Chapter 7
Transient Stability Analysis

Transient stability is the ability of the system to stay in synchronism when subjected to a large disturbance. Due to large disturbances like faults, large load variations, generator outages etc there will be large swings in the electrical power outputs, rotor angles, rotor speeds and bus voltages of the generators in the system. Due to this reason a power system cannot be linearized, like in the case of small signal stability analysis. The system cannot be linearized for large disturbances, hence eigen value analysis cannot be used for assessing the stability as the system is nonlinear. One way of assessing the effect of large disturbances on the system stability is through numerical solution of differential algebraic equations (DAE). The time of interest for transient stability assessment is typically 2 to 10 seconds after a disturbance. During this time usually it is assumed that the turbine and turbine governors will not be acting as they are mechanical systems. Hence, for transient stability analysis usually the turbine and speed governor dynamics are not considered and the input mechanical torque $T_M$ is considered constant.

7.1 Numerical Solution of Differential Algebraic Equations (DAE)

The differential algebraic equations (DAE) given in equations (5.38)-(5.52) and (5.59)-(5.62) in Chapter 5 can be solved through numerical methods. There are two methods by which DAE can be solved [1]-[2], they are:

1. Simultaneous method.
2. Partitioned method.

In simultaneous method both the differential and algebraic equations are solved together in each iteration. Whereas, in partitioned method first the differential equations are solved through numerical integration and then the algebraic equations are solved and this process is repeated till the end of the simulation. Simultaneous method is numerically stable that is the error will not accumulate over the iterations leading to blowing off of the error and making the method unstable. Though the method is stable the computational complexity is much more as compared to the partitioned method. The partitioned method is numerically unstable that is if the time
step of integration is not chosen properly the error will accumulate over the iterations leading to numerical instability. But, the partitioned method is computationally simple as compared to the simultaneous method. The time step of integration depends on the system stiffness, that is the ratio of the highest to lowest time constant or the highest to lowest eigen value of the linearized system. The time step of integration in partitioned method should be less than the lowest time constant else it will lead to numerical instability. In case of simultaneous method even if the time step is greater than the lowest time constant numerically it is stable but the dynamics corresponding to that time constant are not visible in the solution.

7.2 Simultaneous (Implicit) Method

The differential equation given in (5.38) to (5.50) Chapter 5 can be expressed as

\[
\dot{x} = f(x, I_{d-q}, \overline{V}, u)
\]  

(7.1)

Where,

\[
x = \begin{bmatrix} x_1^T \cdots x_{n_g}^T \end{bmatrix}^T, \quad u = \begin{bmatrix} u_1^T \cdots u_{n_g}^T \end{bmatrix}^T, \quad I_{d-q} = \begin{bmatrix} I_{d-q_1}^T \cdots I_{d-q_n}^T \end{bmatrix}^T
\]

\[
x_i = \begin{bmatrix} \delta_i, \omega_i, \psi_{1i}, \psi_{2i}, E_{qi}, E_{di}, E_{qi}, V_R, R_f \end{bmatrix}^T, \quad i = 1, \ldots, n_g
\]

\[
u_i = \begin{bmatrix} V_{ref}, T_{Me} \end{bmatrix}^T
\]

\[
I_{d-q_i} = \begin{bmatrix} I_{di}, I_{qi} \end{bmatrix}^T
\]

\[
\overline{V} = \begin{bmatrix} \overline{V}_1 \cdots \overline{V}_{n} \end{bmatrix}^T = \begin{bmatrix} V_1 e^{i\theta_1} \cdots V_{n} e^{i\theta_n} \end{bmatrix}^T
\]

If saliency is neglected that is \( X_q^* = X_q^\prime \) then the generator current can be written as

\[
I_{di} + jI_{qi} = \left( \frac{1}{R_n + jX_n} \right) \left( (E_{di} + jE_{qi}) e^{j(\delta_i + \frac{\pi}{2})} - V_i e^{i\theta_i} \right), \quad i = 1, 2, \ldots, n_g
\]

(7.3)

or,
Similarly, the algebraic equations can be written as

\[
P_i(\delta_i, I_{d,i}, I_{q,i}, V_i, \theta_i) - \sum_{j=1}^{n} V_j V_j Y_{ij} \cos(\theta_i - \theta_j - \phi_{ij}) = 0 \quad \text{for} \quad i = 1, \ldots, n_g
\]

\[
Q_i(\delta_i, I_{d,i}, I_{q,i}, V_i, \theta_i) - \sum_{j=1}^{n} V_j V_j Y_{ij} \sin(\theta_i - \theta_j - \phi_{ij}) = 0
\]

\[
P_{Li}(V_i) - \sum_{j=1}^{n} V_j V_j Y_{ij} \cos(\theta_i - \theta_j - \phi_{ij}) = 0 \quad \text{for} \quad i = n_g + 1, \ldots, n
\]

or can be expressed as a nonlinear function as shown in (7.6)

\[
g(x, I_{d-q}, \overline{V}) = 0
\]

It has to be noted that solving equation (7.6) is nothing but solving load flow problem. The complete set of DAE can be expressed as

\[
\dot{x} = f(x, I_{d-q}, \overline{V}, u)
\]

\[
I_{d-q} = h(x, \overline{V})
\]

\[
0 = g(x, I_{d-q}, \overline{V})
\]

Equation (7.1) can be numerically integrated through trapezoidal method as

\[
x_{n+1} = x_n + \int_{t_n}^{t_{n+1}} f(x, I_{d-q}, \overline{V}, u) dt
\]

or

\[
x_{n+1} = x_n + \frac{\Delta t}{2} \left( f(x_{n+1}, I_{d-q_{n+1}}, \overline{V}_{n+1}, u_{n+1}) + f(x_n, I_{d-q_{n}}, \overline{V}_{n}, u_{n}) \right)
\]

Where, \( \Delta t = t_{n+1} - t_n \). The \( n+1 \) instant algebraic equations can be written as
\[ I_{d-q_n+1} - h(x_{n+1}, \bar{V}_{n+1}) = 0 \]  
(7.10)

\[ g(x_{n+1}, I_{d-q_n+1}, \bar{V}_{n+1}) = 0 \]  
(7.11)

