\bar{Z}_{Bus}$ matrix is expressed as

$$
\bar{Z}_{Bus} = \begin{bmatrix}
(1) & (2) \\
(1) & j0.32 & \bar{Z}_{12} \\
(2) & \bar{Z}_{21} & \bar{Z}_{22}
\end{bmatrix}
$$

For this element $p = 0$ and $q = 2$ and the set of elements $[\bar{\rho} \bar{\sigma}]$ mutually coupled to this element is $[0 -1(1) 0 -1(2)]$

$$
\bar{Z}_{21} = \bar{Z}_{01} + \frac{\bar{y}_{0-2,0-1(1)} \bar{Z}_{02} - \bar{Z}_{11} \bar{y}_{0-2,0-1(2)}}{\bar{y}_{0-2,0}}
$$

$\bar{Z}_{01}$ and $\bar{Z}_{02}$ are the transfer impedances associated with the reference node and are equal to zero.

$$
\bar{Z}_{21} = \frac{[j0.667 - j0.2667] \cdot [j0.32 - j0.32]}{j2.133} = j0.06
$$
Hence, $\bar{Z}_{12} = \bar{Z}_{21} = j0.06$

$$\bar{Z}_{22} = \bar{Z}_{02} + \frac{1 + [\bar{y}_{0-2,0-1(1)} \bar{y}_{0-2,0-1(2)}]}{\bar{y}_{0-2,0-2}} \left[ \bar{Z}_{02} - \bar{Z}_{12} \right]$$

$$\bar{Z}_{21} = \frac{1 + \left[ j0.667 \ - j0.2667 \right] \times \left[ -j0.32 \right]}{j2.133} = j0.48$$

The modified $[\bar{Z}_{\text{Bus}}]$ matrix is

$$[\bar{Z}_{\text{Bus}}] = \begin{bmatrix}
(1) & (2) \\
(1) & j0.32 & j0.06 \\
(2) & j0.06 & j0.48
\end{bmatrix}$$

**Step 4:** On adding element 4 between $p = 2$ and $q = 3$, a new node, node 3 is created. Hence, this is a branch addition and is shown in Fig. 4.37. The modified $[\bar{Z}_{\text{Bus}}]$ matrix can be written as

![Figure 4.37: Partial network in Step 4](image)

As this element is not mutually coupled to other elements the elements of vector $\bar{y}_{pq,\rho\sigma}$ are zero. Hence, the new elements of $[\bar{Z}_{\text{Bus}}]$ matrix can be calculated, using the expression given in (4.41), as:

**Off-diagonal elements**

$$\bar{Z}_{qi} = \bar{Z}_{pi} \quad \forall \ i = 1, 2, 3 \ i \neq q$$
\[ \bar{Z}_{31} = \bar{Z}_{21} = j0.06 \]
\[ \bar{Z}_{32} = \bar{Z}_{22} = j0.48 \]
\[ \bar{Z}_{13} = \bar{Z}_{31} = j0.06 \]
\[ \bar{Z}_{23} = \bar{Z}_{32} = j0.48 \]

**Diagonal element**

Using the expression of (4.48) with no mutual coupling, the diagonal element can be written as:

\[ \bar{Z}_{qq} = \bar{Z}_{pq} + \bar{z}_{pq,pq} \]

hence,

\[ \bar{Z}_{33} = \bar{Z}_{23} + \bar{z}_{23,23} = j0.48 + j0.4 = j0.88 \]

**Step 5:** Finally add element 5 between nodes \( p = 1 \) and \( q = 3 \). This is an addition of a link hence a temporary row and column are added. Fig. 4.38 shows the final network after the addition of this element. The modified \( \bar{Z}_{Bus}^{(temp)} \) matrix can be written as

![Diagram](image-url)

