Chapter 3

FUNDAMENTAL THEORY OF LOAD COMPENSATION
(Lectures 19-28)

3.1 Introduction

In general, the loads which have poor power factor, unbalance, harmonics, and dc components require compensation. These loads are arc and induction furnaces, sugar plants, steel rolling mills (adjustable speed drives), power electronics based loads, large motors with frequent start and stop etc. All these loads can be classified into three basic categories.

1. Unbalanced ac load
2. Unbalanced ac + non linear load
3. Unbalanced ac + nonlinear ac + dc component of load.

The dc component is generally caused by the usage of half-wave rectifiers. These loads, particularly nonlinear loads generate harmonics as well as fundamental frequency voltage variations. For example arc furnaces generate significant amount of harmonics at the load bus. Other serious loads which degrade power quality are adjustable speed drives which include power electronic circuitry, all power electronics based converters such as thyristor controlled drives, rectifiers, cyclo converters etc.. In general, following aspects are important, while we do provide the load compensation in order to improve the power quality [1].

1. Types of load (unbalance, harmonics and dc component)
2. Real and Reactive power requirements
3. Rate of change of real and reactive power etc.

In this unit, we however, discuss fundamental load compensation techniques for unbalanced linear loads such as combination of resistance, inductance and capacitance and their combinations. The objective here will be to maintain currents balanced and unity factor with their voltages.
3.2 Fundamental Theory of Load Compensation

We shall find some fundamental relationship between supply system, the load, and the compensator. We shall start with the principle of power factor correction, which in its simplest form, and can be studied without reference to supply system [2]–[5].

The supply system, the load, and the compensator can be modeled in various ways. The supply system can be modeled as a Thevenin’s equivalent circuit with an open circuit voltage and a series impedance, (its current or power and reactive power) requirements. The compensator can be modeled as variable impedance or as a variable source (or sink) of reactive current. The choice of model varied according to the requirements. The modeling and analysis done here is on the basis of steady state and phasor quantities are used to note the various parameters in system.

3.2.1 Power Factor and its Correction

Consider a single phase system shown in 3.1(a) shown below. The load admittance is represented

\[ Y_l = G_l + jB_l \]

supplied from a load bus at voltage \( V = V \angle 0 \). The load current is \( I_l \) is given as,

\[
I_l = \frac{V(G_l + jB_l)}{\sqrt{R_l^2 + X_l^2}} = V G_l + jV B_l \\
= I_R + jI_X
\]

According to the above equation, the load current has a two components, i.e. the resistive or in phase component and reactive component or phase quadrature component and are represented by \( I_R \) and \( I_X \) respectively. The current, \( I_X \) will lag 90° for inductive load and it will lead 90° for capacitive load with respect to the reference voltage phasor. This is shown in 3.1(b). The load apparent power can be expressed in terms of bus voltage \( V \) and load current \( I_l \) as given below.

\[
\overline{S_l} = \bar{V} (\bar{I_l})^* \\
= V (I_R + jI_X)^* \\
= V (I_R - jI_X) \\
= V (I_l \cos \phi_l - j I_l \sin \phi_l) \\
= V I_l \cos \phi_l - j V I_l \sin \phi_l \\
= S_l \cos \phi_l - j S_l \sin \phi_l
\]
From (3.1), \( \overline{T}_l = \overline{V} (G_l + jB_l) = V G_l + jV B_l \), equation (3.2) can also be written as following.

\[
\overline{S}_l = \overline{V} (\overline{T}_l)^* \\
= V (V G_l + jV B_l)^* \\
= V (V G_l - jV B_l) \\
= V^2 G_l - jV^2 B_l \\
= P_l + jQ_l
\]  

(3.3)

From equation (3.3), load active \((P_l)\) and reactive power \((Q_l)\) are given as,

\[
P_l = V^2 G_l \\
Q_l = -V^2 B_l
\]  

(3.4)

Now suppose a compensator is connected across the load such that the compensator current, \(I_\gamma\) is equal to \(-I_X\), thus,

\[
\begin{align*}
\overline{T}_\gamma &= \overline{V} Y_\gamma = V (G_\gamma + jB_\gamma) = -I_X \\
&= -j V B_l
\end{align*}
\]  

(3.5)

The above condition implies that \(G_\gamma = 0\) and \(B_\gamma = -B_l\). The source current \(I_s\) can therefore given by,

\[
\begin{align*}
\overline{I}_s &= \overline{T}_l + \overline{T}_\gamma = I_R
\end{align*}
\]  

(3.6)

Therefore due to compensator action, the source supplies only in phase component of the load current. The source power factor is unity. This reduces the rating of the power conductor and losses due to the feeder impedance. The rating of the compensator is given by the following expression.

\[
\begin{align*}
\overline{S}_\gamma &= P_\gamma + jQ_\gamma = \overline{V} (\overline{T}_\gamma)^* \\
&= \overline{V} (-j V B_l)^* \\
&= jV^2 B_l
\end{align*}
\]  

(3.7)

Using (3.4), the above equation indicates the \(P_\gamma = 0\) and \(Q_\gamma = -Q_l\). This is an interesting inference that the compensator generates the reactive power which is equal and opposite to the load reactive and it has no effect on active power of the load. This is shown in Fig. 3.2. Using (3.2) and (3.7), the compensator rating can further be expressed as,

\[
Q_\gamma = -Q_l = -S_l \sin \phi_l = -S_l \sqrt{1 - \cos^2 \phi_l} \text{ VAr}
\]  

(3.8)

From (3.8),

\[
|Q_\gamma| = S_l \sqrt{1 - \cos^2 \phi_l}
\]  

(3.9)

If \(|Q_\gamma| < |Q_l|\) or \(|B_\gamma| < |B_l|\), then load is partially compensated. The compensator of fixed admittance is incapable of following variations in the reactive power requirement of the load. In
practical however a compensator such as a bank of capacitors can be divided into parallel sections, each of switched separately, so that discrete changes in the reactive power compensation can be made according to the load. Some sophisticated compensators can be used to provide smooth and dynamic control of reactive power.

Here voltage of supply is being assumed to be constant. In general if supply voltage varies, the \( Q, \gamma \) will not vary separately with the load and compensator error will be there. In the following discussion, voltage variations are examined and some additional features of the ideal compensator will be studied.

### 3.2.2 Voltage Regulation

Voltage regulation can be defined as the proportional change in voltage magnitude at the load bus due to change in load current (say from no load to full load). The voltage drop is caused due to feeder impedance carrying the load current as illustrated in Fig. 3.3(a). If the supply voltage is represented by Thevenin’s equivalent, then the voltage regulation (VR) is given by,

\[
VR = \frac{|E| - |\mathcal{V}|}{|\mathcal{V}|} = \frac{|E| - |V|}{|V|}
\]  

(3.10)

for \( \mathcal{V} \) being a reference phasor.

In absence of compensator, the source and load currents are same and the voltage drop due to the feeder is given by,

\[
\Delta V = E - V = Z_s I_l
\]  

(3.11)

The feeder impedance, \( Z_s = R_s + jX_s \). The relationship between the load powers and its voltage and current is expressed below.

\[
\overline{S}_l = \mathcal{V} (I_l)^* = P_l + jQ_l
\]  

(3.12)

Since \( \mathcal{V} = V \), the load current is expressed as following.

\[
I_l = \frac{P_l - jQ_l}{V}
\]  

(3.13)
Substituting, \( I_l \) from above equation into (3.11), we get
\[
\Delta V = \bar{E} - \bar{V} = (R_s + jX_s) \left( \frac{P_l - jQ_l}{V} \right) \\
= \frac{R_s P_l + X_s Q_l}{V} + j \frac{X_s P_l - R_s Q_l}{V} \\
= \Delta V_R + j \Delta V_X
\] (3.14)

Thus, the voltage drop across the feeder has two components, one in phase (\( \Delta V_R \)) and another is in phase quadrature (\( \Delta V_X \)). This is illustrated in Fig. 3.3(b).

Fig. 3.3 (a) Single phase system with feeder impedance (b) Phasor diagram

From the above it is evident that load bus voltage (\( \bar{V} \)) is dependent on the value of the feeder impedance, magnitude and phase angle of the load current. In other words, voltage change (\( \Delta V \)) depends upon the real and reactive power flow of the load and the value of the feeder impedance.

Now let us add compensator in parallel with the load as shown in Fig. 3.4(a). The question is: whether it is possible to make \( |\bar{E}| = |\bar{V}| \), in order to achieve zero voltage regulation irrespective of change in the load. The answer is yes, if the compensator consisting of purely reactive components, has enough capacity to supply to required amount of the reactive power. This situation is shown using phasor diagram in Fig. 3.4(b).

The net reactive at the load bus is now \( Q_s = Q_{\gamma} + Q_l \). The compensator reactive power (\( Q_{\gamma} \)) has to be adjusted in such a way as to rotate the phasor \( \Delta V \) until \( |\bar{E}| = |\bar{V}| \).