Equations (7.9)-(7.11) can be rearranged and represented as

\[
\begin{align*}
\{x_{n+1} - \frac{\Delta t}{2} (f(x_{n+1}, I_{d-q_n+1}, \bar{V}_{n+1}, u_{n+1})) \} - \{x_n + \frac{\Delta t}{2} (f(x_n, I_d-q_n, \bar{V}_n, u_n))\} = 0 \\
I_{d-q_{n+1}} - h(x_{n+1}, \bar{V}_{n+1}) = 0 \\
g(x_{n+1}, \bar{V}_{n+1}, I_{d-q_{n+1}}) = 0
\end{align*}
\]  
(7.12)

Let, equation (7.12) be represented, in terms of three nonlinear functions \( F_1, F_2, F_3 \), as

\[
\begin{align*}
F_1(x_{n+1}, I_{d-q_{n+1}}, \bar{V}_{n+1}, u_{n+1}, x_n, I_{d-q_n}, \bar{V}_n, u_n) &= 0 \\
F_2(x_{n+1}, \bar{V}_{n+1}, I_{d-q_{n+1}}) &= 0 \\
F_3(x_{n+1}, \bar{V}_{n+1}, I_{d-q_{n+1}}) &= 0
\end{align*}
\]  
(7.13)

Equation (7.13) can be solved by Newton-Raphson method where the variables are \( x_{n+1}, I_{d-q_{n+1}}, \bar{V}_{n+1}, u_{n+1} \) with initial conditions \( x_n, I_{d-q_n}, \bar{V}_n, u_n \). Since the variables at \( n^{th} \) integration step are not explicitly required for finding the solution of equation (7.13) this method is also called as simultaneous implicit method. The Jacobian of the Newton-Raphson method at \( k^{th} \) iteration at \( n^{th} \) integration step is given in equation (7.14) from which the next iteration values of the variables can be found as given in equation (7.15)-(7.16).

\[
\begin{pmatrix}
J_1 & J_2 & J_3 \\
J_4 & J_5 & J_6 \\
J_7 & J_8 & J_9
\end{pmatrix}
^{(K)}
\begin{Bmatrix}
\Delta x_{n+1} \\
\Delta I_{d-q_{n+1}} \\
\Delta \bar{V}_{n+1}
\end{Bmatrix}
^{(K)}
= \begin{Bmatrix}
F_1 \\
F_2 \\
F_3
\end{Bmatrix}^{(K)}
\]  
(7.14)
\[
\begin{pmatrix}
\Delta x_{n+1} \\
\Delta I_{d-q_{n+1}} \\
\Delta V_{n+1}
\end{pmatrix}^{(K)} = (-J^{(K)})^{-1} \begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix}^{(K)}
\]

(7.15)

\[
\begin{pmatrix}
x_{n+1} \\
I_{d-q_{n+1}} \\
V_{n+1}
\end{pmatrix}^{(K+1)} = \begin{pmatrix}
x_{n+1} \\
I_{d-q_{n+1}} \\
V_{n+1}
\end{pmatrix}^{(K)} + \begin{pmatrix}
\Delta x_{n+1} \\
\Delta I_{d-q_{n+1}} \\
\Delta V_{n+1}
\end{pmatrix}^{(K)}
\]

(7.16)

At each \(n^{th}\) integration step the Newton-Raphson method is applied to solve equation (7.13). Equations (7.14) to (7.16) are iteratively repeated at each \(n^{th}\) integration step till

\[
\begin{bmatrix}
\Delta x_{n+1} \\
\Delta I_{d-q_{n+1}} \\
\Delta V_{n+1}
\end{bmatrix}^{(K)} \leq \varepsilon
\]

(7.17)

Where, \(\varepsilon\) is the error at \(k^{th}\) Newton-Raphson iteration in \(n^{th}\) integration step and depends on the accuracy required. In very dishonest Newton-Raphson method the Jacobian is kept constant for several iteration and also for several time steps. Except at the time of system disturbance Jacobian is considered to be constant.

### 7.3 Partitioned (Explicit) Method

In partitioned explicit method first the differential equations are numerically integrated at \(n^{th}\) integration step and then the algebraic equations are solved. The numerical integration can be done by different methods. Here, we will discuss Euler, modified Euler and Runga-Kutta method of numerical integration.
7.3.1 Euler’s Method

In Euler’s method [3] the integration of equation (7.1) at $n^{th}$ integration step is given as

$$x_{n+1} = x_n + \frac{\Delta t}{2} f(x_n, I_{d-q}, \overline{V}_n, u_n)$$ \hspace{1cm} (7.18)

Where, $\Delta t$ integration time step and should be less than the least time constant of the system in order to have a stable numerical method. Once, the variables $x_{n+1}$ at $n^{th}$ integration step are found then the algebraic equations can be solved by solving equations

$$I_{d-q_{n+1}} - h(x_{n+1}, \overline{V}_{n+1}) = 0$$ \hspace{1cm} (7.19)

$$g(x_{n+1}, I_{d-q_{n+1}}, \overline{V}_{n+1}) = 0$$ \hspace{1cm} (7.20)

Equations (7.19) and (7.20) can be solved using Newton-Raphson method. In the special case of impedance load the power balance equations can be avoided and the linear set of algebraic equations in (7.19) needs to be solved as given in equations (7.21) to (7.23)

$$\begin{bmatrix} I_{G_{n+1}} \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{II} & Y_{IN} \\ Y_{IN}^T & Y_{BUS} \end{bmatrix} \begin{bmatrix} E_{n+1}^* \\ V_{n+1} \end{bmatrix}$$ \hspace{1cm} (7.21)

Where, $n_g$ is the number of generators, $n_b$ is the total number of buses, the admittance matrix is as defined in Chapter 5 equation (5.69) and
\[ I_{Gn+1} = \begin{bmatrix} T_{G1n+1} \\ \vdots \\ T_{Gn,n+1} \end{bmatrix}_{n \times n}, \quad E''_{n+1} = \begin{bmatrix} E''_{1n+1} \\ \vdots \\ E''_{n,n+1} \end{bmatrix}_{n \times n}, \quad V_{Nn+1} = \begin{bmatrix} V_{1n+1} \\ \vdots \\ V_{n,n+1} \end{bmatrix} \]

Since at the \( n^{th} \) integration step the variables \( x_{n+1} \) are known, from equations (7.18), and the vector of internal generator voltages \( E''_{n+1} \) is known hence

\[ I_{Gn+1} = \begin{bmatrix} Y_{II} - Y_{IN} \left[ Y_{BUS} \right]^{-1} Y_{IN}^T \end{bmatrix} E''_{n+1} \]  
(7.22)

\[ V_{n} = -\left[ Y_{BUS} \right]^{-1} Y_{IN}^T E''_{n+1} \]  
(7.23)

The numerical stability of the method can be improved by using modified Euler’s method.