**Figure 4.38:** The complete network after the addition of link in step 5

\[ \bar{Z}_{Bus}^{(temp)} = \begin{bmatrix} (1) & (2) & (3) \\ (1) & j0.32 & j0.06 & j0.06 \\ (2) & j0.06 & j0.48 & j0.48 \\ (3) & j0.06 & j0.48 & j0.88 \end{bmatrix} \]
Since this element is not mutually coupled to other elements, the new elements of $[\bar{Z}_{Bus}^{(temp)}]$ matrix can be calculated, using the expression of (4.55), as:

**Off-diagonal elements**

\[
\bar{Z}_{\ell i} = \bar{Z}_{pi} - \bar{Z}_{qi} \quad \forall \ i = 1, 2, 3
\]

\[
\begin{align*}
\bar{Z}_{1\ell} &= \bar{Z}_{11} - \bar{Z}_{13} = j0.32 - j0.06 = j0.26 = \bar{Z}_{\ell 1} \\
\bar{Z}_{2\ell} &= \bar{Z}_{21} - \bar{Z}_{23} = j0.06 - j0.48 = -j0.42 = \bar{Z}_{\ell 2} \\
\bar{Z}_{3\ell} &= \bar{Z}_{31} - \bar{Z}_{33} = j0.06 - j0.88 = -j0.82 = \bar{Z}_{\ell 3}
\end{align*}
\]

**Diagonal element**

For calculating the diagonal element, the expression given in (4.59) is used. Hence,

\[
\bar{Z}_{\ell\ell} = \bar{Z}_{p\ell} - \bar{Z}_{q\ell} + \bar{z}_{pq,pq}
\]

\[
\bar{Z}_{\ell\ell} = \bar{Z}_{1\ell} - \bar{Z}_{3\ell} + \bar{z}_{13,13} = j0.26 + j0.82 + j0.6 = j1.68
\]

Hence, the temporary $[\bar{Z}_{Bus}^{(temp)}]$ matrix can be written as

\[
[\bar{Z}_{Bus}^{(temp)}] = \begin{bmatrix}
(1) & (2) & (3) & (\ell) \\
j0.32 & j0.06 & j0.06 & j0.26 \\
j0.06 & j0.48 & j0.06 & -j0.42 \\
j0.06 & j0.48 & j0.88 & -j0.82 \\
j0.26 & -j0.42 & -j0.82 & j1.68
\end{bmatrix}
\]

The $\ell^{th}$ row and $\ell^{th}$ column are to be eliminated to restore the size of $\bar{Z}_{Bus}$ to $3 \times 3$. The elimination is done using the relation

\[
\bar{Z}_{Bus} = \bar{Z}_{Bus}^{(temp)} - \frac{\Delta \bar{Z} \ast \Delta \bar{Z}^T}{\bar{Z}_{\ell\ell}}
\]

\[
\Delta \bar{Z}^T = \begin{bmatrix} j0.26 & -j0.42 & -j0.82 \end{bmatrix}
\]

\[
\Delta \bar{Z}_{Bus} = \begin{bmatrix} j0.32 & j0.06 & j0.06 \\
j0.06 & j0.48 & j0.48 \\
j0.06 & j0.48 & j0.88 \end{bmatrix} - \frac{\begin{bmatrix} j0.23 \\
-j0.42 \\
-j0.82 \end{bmatrix} \ast \begin{bmatrix} -j0.26 & -j0.42 & -j0.82 \end{bmatrix}}{j1.68}
\]

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Hence, the final matrix $[\bar{Z}_{Bus}]$ is

$$
[\bar{Z}_{Bus}] = \begin{bmatrix}
(1) & (2) & (3) \\
0.2798 & 0.1250 & 0.1869 \\
0.1250 & 0.3750 & 0.2750 \\
0.1869 & 0.2750 & 0.4798 \\
\end{bmatrix}
$$

After this discussion of formulation of $[\bar{Z}_{Bus}]$ matrix, we are now ready to discuss fault analysis, which we will start from the next lecture.