From (3.14) and Fig. 3.3(b),
\[
E \angle \delta = \left( V + \frac{R_s P_l + X_s Q_s}{V} \right) + j \left( \frac{X_s P_l - R_s Q_s}{V} \right)
\] (3.15)

The above equation implies that,
\[
E^2 = \left( V + \frac{R_s P_l + X_s Q_s}{V} \right)^2 + \left( \frac{X_s P_l - R_s Q_s}{V} \right)^2
\] (3.16)
The above equation can be simplified to,
\[
E^2V^2 = (V^2 + R_s P_l)^2 + X_s^2 Q_s^2 + 2(V^2 + R_s P_l) X_s Q_s + X_s^2 P_l^2 + R_s^2 Q_s^2 - 2X_s P_l R_s Q_s
\]
(3.17)

Above equation, rearranged in the powers of \(Q_s\), is written as following.
\[
(R_s^2 + X_s^2) Q_s^2 + 2V^2 X_s Q_s + (V^2 + R_s P_l)^2 + (X_s P_l)^2 - E^2 V^2 = 0
\]
(3.18)

Thus the above equation is quadratic in \(Q_s\) and can be represented using coefficients of \(Q_s\) as given below.
\[
a Q_s^2 + b Q_s + c = 0
\]
(3.19)

Where \(a = R_s^2 + X_s^2\), \(b = 2V^2 X_s\) and \(c = (V^2 + R_s P_l)^2 + X_s^2 P_l^2 - E^2 V^2\).

Thus the solution of above equation is as following.
\[
Q_s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
(3.20)

In the actual compensator, this value would be determined automatically by control loop. The equation also indicates that, we can find the value of \(Q_s\) by subjecting a condition such as \(E = V\) irrespective of the requirement of the load powers \((P_l, Q_l)\). This leads to the following conclusion that a purely reactive compensator can eliminate supply voltage variation caused by changes in both the real and reactive power of the load, provided that there is sufficient range and rate of \(Q_s\) both in lagging and leading pf. This compensator therefore acts as an ideal voltage regulator. It is mentioned here that we are regulating magnitude of voltage and not its phase angle. In fact its phase angle is continuously varying depending upon the load current.

It is instructive to consider this principle from different point of view. We have seen that compensator can be made to supply all load reactive power and it acts as power factor correction device. If the compensator is designed to compensate power factor, then \(Q_s = Q_l + Q_\gamma = 0\). This implies that \(Q_\gamma = -Q_l\). Substituting \(Q_s = 0\) for \(Q_l\) in (3.14) to achieve this condition, we get the
From above equation, it is observed that $\Delta V$ is independent of $Q_l$. Thus we conclude that a purely reactive compensator cannot maintain both constant voltage and unity power factor simultaneously. Of course the exception to this rule is a trivial case when $P_l = 0$.

### 3.2.3 An Approximation Expression for the Voltage Regulation

Consider a supply system with short circuit capacity ($S_{sc}$) at the load bus. This short circuit capacity can be expressed in terms of short circuit active and reactive powers as given below.

$$S_{sc} = P_{sc} + jQ_{sc} = \overline{E} \overline{T}_{sc}^* = \overline{E} \left( \frac{\overline{E}}{Z_{sc}} \right)^* = \frac{E^2}{Z_{sc}^*}$$  \hspace{1cm} (3.22)

Where $Z_{sc} = R_s + jX_s$ and $T_{sc}$ is the short circuit current. From the above equation

$$|Z_{sc}| = \frac{E^2}{S_{sc}}$$

Therefore,

$$R_s = \frac{E^2}{S_{sc}} \cos \phi_{sc}$$

$$X_s = \frac{E^2}{S_{sc}} \sin \phi_{sc}$$

$$\tan \phi_{sc} = \frac{X_s}{R_s}$$  \hspace{1cm} (3.23)

Substituting above values of $R_s$ and $X_s$, (3.14) can be written in the following form.

$$\frac{\Delta V}{V} = \left( \frac{P_l \cos \phi_{sc} + Q_l \sin \phi_{sc}}{V^2} + j \frac{P_l \sin \phi_{sc} - Q_l \cos \phi_{sc}}{V^2} \right) \frac{E^2}{S_{sc}}$$

$$\frac{\Delta V}{V} = \frac{\Delta V_R}{V} + j \frac{\Delta V_X}{V}$$  \hspace{1cm} (3.24)

Using an approximation that $E \approx V$, the above equation reduces to the following.

$$\frac{\Delta V}{V} = \left( \frac{P_l \cos \phi_{sc} + Q_l \sin \phi_{sc}}{S_{sc}} + j \frac{P_l \sin \phi_{sc} - Q_l \cos \phi_{sc}}{S_{sc}} \right)$$  \hspace{1cm} (3.25)

The above implies that,

$$\frac{\Delta V_R}{V} \approx \frac{P_l \cos \phi_{sc} + Q_l \sin \phi_{sc}}{S_{sc}}$$

$$\frac{\Delta V_X}{V} \approx \frac{P_l \sin \phi_{sc} - Q_l \cos \phi_{sc}}{S_{sc}}$$
Often \((\Delta V_X/V)\) is ignored on the ground that the phase quadrature component contributes negligible to the magnitude of overall phasor. It mainly contributes to the phase angle. Therefore the equation (3.25) is simplified to the following.

\[
\frac{\Delta V}{V} = \frac{\Delta V_R}{V} = \frac{P_l \cos \phi_{sc} + Q_l \sin \phi_{sc}}{S_{sc}}
\]  

(3.26)

Implying that the major change in voltage regulation occurs due to in phase component, \(\Delta V_R\). Although approximate, the above expression is quite useful in terms of short circuit level \((S_{sc})\), \((X_s/R_s)\), active and reactive power of the load.

On the basis of incremental changes in active and reactive powers of the load, i.e., \(\Delta P_l\) and \(\Delta Q_l\) respectively, the above equation can further be written as,

\[
\frac{\Delta V}{V} = \frac{\Delta V_R}{V} = \frac{\Delta P_l \cos \phi_{sc} + \Delta Q_l \sin \phi_{sc}}{S_{sc}}.
\]  

(3.27)

Further, feeder reactance \((X_s)\) is far greater than feeder resistance \((R_s)\), i.e., \(X_s >> R_s\). This implies that \(\phi_{sc} \rightarrow 90^\circ, \sin \phi_{sc} \rightarrow 1\) and \(\cos \phi_{sc} \rightarrow 0\). Using this approximation the voltage regulation is given as following.

\[
\frac{\Delta V}{V} \approx \frac{\Delta V_R}{V} \approx \frac{\Delta Q_l}{S_{sc}} \sin \phi_{sc} \approx \frac{\Delta Q_l}{S_{sc}}.
\]  

(3.28)

That is, per unit voltage change is equal to the ratio of the reactive power swing to the short circuit level of the supply system. Representing \(\Delta V\) approximately by \(E - V\) and assuming linear change in reactive power with the voltage, the equation (3.28) can be written as,

\[
\frac{E - V}{V} \approx \frac{Q_l}{S_{sc}}.
\]  

(3.29)

The above leads to the following expression,

\[
V \approx \frac{E}{(1 + \frac{Q_l}{S_{sc}})} \approx E(1 - \frac{Q_l}{S_{sc}})
\]  

(3.30)

with the assumption that \(Q_l/S_{sc} << 1\). Although above relationship is obtained with approximations, however it is very useful in visualizing the action of compensator on the voltage. The above equation is graphically represented as Fig. 3.5. The nature of voltage variation is drooping with increase in inductive reactive power of the load. This is shown by negative slope \(-E/S_{sc}\) as indicated in the figure.

The above characteristics also explain that when load is capacitive, \(Q_l\) is negative. This makes \(V > E\). This is similar to Ferranti effect due to lightly loaded electric lines.

**Example 3.1** Consider a supply at 10 kV line to neutral voltage with short circuit level of 250 MVA and \(X_s/R_s\) ratio of 5, supplying a star connected load inductive load whose mean power is 25 MW and whose reactive power varies from 0 to 50 MVAR, all quantities per phase.
Fig. 3.5 Voltage variation with reactive power of the load

(a) Find the load bus voltage ($V$) and the voltage drop ($\Delta V$) in the supply feeder. Thus determine load current ($I_l$), power factor and system voltage ($E$).

(b) It is required to maintain the load bus voltage to be same as supply bus voltage i.e. $V=10$ kV. Calculate reactive power supplies by the compensator.

(c) What should be the load bus voltage and compensator current if it is required to maintain the unity power factor at the supply?

**Solution:** The feeder resistance and reactance are computed as following.

$$Z_s = \frac{E_s^2}{S_{sc}} = \left(\frac{10 \text{ kV}}{}\right)^2 / 250 = 0.4 \text{ /phase}$$

It is given that, $X_s / R_s = \tan \phi_{sc} = 5$, therefore $\phi_{sc} = \tan^{-1} 5 = 78.69^\circ$. From this,

$$R_s = Z_s \cos \phi_{sc} = 0.4 \cos(78.69^\circ) = 0.0784 \Omega$$

$$X_s = Z_s \sin \phi_{sc} = 0.4 \sin(78.69^\circ) = 0.3922 \Omega$$

**a) Without compensation** $Q_s = Q_l$, $Q_\gamma = 0$

To know $\Delta V$, first the voltage at the load bus has to be computed. This is done by rearranging (3.18) in powers of voltage $V$. This is given below.

$$(R_s^2 + X_s^2) Q_l^2 + 2 V^2 X_s Q_l + (V^2 + R_s P_l)^2 + X_s^2 P_l^2 - E^2 V^2 = 0$$

Combining the I, II and III terms in the above equation, we get the following.

$$V^4 + \left\{2 (R_s P_l + X_s Q_l) - E^2 \right\} V^2 + (R_s^2 + X_s^2)(Q_l^2 + P_l^2) = 0 \quad (3.31)$$

Now substituting values of $R_s$, $X_s$, $P_l$, $Q_l$ and $E$ in above equation, we get,

$$V^4 + \left\{2 [0.0784 \times 25 + 0.3922 \times 50] - 10^2 \right\} V^2 + (0.0784^2 + 0.3922^2)(25^2 + 50^2) = 0$$
After simplifying the above, we have the following equation.

\[ V^4 - 56.86V^2 + 500 = 0 \]

Therefore

\[
V^2 = \frac{56.86 \pm \sqrt{56.86 - 4 \times 500}}{2}
\]

\[
= 45.985, 10.875
\]

and

\[
V = \pm 6.78 \text{kV}, \pm 3.297 \text{kV}
\]

Since rms value cannot be negative and maximum rms value must be a feasible solution, therefore \( V = 6.78 \text{kV} \).