### 7.3.2 Modified Euler’s Method

In modified Euler’s method [3] first a predictor integration step is used and then a corrector step is used. The predictor integration step is given as

\[ x''_{n+1} = x_n + \frac{\Delta t}{2} f(x_n, I_{d-qn}, \overline{V}_n, u_n) \]  
(7.24)

Where, \( x''_{n+1} \) is the vector of predicted variables. Then algebraic equations are solved either by using equations (7.19)-(7.20) or (7.22)-(7.23) with the predicted variables. The corrector step is given as

\[ x'_{n+1} = x_n + \frac{\Delta t}{2} \left( f(x''_{n+1}, I_{d-qn}^{p}, \overline{V}_{n+1}^{p}, u_{n+1}^{p}) + f(x_n, I_{d-qn}, \overline{V}_n, u_n) \right) \]  
(7.25)
Then algebraic equations are solved either by using equations (7.19)-(7.20) or (7.22)-(7.23) with the corrected variables.

### 7.3.3 Runga-Kutta Fourth Order Method

In Runga-Kutta fourth order method [3] the integration at \( n^{th} \) step is given as

\[
x_{n+1} = x_n + \frac{\Delta t}{6} \left( k_1 + 2k_2 + 2k_3 + k_4 \right)
\]  

(7.26)

Where,

\[
k_1 = f(x_n, I_{d-q_n}, V_n, u_n)  
\]  

(7.27)

\[
x_{n+1}^1 = x_n + \frac{\Delta t}{2} k_1  
\]  

(7.28)

\[
k_2 = f(x_{n+1}^1, I_{d-q_{n+1}}^1, V_{n+1}^1, u_{n+1}^1)  
\]  

(7.29)

\[
x_{n+1}^2 = x_n + \frac{\Delta t}{2} k_2  
\]  

(7.30)

\[
k_3 = f(x_{n+1}^2, I_{d-q_{n+1}}^2, V_{n+1}^2, u_{n+1}^2)  
\]  

(7.31)

\[
x_{n+1}^3 = x_n + k_3  
\]  

(7.32)

\[
k_4 = f(x_{n+1}^3, I_{d-q_{n+1}}^3, V_{n+1}^3, u_{n+1}^3)  
\]  

(7.33)

\( k_1, k_2, k_3, k_4 \) are the slopes at the beginning, two times in the middle and in the end. Euler, modified Euler and Runga-Kutta are also called as explicit methods because the variable \( x_n \) should be known to compute \( x_{n+1} \) whereas as in implicit it is not required.

In case of a three-phase balanced fault at a bus, then the row and column of the system admittance matrix, corresponding to the faulted bus, is removed as the voltage at that bus is zero. The power balance equations at that bus are also removed. This incorporates the effect of a three-phase balanced bus fault. In case the fault at some location on the transmission line then the bus admittance matrix should be changed to include the changed topology with the rest of the procedure remaining same. For an
unbalanced fault this procedure cannot be used. Instead symmetrical component theory can be used as explained in the next section.

### 7.4 Analysis of Unbalanced Faults

In the method of symmetrical components an unsymmetrical set of voltage or current phasors is resolved into symmetrical sets of components [4]-[5]. The unbalanced three-phase system is resolved into three balanced (symmetrical) systems of phasors called the positive, negative and zero sequence components. Let the three-phases of the original system be represented by \(a, b\) and \(c\), and the symmetrical components by 1, 2 and 0. In order to represent these symmetrical components let us define an operator ‘\(a\)’

\[
a = e^{j\theta_0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}
\]  

(7.34)

The operator ‘\(a\)’ has certain properties as given in equation (7.35) to (7.37)

\[
a^2 = e^{j\theta_0}e^{j\theta_0} = e^{j\theta_240}
\]  

(7.35)

\[
a^* = a = e^{-j\theta_0}
\]  

(7.36)

\[
a + a + a^2 = 0
\]  

(7.37)

Each of the three-phase voltages or currents should be expressed as a summation of the positive, negative and zero sequence components. Let \(V_a = V_a \angle 0^\circ\) be the \(a\)-phase voltage and let \(V_{a0}, V_{a1}, V_{a2}\) be the zero, positive and negative sequence components, then

\[
V_a = V_{a0} + V_{a1} + V_{a2}
\]  

(7.38)

Similarly, the other two phase voltages can also be expressed in terms of symmetrical components as

\[
V_b = V_{b0} + V_{b1} + V_{b2}
\]  

(7.39)

\[
V_c = V_{c0} + V_{c1} + V_{c2}
\]  

(7.40)

The positive, negative and zero sequence voltage components are shown in Fig. 7.1.
The positive-sequence voltages have the same phase sequence as the original phasors as shown in Fig. 7.1 (a). They are equal in magnitude and displaced from each other by 120°. The negative-sequence components, equal in magnitude and displaced from each other by 120° in phase, have a phase sequence opposite to that of the original phasors as shown in Fig. 7.1 (b). The zero-sequence components are equal in magnitude and have zero phase displacement as shown in Fig. 7.1 (c). From the definition of symmetrical components and the Fig. 7.1 the following expression can be written

$$\begin{align*}
|\vec{V}_a| &= |\vec{V}_c| = |\vec{V}_{b1}| = |\vec{V}_{a1}| \\
|\vec{V}_{a2}| &= |\vec{V}_{b2}| = |\vec{V}_{c2}| \\
|\vec{V}_{b0}| &= |\vec{V}_{c0}| = |\vec{V}_{a0}| 
\end{align*}$$

(7.41)

(7.42)

(7.43)

The magnitude of voltages of each phase in positive sequence should be same by definition and they should be balanced. Similarly, the voltage magnitudes in negative and zero sequence should be same. Since, the positive sequence components should have same phase sequence as the original phases

$$\begin{align*}
\vec{V}_{b1} &= \vec{V}_{a1}.a^2, & \vec{V}_{c1} &= a\vec{V}_{a1} 
\end{align*}$$

(7.44)

Similarly the negative sequence components should have a phase sequence opposite to that of the original phases hence
\[ \vec{V}_{b2} = a \vec{V}_{a2}, \vec{V}_{c2} = a^2 \vec{V}_{a2} \]  
(7.45)

Equations (7.38) to (7.45) can be expressed in matrix form as

\[
\begin{bmatrix}
\vec{V}_a \\
\vec{V}_b \\
\vec{V}_c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix}
\begin{bmatrix}
\vec{V}_{a0} \\
\vec{V}_{a1} \\
\vec{V}_{a2}
\end{bmatrix}
\]  
(7.46)

or,

\[ \vec{V}_{abc} = T \vec{V}_{012} \]  
(7.47)

Where,

\[ T = \begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix},
T^{-1} = \frac{1}{3}
\begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\]

It can be observed that for a balanced system |\vec{V}_a| = |\vec{V}_{a1}| and \vec{V}_{a2} = \vec{V}_{a0} = 0. The three-phase currents can also be expressed in terms of symmetrical components in a similar way

\[
\begin{bmatrix}
\vec{I}_a \\
\vec{I}_b \\
\vec{I}_c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix}
\begin{bmatrix}
\vec{I}_{a0} \\
\vec{I}_{a1} \\
\vec{I}_{a2}
\end{bmatrix}
\]  
(7.48)

The complex power of the three-phases can be written as

\[ S_{abc} = \vec{V}_{abc}^* \vec{I}_{abc}^* = (T \vec{V}_{012})^* (T \vec{I}_{012})^* 
= \vec{V}_{012}^T \vec{T}^* \vec{T}_{012}^* = 3\vec{V}_{012}^T \vec{T}_{012}^* \]  
(7.49)
It can be observed from (7.49) that the complex power can be computed from symmetrical components as well. Just as the three-phase voltage and currents are expressed in terms of symmetrical components the three-phase impedance can also be expressed in terms of positive negative and zero sequence impedances. Let the self impedance of each phase be \( Z_s \) and the mutual inductance between any two phases be \( Z_m \) then

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
Z_s & Z_m & Z_m \\
Z_m & Z_s & Z_m \\
Z_m & Z_m & Z_s
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

\[
(7.50)
\]

Converting the voltages and currents into the symmetrical components lead to

\[
\begin{bmatrix}
\bar{V}_{a0} \\
\bar{V}_{a1} \\
\bar{V}_{a2}
\end{bmatrix} = \mathbf{T}
\begin{bmatrix}
Z_s & Z_m & Z_m \\
Z_m & Z_s & Z_m \\
Z_m & Z_m & Z_s
\end{bmatrix}
\begin{bmatrix}
\bar{I}_{a0} \\
\bar{I}_{a1} \\
\bar{I}_{a2}
\end{bmatrix}
\]

\[
(7.51)
\]

Hence, the zero sequence impedance is \( Z_0 = Z_s + 2Z_m \) and the positive and negative sequence impedances are \( Z_1 = Z_2 = Z_s - Z_m \). In transformer and transmission lines the positive and negative sequence impedances are same. In case of rotating machines like synchronous machine or induction machines the positive and negative sequence impedances are not same. The zero sequence impedance is present only when there is a path for the zero sequence currents to flow like in a star connected transformer with neutral grounded. In case impedance \( Z_n \) is connected between the neutral and the ground of a transformer then the zero sequence impedance will become \( Z_0 + 3Z_n \). In case of ungrounded neutral zero sequence
currents cannot flow. Hence, network configuration has to be taken into consideration to check the zero sequence current paths.

Now the unsymmetrical faults can be analysed through the symmetrical components defined earlier. In case a fault occurs at a particular location in the system then the rest of the system can be represented as a pre-fault balanced Thevenin’s voltage with a Thevenin’s equivalent impedance in series. Let \( \bar{E}_{abc} \) be the Thevenin’s voltage as seen from the fault location with Thevenin’s impedance \( \bar{Z}_{abc} \). \( \bar{V}_{abc} \), \( \bar{I}_{abc} \) are fault location voltage and current then

\[
\bar{V}_{abc} = \bar{E}_{abc} - \bar{Z}_{abc} \bar{I}_{abc}
\]  

(7.52)

Expressing (7.52) in symmetrical components lead to

\[
\begin{bmatrix}
\bar{V}_{a0} \\
\bar{V}_{a1} \\
\bar{V}_{a2}
\end{bmatrix} = \begin{bmatrix}
0 \\
\bar{E}_a \\
0
\end{bmatrix} - \begin{bmatrix}
\bar{Z}_0 & 0 & 0 \\
0 & \bar{Z}_1 & 0 \\
0 & 0 & \bar{Z}_2
\end{bmatrix} \begin{bmatrix}
\bar{I}_{a0} \\
\bar{I}_{a1} \\
\bar{I}_{a2}
\end{bmatrix}
\]  

(7.53)

Since, the rest of the system is represented as a pre-fault balanced Thevenin’s equivalent voltage source the zero and negative sequence voltages will be zero. This is true for power systems because the synchronous generators will always be producing balanced voltages and hence will only have positive sequence voltages. From (7.53) the fault currents can be computed for different unsymmetrical faults like single line-to-ground, line-to-line and line-to-line-to-ground faults.