Now we can compute \( \Delta V \) using (3.14), as it is given below.

\[
\Delta V = \frac{R_s P_l + X_s Q_l}{V} + \frac{j X_s P_l - R_s Q_l}{V}
\]

\[
= \frac{0.078425 + 0.39250}{6.78} + j \frac{0.392225 - 0.078450}{6.78}
\]

\[
= 3.1814 + j0.8677 \text{kV} = 3.2976 \angle 15.25^\circ \text{kV}
\]

Now the line current can be found out as following.

\[
I_l = \frac{P_l - Q_l}{V} = \frac{25 - j50}{6.782}
\]

\[
= 3.86 - j7.3746 \text{kA}
\]

\[
= 8.242 \angle -63.44^\circ \text{kA}
\]

The power factor of load is \( \cos \left( \tan^{-1} \left( \frac{Q_l}{P_l} \right) \right) = 0.4472 \) lagging. The phasor diagram for this case is similar to what is shown in Fig. 3.3(b).

(b) **Compensator as a voltage regulator**

Now it is required to maintain \( V = E = 10.0 \text{kV} \) at the load bus. For this let their be reactive power \( Q_\gamma \) supplied by the compensator at the load bus. Therefore the net reactive power at the load bus is equal to \( Q_s \), which is given below.

\[ Q_s = Q_l + Q_\gamma \]

Thus from (3.18), we get,

\[
(R_s^2 + X_s^2)Q_s^2 + 2V^2X_sQ_s + (V^2 + R_sP_l)^2 + X_s^2P_l^2 - E^2V^2 = 0
\]

\[
(0.784^2 + 0.3922^2)Q_s^2 + 2 \times 10^2 \times 0.3922 \times Q_s + \left\{ (10^2 + 0.784 \times 25)^2 + 0.3922^2 \times 25^2 - 10^4 \right\} = 0
\]

From the above we have,

\[
0.16Q_s^2 + 78.44Q_s + 491.98 = 0.
\]
Solving the above equation we get,

\[ Q_s = \frac{-78.44 \pm \sqrt{78.44^2 - 4 \times 0.16 \times 491.98}}{2 \times 0.16} = -6.35 \text{ or } -484 \text{ MVAr.} \]

The feasible solution is \( Q_s = -6.35 \) MVAr because it requires less rating of the compensator. Therefore the reactive power of the compensator (\( Q_\gamma \)) is,

\[ Q_\gamma = Q_s - Q_l = -6.35 - 50 = -56.35 \text{ MVAr.} \]

With \( Q_s = -6.35 \) MVAr, the \( \Delta V \) is computed by replacing \( Q_s \) for \( Q_l \) in (3.14) as given below.

\[
\Delta V = \frac{R_s P_l + X_s Q_s}{V} + j \frac{X_s P_l - R_s Q_s}{V} = \frac{0.0784 \times 25 + 0.39225 \times -6.35}{10} + j \frac{0.39225 \times 25 - 0.0784 \times (-6.35)}{10} \\
= \frac{1.96 - 2.4}{10} + j \frac{9.805 + 0.4978}{10} \\
= -0.0532 + j1.030 \text{kV} = 1.03137 \angle 92.95^\circ \text{kV}
\]

Now, we can find supply voltage \( E \) as given below.

\[
E = \Delta V + V = 10 - 0.0532 + j1.030 = 9.9468 + j1.030 = 10 \angle 5.91^\circ \text{kV}
\]

The supply current is,

\[
I_s = \frac{P_l - jQ_s}{V} = \frac{25 - j(-6.35)}{10} = 2.5 + j0.635 \text{kA} = 2.579 \angle 14.25^\circ \text{kA.}
\]

This indicates that power factor is not unity for perfect voltage regulation i.e., \( E = V \). For this case the compensator current is given below.

\[
I_\gamma = \frac{-jQ_\gamma}{V} = \frac{-j(-56.35)}{10} \\
I_\gamma = j5.635 \text{kA}
\]

The load current is computed as below.

\[
I_l = \frac{P_l - jQ_l}{V} = \frac{25 - j50}{10} = 2.5 - j5.0 = I_{IR} + jI_{IX} = 5.59 \angle 63.44^\circ \text{kA}
\]

The phasor diagram is similar to the one shown in Fig. 3.4(b). The phasor diagram shown has interesting features. The voltage at the load bus is maintained to 1.0 pu. It is observed that the reactive power of the compensator \( Q_\gamma \) is not equal to load reactive power (\( Q_l \)). It exceeds by 6.35
MVAr. As a result of this compensation, the voltage regulation is perfect, however power factor is not unity. The phase angle between \( V \) and \( I \) is \( \cos^{-1} 0.969 = 14.25^\circ \) as computed above. Therefore the angle between \( E \) and \( I \) is \( 14.25^\circ - 5.91^\circ = 8.34^\circ \). Thus, source power factor \( (\phi_s) \) is \( \cos(8.34^\circ) = 0.19956 \) leading.

(c) Compensation for unity power factor

To achieve unity power factor at the load bus, the condition \( Q_\gamma = -Q_l \) must be satisfied, which further implies that the net reactive power at the load bus is zero. Therefore substituting \( Q_l = 0 \) in (3.31), we get the following.

\[
V^4 + \left\{2(RsP_l - E^2)\right\}V^2 + (Rs^2 + Xs^2)(P_l^2 + Q_l^2) = 0
\]
\[
V^4 + (2 \times 0.0784 \times 25 - 10^2)V^2 + (0.0784^2 + 0.3922^2)25^2 = 0
\]

From the above,

\[
V^4 + 96.08V^2 + 99.79 = 0
\]

The solution of the above equation is,

\[
V^2 = \frac{96.08 \pm \sqrt{93.97}}{2} = 95.02, 1.052
\]

\[
V = \pm 9.747 \text{kV}, \pm 1.0256 \text{kV}.
\]

Since rms value cannot be negative and maximum rms value must be a feasible solution, therefore \( V = 9.747 \) kV. Thus it is seen that for obtaining unity power factor at the load bus does not ensure desired voltage regulation. Now the other quantities are computed as given below.

\[
I_l = \frac{P_l - jQ_l}{V} = \frac{25 - j50}{9.747} = 2.5648 - j5.129 = 5.7345 \angle -63.43^\circ \text{kA}
\]

Since \( Q_\gamma = -Q_l \), this implies that \( I_\gamma = -jQ_\gamma/V = jQ_l/V = j5.129 \text{kA} \). The voltage drop across the feeder is given as following.

\[
\Delta V = \frac{R_sP_l + X_sQ_l}{V} + j\frac{X_sP_l - R_sQ_l}{V}
\]
\[
= \frac{(0.784 \times 25 + j0.3922 \times 25)}{9.747}
\]
\[
= 0.201 + j1.005 = 1.0249 \angle 5.01^\circ \text{kV}
\]

The phasor diagram for the above case is shown in Fig. 3.6.

The percentage voltage change \( = (10 - 9.748)/10 \times 100 = 2.5 \). Thus we see that power factor improves voltage regulation enormously compared with uncompensated case. In many cases, degree of improvement is adequate and the compensator can be designed to provide reactive power requirement of load rather than as a ideal voltage regulator.
3.3 Some Practical Aspects of Compensator used as Voltage Regulator

In this section, some practical aspects of the compensator in voltage regulation mode will be discussed. The important parameters of the compensator which play significant role in obtaining desired voltage regulation are: Knee point ($V_k$), maximum or rated reactive power $Q_{\gamma_{max}}$ and the compensator gain $K_\gamma$.

The compensator gain $K_\gamma$ is defined as the rate of change of compensator reactive power $Q_\gamma$ with change in the voltage ($V$), as given below.

$$K_\gamma = \frac{dQ_\gamma}{dV}$$  \hfill (3.32)

For linear relationship between $Q_\gamma$ and $V$ with incremental change, the above equation be written as the following.

$$\Delta Q_\gamma = \Delta V K_\gamma$$  \hfill (3.33)

Assuming compensator characteristics to be linear with $Q_\gamma \leq Q_{\gamma_{max}}$ limit, the voltage can be represented as,

$$V = V_k + \frac{Q_\gamma}{K_\gamma}$$  \hfill (3.34)

This is re-written as,

$$Q_\gamma = K_\gamma(V - V_k)$$  \hfill (3.35)
Flat V-Q characteristics imply that $K_\gamma \rightarrow \infty$. That means the compensator which can absorb or generate exactly right amount of reactive power to maintain supply voltage constant as the load varies without any constraint. We shall now see the regulating properties of the compensator, when compensator has finite gain $K_r$ operating on supply system with a finite short circuit level, $S_{sc}$. The further which are made in the following study are: high $X_s/R_s$ ratio and negligible load power fluctuations. The net reactive power at the load bus is sum of the load and the compensator reactive power as given below.

$$Q_l + Q_\gamma = Q_s$$  \hspace{1cm} (3.36)

Using earlier voltage and reactive power relationship from equation (3.30), it can be written as the following.

$$V \simeq E \left(1 - \frac{Q_s}{S_{sc}}\right)$$  \hspace{1cm} (3.37)

The compensator voltage represented by (3.34) and system voltage represented by (3.37) are shown in Fig. 3.7(a) and (b) respectively.