**Single line-to-ground fault**

Let fault be on phase-a then

\[
\bar{V}_a = 0, \quad \bar{I}_b = \bar{I}_c = 0
\]  

(7.54)
Hence,

\[
\bar{V}_{a1} + \bar{V}_{a2} + \bar{V}_{a0} = \bar{V}_a = 0 \tag{7.55}
\]

\[
\bar{I}_{a1} = \bar{I}_{a2} = \bar{I}_{a0} = \frac{\bar{I}_a}{3} \tag{7.56}
\]

Substituting the values of the symmetrical voltages components obtained from (7.53) in (7.55) and also substituting the symmetrical current components in (7.56) in to (7.55) lead to

\[
\bar{E}_a - Z_1 \bar{I}_{a1} - Z_2 \bar{I}_{a2} - Z_0 \bar{I}_{a0} = 0
\]

or

\[
\bar{I}_{a1} = \frac{\bar{E}_a}{Z_1 + Z_2 + Z_0} \tag{7.57}
\]

**Line-to-line fault**

Let the fault be in phase-b and phase-c, then

\[
\bar{V}_b = \bar{V}_c, \quad \bar{I}_b = -\bar{I}_c, \quad \bar{I}_a = 0 \tag{7.58}
\]

Hence,

\[
\bar{V}_{a1} = \bar{V}_{a2} \tag{7.59}
\]

\[
V_{ab} = 0 \tag{7.60}
\]

\[
I_{a1} = -I_{a2} \tag{7.61}
\]

\[
I_{a0} = 0 \tag{7.62}
\]

Substituting (7.59) to (7.62) in (7.53) and solving for the current \( \bar{I}_{a1} \) lead to
\[ I_{a1} = \frac{E_a}{Z_1 + Z_2} \quad (7.63) \]

**Double line-to-ground Fault**

Let the fault be in phase-b and phase-c, then

\[ \bar{V}_b = \bar{V}_c = 0, \; I_a = 0 \quad (7.64) \]

Hence,

\[ \bar{V}_{a0} = \bar{V}_{a1} = \bar{V}_{a2} = \frac{1}{3} \bar{V}_a \quad (7.65) \]

\[ I_{a1} + I_{a2} + I_{a0} = 0 \quad (7.66) \]

Substituting equation (7.53) in equation (7.65) lead to

\[ E_a - Z_1 I_{a1} = -Z_2 I_{a2} = -Z_0 I_{a0} \quad (7.67) \]

Eliminating \( I_{a2}, I_{a0} \) from (7.66) using (7.67) leads to

\[ I_{a1} + \frac{E_a - Z_1 I_{a1}}{-Z_2} + \frac{E_a - Z_1 I_{a1}}{-Z_0} = 0 \quad (7.68) \]

Solving (7.68) for \( I_{a1} \) gives

\[ I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0}} \quad (7.69) \]

Equations (7.57), (7.63) and (7.69) it can be observed that the power system during unbalanced fault can still be represented as a positive sequence network with
appropriate effective impedance connected at the fault location and rest of the transient analysis can be done as explained before. The effective impedance in different fault conditions is

Single line-to-ground fault \( Z_{ef} = Z_1 + Z_2 + Z_0 \)

Line-to-line fault \( Z_{ef} = Z_1 + Z_2 \)

Double line-to-ground fault \( Z_{ef} = Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0} \).

Transient stability of a system is assessed for a large disturbance like fault, either balanced or unbalanced, by solving the DAE through numerical methods mentioned above for the period of interest typically 2 to 10 seconds. If the system does not settle down to post fault stable operating point within the time of interest then the system is said to be unstable. If the system settles down to a stable operating point then the system is said to be stable. It is necessary that both small signal and transient stability analysis should be carried out for any system to assess the health of the system.

7.5 Direct Method of Transient Stability Analysis

Instead of finding the numerical solutions of DAE for a given time period the transient stability can be assessed directly through Lyapunov direct method of stability. Lyapunov stability theorem is stated as following.

For a dynamic system

\[ \dot{X} = f(X) \quad (7.70) \]

if there exist a positive definite continuous function \( V(X) \), whose first partial derivative with respect to the state variable exist, then if the total derivative \( \dot{V}(X) \) is negative semi definite then the system is said to be stable. The function \( V(X) \) is called as Lyapunov energy function. In [6], a method was proposed to estimate the transient stability of a system using Lyapunov energy function. The energy function
A method for assessing transient stability can be explained through an analogy of a ball in a bowl as shown in Fig. 7.2.

The Ball resting in the bowl is called as a stable equilibrium point (SEP). If due to some disturbance some kinetic energy is imparted to the ball then the ball will try to move in the direction of arrow. If at the edge of the bowl the kinetic energy is not fully consumed and converted to potential energy (since it is gaining height against gravity). Then the ball will fly off the edge else it will come back to its equilibrium position. The surface inside the bowl represents the potential energy surface, and the rim of the bowl represents the potential energy boundary surface (PEBS).

Hence if at PEBS the kinetic energy of the ball is greater than the potential energy acquired then ball will jump off the bowl or in to instability region. If the kinetic energy of the ball is less than or equal to the potential energy acquired then the ball will lies inside the bowl or is in the region of stability. Hence two quantities are required to determine if the ball will enter the instability region:

(a). The initial kinetic energy injected
(b). The height of the rim at the crossing point

These can be applied to power systems, for a given fault, whether the system kinetic energy during the fault can be dissipated or converted into potential energy.
before reaching a critical point beyond which the system will become unstable. This is called as transient energy function. We need some function which can adequately define the transient energy of the system and also the critical energy required to destabilize the system. This can be explained for a single machine connected to an infinite bus system.