![Diagram](image)

Fig. 3.7 (a) Voltage characteristics of compensator (b) System voltage characteristics

Differentiating $V$ with respect to $Q_s$, we get, intrinsic sensitivity of the supply voltage with variation in $Q_s$ as given below.

$$\frac{dV}{dQ_s} = -\frac{E}{S_{sc}}$$  \hspace{1cm} (3.38)

It is seen from the above equation that high value of short circuit level $S_{sc}$ reduces the voltage sensitivity, making voltage variation flat irrespective of $Q_l$. With compensator replacing $Q_s = Q_\gamma + Q_l$ in (3.37), we have the following.

$$V \simeq E \left(1 - \frac{Q_l + Q_\gamma}{S_{sc}}\right)$$  \hspace{1cm} (3.39)
Substituting $Q_{\gamma}$ from (3.35), we get the following equation.

$$V \simeq E \left[ \frac{1 + K_{\gamma} V_k / S_{sc}}{1 + E K_{\gamma} / S_{sc}} - \frac{Q_l / S_{sc}}{1 + E K_{\gamma} / S_{sc}} \right]$$ (3.40)

Although approximate, above equation gives the effects of all the major parameters such as load reactive power, the compensator characteristics $V_{\gamma}$ and $K_{\gamma}$ and the system characteristics $E$ and $S_{sc}$. As we discussed, V-Q characteristics is flat for high or infinite value $K_{\gamma}$. However the higher value of the gain $K_{\gamma}$ means large rating and quick rate of change of the reactive power with variation in the system voltage. This makes cost of the compensator high.

The compensator has two effects as seen from (3.40), i.e., it alters the no load supply voltage ($E$) and it modifies the sensitivity of supply point voltage to the variation in the load reactive power. Differentiating (3.40) with respect to $Q_l$, we get,

$$\frac{dV}{dQ_l} = -\frac{E/S_{sc}}{1 + K_{\gamma} E/S_{sc}}$$ (3.41)

which is voltage sensitivity of supply point voltage to the load reactive power. It can be seen that the voltage sensitivity is reduced as compared to the voltage sensitivity without compensator as indicated in (3.38).

It is useful to express the slope ($-E/S_{sc}$) by a term in a form similar to $K_{\gamma} = dQ_{\gamma}/dV$, as given below.

$$K_s = -\frac{S_{sc}}{E}$$

Thus, $\frac{1}{K_s} = -\frac{E}{S_{sc}}$ (3.42)

Substituting $V$ from (3.39) into (3.35), the following is obtained.

$$Q_{\gamma} = K_{\gamma} \left[ E \left( 1 - \frac{Q_l + Q_{\gamma}}{S_{sc}} \right) - V_k \right]$$ (3.43)

Collecting the coefficients of $Q_{\gamma}$ from both sides of the above equation, we get

$$Q_{\gamma} = \frac{K_{\gamma}}{1 + K_{\gamma} (E/S_{sc})} \left[ E \left( 1 - \frac{Q_l}{S_{sc}} \right) - V_k \right]$$ (3.44)

Setting knee voltage $V_k$ of the compensator equal to system voltage $E$ i.e., $V_k = E$, the above equation is simplified to,

$$Q_{\gamma} = -\frac{K_{\gamma} (E/S_{sc})}{1 + K_{\gamma} (E/S_{sc})} Q_l$$

$$= -\left( \frac{K_{\gamma}/K_s}{1 + K_{\gamma}/K_s} \right) Q_l.$$ (3.45)
From the above equation it is observed that, when compensator gain $K_\gamma \to \infty$, $Q_\gamma \to -Q_l$. This indicates perfect compensation of the load reactive power in order to regulate the load bus voltage.

**Example 3.2** Consider a three-phase system with line-line voltage 11 kV and short circuit capacity of 480 MVA. With compensator gain of 100 pu determine voltage sensitivity with and without compensator. For each case, if a load reactive power changes by 10 MVARs, find out the change in load bus voltage assuming linear relationship between V-Q characteristics. Also find relationship between compensator and load reactive powers.

**Solution:** The voltage sensitivity can be computed using the following equation.

$$\frac{dV}{dQ_l} = -\frac{E/S_{sc}}{1 + K_\gamma E/S_{sc}}$$

Without compensator $K_\gamma = 0$, $E = (11/\sqrt{3}) = 6.35$ kV and $S_{sc} = 480/3 = 160$ MVA. Substituting these values in the above equation, the voltage sensitivity is given below.

$$\frac{dV}{dQ_l} = -\frac{6.35/160}{1 + 0 \times 6.35/160} = -0.039$$

The change in voltage due to variation of reactive power by 10 MVARs, $\Delta V = -0.039 \times 10 = -0.39$ kV.

With compensator, $K_\gamma = 100$

$$\frac{dV}{dQ_l} = -\frac{6.35/160}{1 + 100 \times 6.35/160} = -0.0078$$

The change in voltage due to variation of reactive power by 10 MVARs, $\Delta V = -0.0078 \times 10 = -0.078$ kV.

Thus it is seen that, with finite compensator gain their is quite reduction in the voltage sensitivity, which means that the load bus is fairly constant for considerable change in the load reactive power. The compensator reactive power $Q_\gamma$ and load reactive power $Q_l$ are related by equation (3.45) and is given below.

$$Q_\gamma = -\frac{K_\gamma (E/S_{sc})}{1 + K_\gamma (E/S_{sc})} Q_l = -\frac{100 \times (6.35/160)}{1 + 100 \times (6.35/160)} Q_l$$

$$= -0.79 Q_l$$

It can be observed that when compensator gain, $(Q_\gamma)$ is quite large, then compensator reactive power $Q_\gamma$ is equal and opposite to that of load reactive power i.e., $Q_\gamma = -Q_l$. It is further observed that due to finite compensator gain i.e., $K_\gamma = 100$, reactive power is partially compensated The compensator reactive power varies from 0 to 7.9 MVAR for 0 to 10 MVAR change in the load reactive power.

### 3.4 Phase Balancing and Power Factor Correction of Unbalanced Loads

So far we have discussed voltage regulation and power factor correction for single phase systems. In this section we will focus on balancing of three-phase unbalanced loads. In considering unbalanced loads, both load and compensator are modeled in terms of their admittances and impedances.
3.4.1 Three-phase Unbalanced Loads

Consider a three-phase three-wire system supplying unbalanced load as shown in Fig. 3.8.

Applying Kirchoff’s voltage law for the two loops shown in the figure, we have the following equations.

\[-V_{an} + Z_a I_1 + Z_b (I_1 - I_2) + V_{bn} = 0\]
\[-V_{bn} + Z_b I_2 + Z_b (I_2 - I_1) + V_{cn} = 0\]  \hspace{1cm} (3.46)

Rearranging above, we get the following.

\[V_{an} - V_{bn} = (Z_a + Z_b) I_1 - Z_b I_2\]
\[V_{bn} - V_{cn} = (Z_b + Z_c) I_2 - Z_b I_1\]  \hspace{1cm} (3.47)

The above can be represented in matrix form as given below.

\[
\begin{bmatrix}
V_{an} - V_{bn} \\
V_{bn} - V_{cn}
\end{bmatrix} =
\begin{bmatrix}
(Z_a + Z_b) & -Z_b \\
-Z_b & (Z_b + Z_c)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]  \hspace{1cm} (3.48)

Therefore the currents are given as below.

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\frac{1}{\Delta Z}
\begin{bmatrix}
(Z_b + Z_c) & Z_b \\
Z_b & (Z_a + Z_b)
\end{bmatrix}
\begin{bmatrix}
V_{an} - V_{bn} \\
V_{bn} - V_{cn}
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\frac{1}{\Delta Z}
\begin{bmatrix}
(Z_b + Z_c) & Z_b \\
Z_b & (Z_a + Z_b)
\end{bmatrix}
\begin{bmatrix}
V_{an} - V_{bn} \\
V_{bn} - V_{cn}
\end{bmatrix}
\]  \hspace{1cm} (3.49)

Where, \(\Delta Z = (Z_b + Z_c)(Z_a + Z_b) - Z_b^2 = Z_a Z_b + Z_b Z_c + Z_c Z_a\). The current \(I_1\) is given below.

\[
I_1 = \frac{1}{\Delta Z} \left[ (Z_b + Z_c) (V_{an} - V_{bn}) + Z_b (V_{bn} - V_{cn}) \right]
\]
\[
= \frac{1}{\Delta Z} \left[ (Z_b + Z_c) V_{an} - Z_c V_{bn} - Z_b V_{cn} \right]
\]  \hspace{1cm} (3.50)
Similarly,
\[ I_2 = \frac{1}{\Delta Z} \left[ Z_b (\nabla_{an} - \nabla_{bn}) + (Z_a + Z_b) (\nabla_{bn} - \nabla_{cn}) \right] \]
\[ = \frac{1}{\Delta Z} \left[ Z_b \nabla_{an} + Z_a \nabla_{bn} - (Z_a + Z_b) \nabla_{cn} \right] \quad (3.51) \]