The swing equation of the generator connected to infinite bus is given as

\[
\frac{2H}{\omega_{\text{base}}} \frac{d^2 \delta}{dt^2} = (P_m - P_{\text{max}} \sin \delta)
\]  

(7.71)

Let a potential energy function be defined as a function of the rotor angle \(\delta\) as

\[
V_{PE}(\delta) = -P_m \delta - P_{\text{max}} \cos(\delta)
\]  

(7.72)

Hence,

\[
\frac{2H}{\omega_{\text{base}}} \frac{d^2 \delta}{dt^2} = (P_m - P_{\text{max}} \sin \delta) = -\frac{dV_{PE}(\delta)}{d\delta}
\]

(7.73)

Multiplying equation (7.73) with \(\dot{\delta}\) on both the sides and rearranging leads to, assuming \(M = \frac{2H}{\omega_s}\),

\[
\frac{d}{dt} \left( \frac{1}{2} M \left( \frac{d\delta}{dt} \right)^2 \right) + \frac{dV_{PE}(\delta)}{dt} = 0
\]

(7.74)

Let, \(\omega' = \omega - \omega_{\text{base}}\) then

\[
\frac{d\delta}{dt} = \omega'
\]

(7.75)

\[
\frac{d}{dt} \left( \frac{1}{2} M (\omega')^2 + \frac{dV_{PE}(\delta)}{dt} \right) = 0
\]

(7.76)
or

\[
\frac{d}{dt} \left( V(\delta, \omega') \right) = 0
\]  

(7.77)

The function \( V(\delta, \omega') \) is the sum of change in kinetic energy and the potential energy gained. The potential energy function \( V_{PE}(\delta) \) can also be expressed as a change in the potential energy from the steady state. If \( \delta_0 \) is the steady state angle then

\[
V_{PE}(\delta, \delta_0) = -P_m (\delta - \delta_0) - P_{\text{max}} \left( \cos(\delta) - \cos(\delta_0) \right)
\]  

(7.78)

Now defining the total energy of the system as an energy function

\[
V(\delta, \omega') = \frac{1}{2} M (\omega')^2 - P_m (\delta - \delta_0) - P_{\text{max}} \left( \cos(\delta) - \cos(\delta_0) \right)
\]  

(7.79)

The swing curve along with the energy function given in (7.79) is given in Fig. 7.3. A three-phase bolted fault is applied due to which the rotor angle starts increasing from \( \delta_0 \). The fault is cleared at a critical clearing angle \( \delta_c \) then if the kinetic energy gained during fault is absorbed by the system before reaching the point \( \delta_u = \pi - \delta_0 \) then the system is stable. The transient energy at the angle \( \delta_u \) is given as

\[
V_{cr}(\delta_u, \omega') = \frac{1}{2} M (\omega')^2 - P_m (\delta_u - \delta_0) - P_{\text{max}} \left( \cos(\delta_u) - \cos(\delta_0) \right)
\]  

(7.80)

The energy function is labelled as \( V_{cr}(\delta_u, \omega') \) because this is the maximum energy that the system can have without becoming unstable. If the energy exceeds this critical energy then the system is unstable. Hence, the system stability can be assessed by computing the transient energy at critical clearing angle and checking if it less than \( V_{cr}(\delta_u, \omega') \) that is
Fig. 7.3: Swing curve along with energy function

\[ V_{cl}(\delta_c, \omega') = \frac{1}{2} M (\omega')^2 - P_m (\delta_c - \delta_0) - P_{max} \left( \cos(\delta_c) - \cos(\delta_0) \right) \]  \hspace{1cm} (7.81)

\[ V_{cl}(\delta_c, \omega') < V_{cr}(\delta_u, \omega') \quad \text{stable} \]

\[ V_{cl}(\delta_c, \omega') > V_{cr}(\delta_u, \omega') \quad \text{unstable} \]  \hspace{1cm} (7.82)
Multi-Machine stability

In a similar way the transient stability of multi-machine can be assessed through transient energy function method [7]. Let the swing equation of an \(i^{th}\) generator be given as

\[
M_i \frac{d^2 \delta_i}{dt^2} = (P_{mi} - P_{maxi} \sin \delta_i) \quad i = 1, ..., n_g
\]  

(7.83)

If we eliminate all the terminal buses and load buses, except the generator internal nodes, then

\[
I_G = Y_{red} E_G
\]  

(7.84)

Where, \(Y_{red}\) is the reduced admittance matrix consisting self and transfer admittances of the internal generator nodes. The vector of internal voltages is represented as \(E_G\) and the vector of generator currents as \(I_G\). The real power output of an \(i^{th}\) generator is given as

\[
P_{gi} = \text{Real} \left( E_{gi} (I_{gi})^* \right) = \text{Real} \left( E_{gi} (Y_{red} E_G)^* \right)
\]

\[
= \text{Real} \left( E_{gi} \sum_{k=1}^{n_g} (G_{ik} - jB_{ik}) E_{gi}^* \right)
\]

\[
= E_{gi}^2 G_{ii} + \sum_{j=1}^{n_g} E_{gi} E_{Gk} (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})
\]

(7.85)

Let, \(P_{mi} = P_{mi} - E_{gi}^2 G_{ii}\), then

\[
M_i \frac{d^2 \delta_i}{dt^2} = \left( P_{mi} - \sum_{j=1}^{n_g} E_{Gj} E_{Gk} (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \right) \quad i = 1, ..., n_g
\]  

(7.86)
Defining all the rotor angles and speed in terms of Centre Of Inertia (COI) as

\[
\delta_0 = \frac{1}{M_T} \sum_{i=1}^{n_r} M_i \delta_i
\]

(7.87)

\[
\omega_0 = \frac{1}{M_T} \sum_{i=1}^{n_r} M_i \omega_i
\]

\[
\theta_i = \delta_i - \delta_0
\]

(7.88)

\[
\tilde{\omega}_i = \omega_i - \omega_0
\]

Where, \( M_T = \sum_{i=1}^{m} M_i \). Equation (7.86) can be represented in terms of COI variables as

\[
M_i \frac{d^2 \delta_0}{dt^2} + M_i \frac{d^2 \delta_0}{dt^2} = P_M + \sum_{k=1}^{n_r} \left( C_{ik} \sin(\theta_k) + D_{ik} \cos(\theta_k) \right)
\]

(7.89)

\[
M_i \frac{d^2 \delta_0}{dt^2} = \frac{M_i}{M_T} \sum_{i=1}^{n_r} M_i \frac{d^2 \delta_i}{dt^2}
\]

\[
= \frac{M_i}{M_T} \sum_{i=1}^{n_r} \left( P_{M,i} - \sum_{k=1}^{n_r} \left( G_{ik} E_{G_k} \left( G_k \cos(\theta_i) + B_{ik} \sin(\theta_i) \right) \right) \right)
\]