Now,
\[ I_a = I_1 = \frac{1}{\Delta Z} \left[ (Z_b + Z_c) \nabla_{an} - Z_c \nabla_{bn} - Z_b \nabla_{cn} \right] \]
\[ I_b = I_2 - I_1 \]
\[ = \frac{1}{\Delta Z} \left[ Z_b \nabla_{an} + Z_a \nabla_{bn} - (Z_a + Z_b) \nabla_{cn} - (Z_b + Z_c) \nabla_{an} + Z_c \nabla_{bn} + Z_b \nabla_{cn} \right] \]
\[ = \frac{[ (Z_c + Z_a) \nabla_{bn} - Z_a \nabla_{cn} - Z_c \nabla_{an} ]}{\Delta Z} \quad (3.52) \]

and
\[ I_c = -I_2 = -I_b - I_a = \frac{(Z_a + Z_b) \nabla_{cn} - Z_b \nabla_{an} - Z_a \nabla_{bn}}{\Delta Z} \quad (3.53) \]

Alternatively phase currents can be expressed as following.
\[ I_a = \frac{\nabla_{an} - \nabla_{Nn}}{Z_a} \]
\[ I_b = \frac{\nabla_{bn} - \nabla_{Nn}}{Z_b} \]
\[ I_c = \frac{\nabla_{cn} - \nabla_{Nn}}{Z_c} \quad (3.54) \]

Applying Kirchoff’s current law at node \( N \), we get \( I_a + I_b + I_c = 0 \). Therefore from the above equation,
\[ \frac{\nabla_{an} - \nabla_{Nn}}{Z_a} + \frac{\nabla_{bn} - \nabla_{Nn}}{Z_b} + \frac{\nabla_{cn} - \nabla_{Nn}}{Z_c} = 0. \quad (3.55) \]

Which implies that,
\[ \frac{\nabla_{an}}{Z_a} + \frac{\nabla_{bn}}{Z_b} + \frac{\nabla_{cn}}{Z_c} = \left[ \frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_c} \right] \nabla_{Nn} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a Z_b Z_c} \nabla_{Nn} \quad (3.56) \]

From the above equation the voltage between the load and system neutral can be found. It is given below.
\[ \nabla_{Nn} = \frac{Z_a Z_b Z_c}{\Delta Z} \left[ \frac{\nabla_{an}}{Z_a} + \frac{\nabla_{bn}}{Z_b} + \frac{\nabla_{cn}}{Z_c} \right] \]
\[ = \frac{1}{Z_a + \frac{1}{Z_b} + \frac{1}{Z_c}} \left[ \frac{\nabla_{an}}{Z_a} + \frac{\nabla_{bn}}{Z_b} + \frac{\nabla_{cn}}{Z_c} \right] \quad (3.57) \]
Some interesting points are observed from the above formulation.

1. If both source voltage and load impedances are balanced i.e., \( Z_a = Z_b = Z_c = Z \), then
   \[ V_{Nn} = \frac{1}{3} (V_{an} + V_{bn} + V_{cn}) = 0. \]
   Thus their will not be any voltage between two neutrals.

2. If supply voltage are balanced and load impedances are unbalanced, then \( V_{Nn} \neq 0 \) and is given by the above equation.

3. If supply voltages are not balanced but load impedances are identical, then
   \[ V_{Nn} = \frac{1}{3} (V_{an} + V_{bn} + V_{cn}). \]
   This equivalent to zero sequence voltage \( V_0 \).

It is interesting to note that if the two neutrals are connected together i.e., \( V_{Nn} = 0 \), then each phase become independent through neutral. Such configuration is called three-phase four-wire system. In general, three-phase four-wire system has following properties.

\[
\begin{align*}
V_{Nn} &= 0 \\
I_a + I_b + I_c &= I_{Nn} \neq 0
\end{align*}
\] (3.58)

The current \( I_{Nn} \) is equivalent to zero sequence current (\( I_0 \)) and it will flow in the neutral wire.

For three-phase three-wire system, the zero sequence current is always zero and therefore following properties are satisfied.

\[
\begin{align*}
V_{Nn} &\neq 0 \\
I_a + I_b + I_c &= 0
\end{align*}
\] (3.59)

Thus, it is interesting to observe that three-phase three-wire and three-phase four-wire system have dual properties in regard to neutral voltage and current.

### 3.4.2 Representation of Three-phase Delta Connected Unbalanced Load

A three-phase delta connected unbalanced and its equivalent star connected load are shown in Fig. 3.9(a) and (b) respectively. The three-phase load is represented by line-line admittances as given below.

![Fig. 3.9 (a) An unbalanced delta connected load (b) Its equivalent star connected load](image)
\[
\begin{align*}
Y_{l}^{ab} &= G_{l}^{ab} + jB_{l}^{ab} \\
Y_{l}^{bc} &= G_{l}^{bc} + jB_{l}^{bc} \\
Y_{l}^{ca} &= G_{l}^{ca} + jB_{l}^{ca}
\end{align*}
\] (3.60)

The delta connected load can be equivalently converted to star connected load using following expressions.

\[
\begin{align*}
Z_{l}^{a} &= \frac{Z_{l}^{ab} Z_{l}^{ca}}{Z_{l}^{ab} + Z_{l}^{bc} + Z_{l}^{ca}} \\
Z_{l}^{b} &= \frac{Z_{l}^{bc} Z_{l}^{ab}}{Z_{l}^{ab} + Z_{l}^{bc} + Z_{l}^{ca}} \\
Z_{l}^{c} &= \frac{Z_{l}^{ca} Z_{l}^{bc}}{Z_{l}^{ab} + Z_{l}^{bc} + Z_{l}^{ca}}
\end{align*}
\] (3.61)

Where \( Z_{l}^{ab} = 1/Y_{l}^{ab}, Z_{l}^{bc} = 1/Y_{l}^{bc} \) and \( Z_{l}^{ca} = 1/Y_{l}^{ca} \). The above equation can also be written in admittance form

\[
\begin{align*}
Y_{l}^{a} &= \frac{Y_{l}^{ab} Y_{l}^{bc} + Y_{l}^{bc} Y_{l}^{ca} + Y_{l}^{ca} Y_{l}^{ab}}{Y_{l}^{bc}} \\
Y_{l}^{b} &= \frac{Y_{l}^{ab} Y_{l}^{bc} + Y_{l}^{bc} Y_{l}^{ca} + Y_{l}^{ca} Y_{l}^{ab}}{Y_{l}^{ca}} \\
Y_{l}^{c} &= \frac{Y_{l}^{ab} Y_{l}^{bc} + Y_{l}^{bc} Y_{l}^{ca} + Y_{l}^{ca} Y_{l}^{ab}}{Y_{l}^{ab}}
\end{align*}
\] (3.62)

**Example 3.3** Consider three-phase system supply a delta connected unbalanced load with \( Z_{l}^{a} = R_{a} = 10 \Omega, Z_{l}^{b} = R_{b} = 15 \Omega \) and \( Z_{l}^{c} = R_{c} = 30 \Omega \) as shown in Fig. 3.8. Determine the voltage between neutrals and find the phase currents. Assume a balance supply voltage with rms value of 230 V. Find out the vector and arithmetic power factor. Comment upon the results.
Solution: The voltage between neutrals $V_{Nn}$ is given as following.

\[
V_{Nn} = \frac{R_a R_b R_c}{R_a R_b + R_b R_c + R_c R_a} \left[ \frac{V_{an}}{R_a} + \frac{V_{bn}}{R_b} + \frac{V_{cn}}{R_c} \right]
\]

\[
= \frac{10 \times 15 \times 30}{10 \times 15 + 15 \times 30 + 30 \times 10} \left[ \frac{V \angle 0^\circ}{10} + \frac{V \angle -120^\circ}{15} + \frac{V \angle 120^\circ}{30} \right]
\]

\[
= \frac{4500}{900} \left[ 3V \angle 0^\circ + 2V \angle -120^\circ + V \angle 120^\circ \right]
\]

\[
= \frac{4500}{900} \frac{1}{30} V \left[ 3 + 2 \left( -\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) + \left( -\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right]
\]

\[
= \frac{V}{6} \left[ 3 - 1 - \frac{1}{2} - j2 \times \frac{\sqrt{3}}{2} + j\frac{\sqrt{3}}{2} \right]
\]

\[
= \frac{V}{6} \left[ 3 - j\frac{\sqrt{3}}{2} \right] = V \left[ 1 - j \frac{1}{4\sqrt{3}} \right] = V [0.25 - j0.1443]
\]

\[
V_{Nn} = \frac{V}{2\sqrt{3}} \angle -30^\circ = 66.39\angle -30^\circ \text{ Volts}
\]

Knowing this voltage, we can find phase currents as following.

\[
I_a = \frac{V_{an} - V_{Nn}}{R_a} = \frac{V \angle 0^\circ - V/(2\sqrt{3}) \angle -30^\circ}{10}
\]

\[
= \frac{V [1 - 0.25 + j0.1443]}{10}
\]

\[
= 230 \times [0.075 + j0.01443]
\]

\[
= 17.56 \angle 10.89^\circ \text{ Amps}
\]

Similarly,

\[
I_b = \frac{V_{bn} - V_{Nn}}{R_b} = \frac{V \angle -120^\circ - V/(2\sqrt{3}) \angle -30^\circ}{15}
\]

\[
= 230 \times [-0.05 - j0.04811]
\]

\[
= 15.94 \angle -136.1^\circ \text{ Amps}
\]

and

\[
I_c = \frac{V_{cn} - V_{Nn}}{Z_c} = \frac{V \angle 120^\circ - V/(2\sqrt{3}) \angle -30^\circ}{30}
\]

\[
= 230 \times [-0.025 + j0.03367]
\]

\[
= 9.64 \angle 126.58^\circ \text{ Amps}
\]

89
It can been seen that \( I_a + I_b + I_c = 0 \). The phase powers are computed as below.

\[
\begin{align*}
S_a &= V_a (I_a)^* = P_a + jQ_a = 230 \times 17.56 \angle -10.81^\circ = 3976.12 - j757.48 \text{ VA} \\
S_b &= V_b (I_b)^* = P_b + jQ_b = 230 \times 15.94 \angle (-120^\circ + 136.1^\circ) = 3522.4 + j1016.69 \text{ VA} \\
S_c &= V_c (I_c)^* = P_c + jQ_c = 230 \times 9.64 \angle (120^\circ - 126.58^\circ) = 2202.59 - j254.06 \text{ VA}
\end{align*}
\]

From the above the total apparent power \( S_V = S_a + S_b + S_c = 9692.11 + j0 \text{ VA} \). Therefore, 
\( S_V = |S_a + S_b + S_c| = 9692.11 \text{ VA} \).