(7.90)

\[
= \frac{M_i}{M_T} \left( \sum_{i=1}^{n_r} P_{M,i} - \sum_{i=1}^{n_r} \sum_{k=1}^{n_r} G_{ik} E_{G_k} \left( G_k \cos(\theta_i) + B_{ik} \sin(\theta_i) \right) \right)
\]

\[
= \frac{M_i}{M_T} P_{COI}
\]

Hence, equation (7.89) can be written as

\[
M_i \frac{d^2 \theta_i}{dt^2} = P_{M,i} - \sum_{k=1}^{n_r} \left( C_{ik} \sin(\theta_k) + D_{ik} \cos(\theta_k) \right) - \frac{M_i}{M_T} P_{COI}
\]

(7.91)
Since, $\sin(\theta_{ij}) = -\sin(\theta_{ji})$, and $\cos(\theta_{ik}) = \cos(\theta_{ki})$ the expression for $P_{COI}$ is further simplified as

$$P_{COI} = \sum_{i=1}^{n_g} P_{mi}^\prime - 2\sum_{i=1}^{n_g} \sum_{k=1}^{n_g} E_{gi} E_{gi} G_{ik} \cos(\theta_{ik}) \tag{7.92}$$

The energy function for each generator can be defined as

$$V_i(\theta_i, \bar{\theta}_i) = \frac{1}{2} M_i \bar{\omega}_i^2 - \int_{\theta_i^S}^{\theta_i} f_i(\theta) d\theta_i \quad \text{for } i=1,\ldots,n_g \tag{7.93}$$

Where, $\theta_i^S$ is the $i^{th}$ generator rotor angle in COI. The function $f_i(\theta)$ is given as

$$f_i(\theta) = P_{mi}^\prime - \sum_{k=1}^{n_g} (C_{ik} \sin(\theta_{ik}) + D_{ik} \cos(\theta_{ik})) - \frac{M_i}{M_R} P_{COI} \tag{7.94}$$

Summing the energy function defined in (7.93) for all the generators lead to

$$V(\theta, \bar{\theta}) = \sum_{i=1}^{n_g} \frac{1}{2} M_i \bar{\omega}_i^2 - \sum_{i=1}^{n_g} P_{mi}^\prime (\theta_i^S - \theta_i^S) - \sum_{i=1}^{n_g} \sum_{k=1}^{n_g} (C_{ij} \cos \theta_{ij} - C_{ij} \cos \theta_{ij}^S)$$

$$- \sum_{i=1}^{n_g} \sum_{k=1}^{n_g} \int_{0_i}^{\theta_i} D_{ik} \cos \theta_{ik} d\theta_i - \sum_{i=1}^{n_g} \frac{M_i}{M_R} \int_{0_i}^{\theta_i} P_{COI} d\theta_i \tag{7.95}$$

Where, the first term $\frac{1}{2} \sum M_i \bar{\omega}_i^2$ is the change in rotor kinetic energy of all the generators in COI reference frame. The rest of the terms are nothing but the potential energy of the system. The method of assessing the transient stability of the system through equation (7.95) can be done by doing a time domain simulation with a sustained fault until the energy function in equation (7.95) reaches its maximum value.
that value is the critical energy which the system can withstand and be stable. Now compute the energy function given in (7.95) for a clearing angle and if the energy is less than the critical energy then the system is stable else the system is not stable. There is some recent literature, which instead of taking simple classical model of generator for finding the transient energy function takes a detailed model along with the generator exciter. The interested readers may go through the reference [6], [7].

7.6 Methods of Improving Transient Stability

1. **High Speed fault clearing:** The amount of kinetic energy gained during a fault is directly proportional to the fault duration hence faster the clearing better the stability. Fast acting circuit breakers can be used for clearing the fault within 2 cycles through better communication and fast acting relays.

2. **Reduction of transmission system reactance:** The stability improves with reduction in system reactance as it enhances the synchronizing torque. This can be done by series compensation of transmission lines. Also transformers with lower leakage reactance can be used.

3. **Regulated Shunt Compensation:** Shunt Compensators like SVC, STATCOM can be used for improving the voltage profile there by synchronizing power among inter-connected generators.

4. **Dynamic braking:** During a fault the input mechanical power to the generator is greater than the output electrical power leading to acceleration of the generator rotor. Instead the excess power available due to mismatch between the input and output powers can be absorbed by resistors, which can be connected to the generator terminals during the fault. These are called as braking resistors as they will decrease the acceleration of the rotor by absorbing the excess power. The disadvantage is that the power is lost in the form of heat dissipation of the resistors.

5. **Independent pole operation of circuit breakers:** Instead of switching out all the three phases for every fault only the faulted phase may be switched out. This will significantly improve the stability.

6. **Single pole switching:** Again instead of tripping all 3 phase and then re-closing all three simultaneously each phase can be tripped and re-closed with
0.5 to 1.5 seconds. So that other two phase are still working in case of single line to ground fault as they are the most common faults.

7. **Fast valve operation:** It is a technique applicable to thermal units to assist in maintaining power system stability. It involves rapid closing and opening of steam valves in a prescribed manner to reduce the generator accelerating or decelerating power following the reception of a severe transmission system fault.

8. **Tripping of generator:** Selective tripping of generating unit for severe transmission system contingencies has been used as method of improving system stability for many years.

9. **Using supplementary damping controllers:** Like power system stabilizer or supplementary feedback controllers to FACTS devices can improve small signal stability.

10. **Using HVDC links:** HVDC links can also improve small signal stability. By modulating power in the DC link the small signal oscillations can be damped.
Example Problems

E1. A two area test system consisting of four generators and eleven buses is shown in the figure E1.1 below.