The total arithmetic apparent power \( S_A = |S_a| + |S_b| + |S_c| = 9922.2 \text{ VA} \). Therefore, the arithmetic and vector apparent power factors are given by,

\[
\begin{align*}
p_{fA} &= \frac{P}{S_A} = \frac{9692.11}{9922.2} = 0.9768 \\
p_{fV} &= \frac{P}{S_V} = \frac{9622.11}{9622.11} = 1.00.
\end{align*}
\]

It is interesting to note that although the load in each phase is resistive but each phase has some reactive power. This is due to unbalance of the load currents. This apparently increases the rating of power conductors for given amount of power transfer. It is also to be noted that the net reactive power \( Q = Q_a + Q_b + Q_c = 0 \) leading to the unity vector apparent power factor . However the arithmetic apparent power factor is less than unity showing the effect of the unbalance loads on the power factor.

### 3.4.3 An Alternate Approach to Determine Phase Currents and Powers

In this section, an alternate approach will be discussed to solve phase currents and powers directly without computing the neutral voltage for the system shown in Fig. 3.9(a). First we express three-phase voltage in the following form.

\[
\begin{align*}
\overline{V}_a &= V \angle 0^\circ \\
\overline{V}_b &= V \angle -120^\circ = \alpha^2 V \\
\overline{V}_c &= V \angle 120^\circ = \alpha V
\end{align*}
\]

Where, in above equation, \( \alpha \) is known as complex operator and value of \( \alpha \) and \( \alpha^2 \) are given below.

\[
\begin{align*}
\alpha &= e^{j2\pi/3} = 1\angle 120^\circ = -1/2 + j\sqrt{3}/2 \\
\alpha^2 &= e^{j4\pi/3} = 1\angle 240 = 1\angle -120 = -1/2 - j\sqrt{3}/2
\end{align*}
\]

Also note the following property,

\[
1 + \alpha + \alpha^2 = 0. \tag{3.65}
\]

Using the above, the line to line voltages can be expressed as following.
\[
\begin{align*}
\bar{V}_{ab} &= \bar{V}_a - \bar{V}_b = (1 - \alpha^2)V \\
\bar{V}_{bc} &= \bar{V}_b - \bar{V}_c = (\alpha^2 - \alpha)V \\
\bar{V}_{ca} &= \bar{V}_c - \bar{V}_a = (\alpha - 1)V
\end{align*}
\] (3.66)

Therefore, currents in line \(ab\), \(bc\) and \(ca\) are given as,

\[
\begin{align*}
I_{abl} &= Y_{abl} V_{ab} = Y_{abl} (1 - \alpha^2)V \\
I_{bcl} &= Y_{bcl} V_{bc} = Y_{bcl} (\alpha^2 - \alpha)V \\
I_{cal} &= Y_{cal} V_{ca} = Y_{cal} (\alpha - 1)V
\end{align*}
\] (3.67)

Hence line currents are given as,

\[
\begin{align*}
I_{al} &= I_{abl} - I_{cal} = [Y_{abl} (1 - \alpha^2) - Y_{cal} (\alpha - 1)]V \\
I_{bl} &= I_{bcl} - I_{abl} = [Y_{bcl} (\alpha^2 - \alpha) - Y_{abl} (1 - \alpha^2)]V \\
I_{cl} &= I_{cal} - I_{bcl} = [Y_{cal} (\alpha - 1) - Y_{bcl} (\alpha^2 - \alpha)]V
\end{align*}
\] (3.68)

**Example 3.4** Compute line currents by using above expressions directly for the problem in Example 3.3.

**Solution**: To compute line currents directly from the above expressions, we need to compute \(Y_{abl}^{\prime}\). These are given below

\[
\begin{align*}
Y_{abl}^{\prime} &= \frac{1}{Z_{abl}} = \frac{Z_c}{Z_a^c Z_b^c + Z_a^c Z_c^b Z_a^b Z_c^c} \\
Y_{bcl}^{\prime} &= \frac{1}{Z_{bcl}} = \frac{Z_a^c}{Z_a^c Z_b^c + Z_a^c Z_b^c Z_c^b Z_b^c Z_c^c} \\
Y_{cal}^{\prime} &= \frac{1}{Z_{cal}} = \frac{Z_b^c}{Z_b^c Z_a^c + Z_b^c Z_a^c Z_c^b Z_a^c Z_b^c Z_c^c}
\end{align*}
\] (3.69)

Substituting, \(Z_a^l = R_a = 10\ \Omega\), \(Z_b^l = R_b = 15\ \Omega\) and \(Z_c^l = R_c = 30\ \Omega\) into above equation, we get the following.

\[
\begin{align*}
Y_{abl}^{\prime} &= G_{abl} = \frac{1}{30\ \Omega} \\
Y_{bcl}^{\prime} &= G_{bcl} = \frac{1}{90\ \Omega} \\
Y_{cal}^{\prime} &= G_{cal} = \frac{1}{60\ \Omega}
\end{align*}
\]

Substituting above values of the admittances in (3.68), line currents are computed as below.
\[
T_a = \left[ \frac{1}{30} \left\{ 1 - \left( -\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right\} - \frac{1}{60} \left\{ \left( -\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) - 1 \right\} \right] V
\]
\[
= V \left( 0.075 + j0.0144 \right)
\]
\[
= 0.07637 V\angle10.89^o
\]
\[
= 17.56\angle10.89^o \text{ Amps, for } V=230 \text{ V}
\]

Similarly for Phase-\(b\) current,
\[
T_b = \left[ \frac{1}{90} \left\{ \left( -\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) - \left( -\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right\} - \frac{1}{30} \left\{ 1 - \left( -\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right\} \right] V
\]
\[
= V \left( -0.05 - j0.0481 \right)
\]
\[
= 0.06933 V\angle-136.1^o
\]
\[
= 15.94\angle-136.91^o \text{ Amps, for } V=230 \text{ V}
\]

Similarly for Phase-\(c\) current,
\[
T_c = \left[ \frac{1}{60} \left\{ \left( -\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) - 1 \right\} / \left( -\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right] - \frac{1}{30} \left\{ 1 - \left( -\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right\} \right] V
\]
\[
= V \left( -0.025 + j0.0336 \right)
\]
\[
= 0.04194 V\angle126.58^o
\]
\[
= 9.64\angle126.58^o \text{ Amps, for } V=230 \text{ V}
\]

Thus it is found that the above values are similar to what have been found in previous Example 3.3. The other quantities such as powers and power factors are same.

### 3.4.4 An Example of Balancing an Unbalanced Delta Connected Load

An unbalanced delta connected load is shown in Fig. 3.10(a). As can be seen from the figure that between phase-\(a\) and \(b\) there is admittance \(Y_{ab}^\gamma = G_{ab}^\gamma\) and other two branches are open. This is an example of extreme unbalanced load. Obviously for this load, line currents will be extremely unbalanced. Now we aim to make these line currents to be balanced and in phase with their phase voltages. So, let us assume that we add admittances \(Y_{ab}^\gamma\), \(Y_{bc}^\gamma\) and \(Y_{ca}^\gamma\) between phases \(ab\), \(bc\) and \(ca\) respectively as shown in Fig. 3.10(b) and (c). Let values of compensator susceptances are given by,

\[
Y_{ab}^\gamma = 0
\]
\[
Y_{bc}^\gamma = jG_{ab}^\gamma / \sqrt{3}
\]
\[
Y_{ca}^\gamma = -jG_{ab}^\gamma / \sqrt{3}
\]
Thus total admittances between lines are given by,

\[ Y_{ab} = Y_{i}^{ab} + Y_{\gamma}^{ab} = G_{i}^{ab} + 0 = G_{i}^{ab} \]

\[ Y_{bc} = Y_{i}^{bc} + Y_{\gamma}^{bc} = 0 + jG_{i}^{ab}/\sqrt{3} = jG_{i}^{ab}/\sqrt{3} \]

\[ Y_{ca} = Y_{i}^{ca} + Y_{\gamma}^{ca} = 0 - jG_{i}^{ab}/\sqrt{3} = -jG_{i}^{ab}/\sqrt{3}. \]

Therefore the impedances between load lines are given by,

\[ Z_{ab} = \frac{1}{Y_{ab}} = \frac{1}{G_{i}^{ab}} \]

\[ Z_{bc} = \frac{1}{Y_{bc}} = \frac{-j\sqrt{3}}{G_{i}^{ab}} \]

\[ Z_{ca} = \frac{1}{Y_{ca}} = \frac{j\sqrt{3}}{G_{i}^{ab}} \]

Note that \( Z_{ab} + Z_{bc} + Z_{ca} = 1/G_{i}^{ab} - j\sqrt{3}/G_{i}^{ab} + j\sqrt{3}/G_{i}^{ab} = 1/G_{i}^{ab} \).