Fig. E1.1: Single line diagram of two area test system

The generators G1, G2 are in area-1 and generators G3, G4 are in area-2. All the four generators are of same rating that is three-phase 60 Hz, 20 kV and 900 MVA. The transmission line parameters are defined on the base of 230 kV, 100 MVA. The system base is taken as 100 MVA for load flow analysis. At bus-7 and bus-9 two capacitors of 200 MVAr and 350 MVAr are connected, respectively. The bus data is given in Table E1.1, line data is given in Table E1.2, line flow data is given in Table E1.3. A net real power of 200 MW is being transferred from area-1 to area-2, which is from bus-7 to bus-8, as can be observed from Table E1.3. The generators parameters on 900 MVA, 20 kV bases are given below.

\[ R_s = 0, X_{ls} = 0.2, X_d = 1.8, X'_d = 0.3, X''_d = 0.25, T_d^0 = 8 \text{ s}, T_d'' = 0.03 \text{ s} \]

\[ X_q = 1.7, X'_q = 0.55, X''_q = 0.24, T_q^0 = 0.4 \text{ s}, T_q''_0 = 0.05 \text{ s}, H = 6.5 \text{ s} \]

The static high gain exciters are used at all the four generators and the parameters are given as \( K_A = 200, T_R = 0.01 \).
Table E1.1: Bus data

<table>
<thead>
<tr>
<th>BUS</th>
<th>VOLTAGE (pu)</th>
<th>ANGLE (degrees)</th>
<th>GENERATION (pu)</th>
<th>LOAD (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>REAL</td>
<td>REACTIVE</td>
</tr>
<tr>
<td>1</td>
<td>1.03</td>
<td>18.5</td>
<td>7</td>
<td>1.79</td>
</tr>
<tr>
<td>2</td>
<td>1.01</td>
<td>8.765</td>
<td>7</td>
<td>2.20</td>
</tr>
<tr>
<td>3</td>
<td>1.01</td>
<td>-18.4439</td>
<td>7</td>
<td>1.85</td>
</tr>
<tr>
<td>4</td>
<td>1.03</td>
<td>-8.273</td>
<td>7.19</td>
<td>1.70</td>
</tr>
<tr>
<td>5</td>
<td>1.0074</td>
<td>12.0365</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.9805</td>
<td>1.9858</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.9653</td>
<td>-6.3737</td>
<td>0</td>
<td>0</td>
</tr>
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</tr>
<tr>
<td>9</td>
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<td>-33.5456</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.9862</td>
<td>-25.1838</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>1.0093</td>
<td>-14.9053</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table E1.2: Line data (pu)

<table>
<thead>
<tr>
<th>From Bus</th>
<th>To Bus</th>
<th>Resistance</th>
<th>Reactance</th>
<th>Line charging</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0</td>
<td>0.0167</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td>0.0167</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.001</td>
<td>0.01</td>
<td>0.0175</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>0.011</td>
<td>0.11</td>
<td>0.1925</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>0.011</td>
<td>0.11</td>
<td>0.1925</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.0025</td>
<td>0.025</td>
<td>0.0437</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>0</td>
<td>0.0167</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0</td>
<td>0.0167</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.011</td>
<td>0.11</td>
<td>0.1925</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>0.011</td>
<td>0.11</td>
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</tr>
<tr>
<td>9</td>
<td>10</td>
<td>0.001</td>
<td>0.01</td>
<td>0.0175</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>0.0025</td>
<td>0.025</td>
<td>0.0437</td>
</tr>
</tbody>
</table>
(a) A three phase-to-ground fault is applied on line connecting bus-7 and bus-8. The fault is near to bus-7. The fault is cleared after 160 ms followed by tripping of the line between bus-7 and bus-8. After fault clearance only one line is present between bus-7 and bus-8. Plot the generator G1-G4 torque angles with respect to CoI, speed of the rotors and electrical power output. Comment on the system stability.

(b) Power system stabilizers (PSS) are used at all the four generators and the parameters are given as

\[ K_{PSS} = 50, T_W = 10 \text{s}, T_1 = 0.05 \text{s}, T_2 = 0.02 \text{s}, T_3 = 3.0 \text{s}, T_4 = 5.4 \text{s} \]

For the same fault as in case (a), plot generator G1-G4 torque angles with respect to CoI, speed of the rotors and electrical power output. Comment on the system stability.

Sol:

Each generator is represented by a 7th order model. There are 28 dynamic states. Modified Euler’s method has been used for transient simulations. A time step of 0.005 seconds been used. The transient simulation is carried out for 15 seconds. Fig. E1.2 show the voltage of the faulted bus, bus-7. The fault is applied at 2.10 seconds and cleared at 2.26 seconds followed by tripping of the line connected between bus-7 and bus-8. It can be observed from Fig. E1.2 that at the moment of fault the voltage becomes zero and after the fault is cleared shoots back to the pre-fault value and starts oscillating around the pre-fault value.
Fig. E1.2: Voltage at the faulted bus, bus-7 without PSS

The generator torque angles, rotor speeds and electrical power outputs are given in Figs. E1.3 –E1.5.

Fig. E1.3: Generator torque angles without PSS
Fig. E1. 4: Generator rotor speeds without PSS

Fig. E1. 5: Generator electrical power outputs without PSS
It can be clearly observed from Figs. E1.3 to E1.5 that there are inter area oscillations between the generators G1, G2 in area-1 and generator G3, G4 in area-2. The torque angles and the rotor speeds of generators G1, G2 are oscillating together against generator G3, G4 torque and rotor speeds. When, the torque angles and rotor speed of G1, G2 are maximum then the torque angles and rotor speed of G3, G4 are at minimum. The same pattern can be observed in the electrical power output as well. It can also be observed that the oscillations are not settling down and in fact the system has sustained oscillations.

(b)

The faulted bus, bus-7 voltage, with PSS connected to all the four generators, is shown in Fig. E1.6. It can be clearly observed that the oscillations in the bus voltage settle down at about 10 seconds.

![Fig. E1.6: Voltage at the fault bus, bus-7 with PSS](image)

The generator torque angles, rotor speeds and electrical power outputs with PSS are given in Figs. E1.7–E1.9. It can be observed from Figs. E1.7 to E1.9, that thought the
generators G1, G2 are oscillating against generator G3, G4 the oscillations are damped out and approach a steady state value at about 10 seconds.

Fig. E1.7: Generator torque angle with PSS

Fig. E1.8: Generator rotor speeds with PSS
It has to be noted that the system, with PSS, settles down at a new operating point as the system topology changes after the fault. The effect of PSS on the system stability was in fact captured by small-signal stability analysis done on the two area test system in Chapter 6, Example 3. It was observed in Example 3 of Chapter 6 that the damping of the critical inter area mode improves with the inclusion of PSS significantly and this effect is clearly visible in the transient simulation.
References