The impedances, \( Z_a, Z_b \) and \( Z_c \) of equivalent star connected load are given as follows.

\[ Z_a = \frac{Z_{ab} \times Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \]

\[ = \left( \frac{1}{G_{i}^{ab}} \times \frac{j\sqrt{3}}{G_{i}^{ab}} \right)/\left( \frac{1}{G_{i}^{ab}} \right) \]

\[ = \frac{j\sqrt{3}}{G_{i}^{ab}} \]
\[ Z_b = \frac{Z^{bc} \times Z^{ab}}{Z^{ab} + Z^{bc} + Z^{ca}} \]
\[ = \left( \frac{1}{G_{lb}^{ab}} \times \frac{-j \sqrt{3}}{G_{lb}^{ab}} \right) / \left( \frac{1}{G_{lb}^{ab}} \right) \]
\[ = \frac{-j \sqrt{3}}{G_{lb}^{ab}} \]

\[ Z_c = \frac{Z^{ca} \times Z^{bc}}{Z^{ab} + Z^{bc} + Z^{ca}} \]
\[ = \left( \frac{-j \sqrt{3}}{G_{lb}^{ab}} \times \frac{j \sqrt{3}}{G_{lb}^{ab}} \right) / \left( \frac{1}{G_{lb}^{ab}} \right) \]
\[ = \frac{3}{G_{lb}^{ab}} \]

The above impedances seen from the load side are shown in Fig. 3.11(a) below. Using (3.57),

\[ V_{Nn} = \frac{1}{Z_a + \frac{1}{Z_b} + \frac{1}{Z_c}} \left[ \frac{V_{an}}{Z_a} + \frac{V_{bn}}{Z_b} + \frac{V_{cn}}{Z_c} \right] \]
\[ = \frac{1}{\left( \frac{j \sqrt{3}}{G_{lb}^{ab}} \right) + \left( \frac{1}{G_{lb}^{ab}} \right) + \left( -j \sqrt{3} / G_{lb}^{ab} \right)} \left[ \frac{V \angle 0^\circ}{G_{lb}^{ab}} + \frac{V \angle -120^\circ}{G_{lb}^{ab}} \frac{V \angle 120^\circ}{G_{lb}^{ab}} \right] \]
\[ = 3 V \frac{G_{lb}^{ab}}{G_{lb}^{ab}} \left( \frac{1}{3} - j \frac{\sqrt{3}}{ \sqrt{3}} \right) \]
\[ = 2 V \left( \frac{1}{2} - j \frac{\sqrt{3}}{2} \right) \]
\[ = 2 V \angle -60^\circ \]

Fig. 3.11 Compensated system (a) Load side (b) Source side

The voltage between load and system neutral of delta equivalent star load as shown in Fig. 3.11, is computed as below.
Using above value of neutral voltage the line currents are computed as following.

\[ I_a = \frac{V_{an} - V_{Nn}}{Z_a} \]
\[ = \frac{[V \angle 0^\circ - 2V \angle -60^\circ]}{j\sqrt{3}G_{ab}} \]
\[ = G_{ab}^a V = G_{ab}^a V_a \]

\[ I_b = \frac{V_{bn} - V_{Nn}}{Z_b} \]
\[ = \frac{[V \angle -120^\circ - 2V \angle -60^\circ]}{-j\sqrt{3}G_{ab}} \]
\[ = G_{ab}^b V \angle 240^\circ \]
\[ = G_{ab}^b V_b \]

\[ I_c = \frac{V_{cn} - V_{Nn}}{Z_c} \]
\[ = \frac{[V \angle 120^\circ - 2V \angle -60^\circ]}{3G_{ab}} \]
\[ = G_{ab}^c V \angle 120^\circ \]
\[ = G_{ab}^c V_c \]

From the above example, it is seen that the currents in each phase are balanced and in phase with their respective voltages. This is equivalently shown in Fig. 3.11(b). It is to be mentioned here that the two neutrals in Fig. 3.11 are not same. In Fig. 3.11(b), the neutral \( N \) is same as the system neutral as shown in Fig. 3.8, whereas in Fig. 3.11(a), \( V_{Nn} = 2V \angle -60^\circ \). However the reader may be curious to know why \( Y_{\gamma}^{ab} = 0 \), \( Y_{\gamma}^{bc} = jG_{ab}/\sqrt{3} \) and \( \gamma^{ca} = -jG_{ab}/\sqrt{3} \) have been chosen as compensator admittance values. The answer of the question can be found by going following sections.

### 3.5 A Generalized Approach for Load Compensation using Symmetrical Components

In the previous section, we have expressed line currents \( I_a, I_b \) and \( I_c \), in terms load admittances and the voltage \( V \) for a delta connected unbalanced load as shown in Fig 3.12(a). For the sake of completeness, theses are reproduced below.
\[ T_{al} = T_{abl} - T_{cal} = [Y_{ab}^t(1 - \alpha^2) - Y_{ca}^t(\alpha - 1)]V \]
\[ T_{bl} = T_{bcl} - T_{abl} = [Y_{bc}^t(\alpha^2 - \alpha) - Y_{ab}^t(1 - \alpha^2)]V \]
\[ T_{cl} = T_{cal} - T_{bcl} = [Y_{ca}^t(\alpha - 1) - Y_{bc}^t(\alpha^2 - \alpha)]V \]  

Since loads currents are unbalanced, these will have positive and negative currents. The zero sequence current will be zero as it is three-phase and three-wire system. These symmetrical components of the load currents are expressed as following.

\[
\begin{bmatrix}
I_{0l} \\
I_{1l} \\
I_{2l}
\end{bmatrix} = \frac{1}{\sqrt{3}} 
\begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & a^2 \\
1 & a^2 & \alpha
\end{bmatrix} 
\begin{bmatrix}
I_{al} \\
I_{bl} \\
I_{cl}
\end{bmatrix}
\]

In equation (3.71), a factor of \(1/\sqrt{3}\) is considered to have unitary symmetrical transformation. From the above equation, zero sequence current is given below.

\[ I_{0l} = (I_{al} + I_{bl} + I_{cl}) / \sqrt{3} \]

The positive sequence current is as follows.

\[
T_{1l} = \frac{1}{\sqrt{3}} \left[ I_{al} + \alpha I_{bl} + \alpha^2 I_{cl} \right] \\
= \frac{1}{\sqrt{3}} \left[ Y_{ab}^t(1 - \alpha^2) - Y_{ca}^t(\alpha - 1) + \alpha \left\{ Y_{bc}^t(\alpha^2 - \alpha) - Y_{ab}^t(1 - \alpha^2) \right\} \\
+ \alpha^2 \left\{ Y_{ca}^t(\alpha - 1) - Y_{bc}^t(\alpha - \alpha^2) \right\} \right] V \\
= \frac{1}{\sqrt{3}} \left[ Y_{ab}^t - \alpha^2 Y_{ab}^t + Y_{ca}^t - \alpha Y_{ca}^t + \alpha Y_{bc}^t - \alpha^2 Y_{bc}^t - \alpha Y_{ab}^t - \alpha^3 Y_{ab}^t + \alpha^3 Y_{ca}^t - \alpha^2 Y_{bc}^t \\
- \alpha Y_{bc}^t + \alpha^3 Y_{bc}^t \right] V \\
= (Y_{ab}^t + Y_{bc}^t + Y_{ca}^t) V \sqrt{3} \]
Similarly negative sequence component of the current is,

\[ I_{2l} = \frac{1}{\sqrt{3}} \left[ I_{al} + \alpha^2 I_{bl} + \alpha I_{cl} \right] \]

\[ = \frac{1}{\sqrt{3}} \left[ Y_{ab}^l (1 - \alpha^2) - Y_{ca}^l (\alpha - 1) + \alpha^2 \left\{ Y_{bc}^l (\alpha^2 - \alpha) - Y_{ab}^l (1 - \alpha^2) \right\} \right] \bar{V} \]

\[ + \alpha \left\{ Y_{ca}^l (\alpha - 1) - Y_{bc}^l (\alpha^2 - \alpha) \right\} \bar{V} \]

\[ = \frac{1}{\sqrt{3}} \left[ Y_{ab}^l - \alpha^2 Y_{bc}^l - \alpha Y_{ca}^l + Y_{ca}^l + \alpha^2 Y_{bc}^l + \alpha^4 Y_{bc}^l - \alpha Y_{bc}^l \right] \bar{V} \]

\[ + \alpha^4 Y_{ab}^l + \alpha^2 Y_{ca}^l - Y_{ca}^l + \alpha Y_{ca}^l + \alpha^2 Y_{bc}^l \]

\[ = \frac{1}{\sqrt{3}} \left[ -3\alpha^2 Y_{ab}^l - 3Y_{bc}^l - 3\alpha Y_{ca}^l \right] \bar{V} \]

\[ = -[\alpha^2 Y_{ab}^l + Y_{bc}^l + \alpha Y_{ca}^l] \sqrt{3} \bar{V} \]

From the above, it can be written that,

\[ I_{0l} = 0 \]

\[ I_{1l} = (Y_{ab}^l + Y_{bc}^l + Y_{ca}^l) \sqrt{3} \bar{V} \quad (3.72) \]

\[ I_{2l} = -\left(\alpha^2 Y_{ab}^l + Y_{bc}^l + \alpha Y_{ca}^l \right) \sqrt{3} \bar{V} \]

When compensator is used, three delta branches \( Y_{ab}^\gamma, Y_{bc}^\gamma \) and \( Y_{ca}^\gamma \) are added as shown in Fig. 3.12(b). Using above analysis, the sequence components of the compensator currents can be given as below.

\[ I_{0\gamma} = 0 \]

\[ I_{1\gamma} = (Y_{ab}^\gamma + Y_{bc}^\gamma + Y_{ca}^\gamma) \sqrt{3} V \quad (3.73) \]

\[ I_{2\gamma} = -\left(\alpha^2 Y_{ab}^\gamma + Y_{bc}^\gamma + \alpha Y_{ca}^\gamma \right) \sqrt{3} V \]

Since, compensator currents are purely reactive, i.e., \( G_{ab}^\gamma = G_{bc}^\gamma = G_{ca}^\gamma = 0 \),

\[ Y_{ab}^\gamma = G_{ab}^\gamma + jB_{ab}^\gamma = j B_{ab}^\gamma \]

\[ Y_{bc}^\gamma = G_{bc}^\gamma + jB_{bc}^\gamma = j B_{bc}^\gamma \]

\[ Y_{ca}^\gamma = G_{ca}^\gamma + jB_{ca}^\gamma = j B_{ca}^\gamma \quad (3.74) \]

Using above, the compensated sequence currents can be written as,

\[ I_{0\gamma} = 0 \]

\[ I_{1\gamma} = j \left( B_{ab}^\gamma + B_{bc}^\gamma + B_{ca}^\gamma \right) \sqrt{3} V \quad (3.75) \]

\[ I_{2\gamma} = -j (\alpha^2 B_{ab}^\gamma + B_{bc}^\gamma + \alpha B_{ca}^\gamma) \sqrt{3} V \]

Knowing nature of compensator and load currents, we can set compensation objectives as following.

1. All negative sequence component of the load current must be supplied from the compensator negative current, i.e.,

\[ \bar{I}_{2l} = -\bar{I}_{2\gamma} \quad (3.76) \]
The above further implies that,

\[ \text{Re} \left( I_{2l} \right) + j \text{Im} \left( I_{2l} \right) = -\text{Re} \left( I_{2\gamma} \right) - j \text{Im} \left( I_{2\gamma} \right) \quad (3.77) \]

2. The total positive sequence current, which is source current should have desired power factor from the source, i.e.,

\[ \frac{\text{Im} \left( I_{1l} + I_{1\gamma} \right)}{\text{Re} \left( I_{1l} + I_{1\gamma} \right)} = \tan \phi = \beta \quad (3.78) \]

Where, \( \phi \) is the desired phase angle between the line currents and the supply voltages. The above equation thus implies that,

\[ \text{Im} \left( I_{1l} + I_{1\gamma} \right) = \beta \text{Re} \left( I_{1l} + I_{1\gamma} \right) \quad (3.79) \]

Since \( \text{Re} \left( I_{1\gamma} \right) = 0 \), the above equation is rewritten as following.

\[ \text{Im} \left( I_{1l} \right) - \beta \text{Re} \left( I_{1l} \right) = -\text{Im} \left( I_{1\gamma} \right) \quad (3.80) \]

The equation (3.77) gives two conditions and equation (3.79) gives one condition. There are three unknown variables, i.e., \( B_{ab}^{\gamma} \), \( B_{bc}^{\gamma} \) and \( B_{ca}^{\gamma} \) and three conditions. Therefore the unknown variables can be solved. This is described in the following section. Using (3.75), the current \( I_{2\gamma} \) is expressed as following.

\[ I_{2\gamma} = -j [\alpha^2 B_{ab}^{\gamma} + B_{bc}^{\gamma} + \alpha B_{ca}^{\gamma}] \sqrt{3} V \]

\[ = -j \left[ \left( -\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) B_{ab}^{\gamma} + B_{bc}^{\gamma} + \left( -\frac{1}{2} + j \frac{\sqrt{3}}{2} \right) B_{ca}^{\gamma} \right] \sqrt{3} V \]

\[ = \left[ \left( -\frac{\sqrt{3}}{2} B_{ab}^{\gamma} + \frac{\sqrt{3}}{2} B_{ca}^{\gamma} \right) - j \left( -\frac{1}{2} B_{ab}^{\gamma} + B_{bc}^{\gamma} - \frac{1}{2} B_{ca}^{\gamma} \right) \right] \sqrt{3} V \quad (3.81) \]

Thus the above equation implies that

\[ \left( \frac{\sqrt{3}}{2} B_{ab}^{\gamma} - \frac{\sqrt{3}}{2} B_{ca}^{\gamma} \right) = -\frac{1}{\sqrt{3}V} \text{Re} \left( I_{2\gamma} \right) = \frac{1}{\sqrt{3}V} \text{Re} \left( I_{2l} \right) \quad (3.82) \]

and,

\[ \left( -\frac{1}{2} B_{ab}^{\gamma} + B_{bc}^{\gamma} - \frac{1}{2} B_{ca}^{\gamma} \right) = -\frac{1}{\sqrt{3}V} \text{Im} \left( I_{2\gamma} \right) = \frac{1}{\sqrt{3}V} \text{Im} \left( I_{2l} \right) \quad (3.83) \]

Or

\[ (-B_{ab}^{\gamma} + 2 B_{bc}^{\gamma} - B_{ca}^{\gamma}) = \frac{1}{\sqrt{3}V} 2 \text{Im} \left( I_{2l} \right) \quad (3.84) \]

98
From (3.75), Im\( (\bar{T}_{1\gamma}) \) can be written as,

\[
\text{Im} (\bar{T}_{1\gamma}) = (B_{\gamma}^{ab} + B_{\gamma}^{bc} + B_{\gamma}^{ca}) \sqrt{3} V.
\] (3.85)

Substituting \( \text{Im} (\bar{T}_{1\gamma}) \) from above equation into (3.85), we get the following.

\[
-(B_{\gamma}^{ab} + B_{\gamma}^{bc} + B_{\gamma}^{ca}) = -\frac{1}{\sqrt{3}V} \text{Im} (\bar{T}_{1\gamma}) = \frac{1}{\sqrt{3}V} \{ \text{Im} (\bar{T}_{1\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma}) \}
\] (3.86)

Subtracting (3.86) from (??), the following is obtained,

\[
B_{\gamma}^{bc} = \frac{-1}{3\sqrt{3}V} [\text{Im} (\bar{T}_{1\gamma}) - 2 \text{Im} (\bar{T}_{2\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma})].
\] (3.87)

Now, from (3.82) we have

\[
-\frac{1}{2} B_{\gamma}^{ab} - \frac{1}{2} B_{\gamma}^{ca} = \frac{1}{\sqrt{3}V} \text{Im} (\bar{T}_{2\gamma}) - B_{\gamma}^{bc}
\]

\[
= \frac{1}{\sqrt{3}V} \text{Im} (\bar{T}_{2\gamma}) - \left[ \frac{-1}{3\sqrt{3}V} \{ \text{Im} (\bar{T}_{1\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma}) - 2 \text{Im} (\bar{T}_{2\gamma}) \} \right]
\]

\[
= \frac{1}{3\sqrt{3}V} \{ \text{Im} (\bar{T}_{2\gamma}) + \text{Im} (\bar{T}_{1\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma}) \}
\] (3.88)

Reconsidering (3.88) and (3.83), we have

\[
-\frac{1}{2} B_{\gamma}^{ab} - \frac{1}{2} B_{\gamma}^{ca} = \frac{1}{3\sqrt{3}V} \{ \text{Im} (\bar{T}_{1\gamma}) + \text{Im} (\bar{T}_{2\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma}) \}
\]

\[
\frac{1}{2} B_{\gamma}^{ab} - \frac{1}{2} B_{\gamma}^{ca} = \frac{1}{3\sqrt{3}V} [\sqrt{3} \text{Re} (\bar{T}_{2\gamma})]
\]

Adding above equations, we get

\[
B_{\gamma}^{ca} = \frac{-1}{3\sqrt{3}V} [\text{Im} (\bar{T}_{1\gamma}) + \text{Im} (\bar{T}_{2\gamma}) + \sqrt{3} \text{Re} (\bar{T}_{2\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma})].
\] (3.89)

Therefore,

\[
B_{\gamma}^{ab} = B_{\gamma}^{ca} + \frac{2}{3\sqrt{3}V} [\sqrt{3} \text{Re} (\bar{T}_{2\gamma})]
\]

\[
= \frac{-1}{3\sqrt{3}V} [\text{Im} (\bar{T}_{1\gamma}) + \text{Im} (\bar{T}_{2\gamma}) + \sqrt{3} \text{Re} (\bar{T}_{2\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma}) - 2 \sqrt{3} \text{Re} (\bar{T}_{2\gamma})]
\]

\[
= \frac{-1}{3\sqrt{3}V} [\text{Im} (\bar{T}_{1\gamma}) + \text{Im} (\bar{T}_{2\gamma}) - \sqrt{3} \text{Re} (\bar{T}_{2\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma})]
\] (3.90)

Similarly, \( B_{\gamma}^{bc} \) can be written as in the following.

\[
B_{\gamma}^{bc} = -\frac{1}{3\sqrt{3}V} [\text{Im} (\bar{T}_{1\gamma}) - 2\text{Im} (\bar{T}_{2\gamma}) - \beta \text{Re} (\bar{T}_{1\gamma})]
\] (3.91)

\[
(3.92)
\]
From the above, the compensator susceptances in terms of real and imaginary parts of the load current can be written as following.

\[
B_{\gamma}^{ab} = \frac{-1}{3 \sqrt{3V}} [\text{Im}(I_{ul}) + \text{Im}(I_{2l}) - \sqrt{3} \text{Re}(I_{2l}) - \beta \text{Re}(I_{ul})]
\]

\[
B_{\gamma}^{bc} = \frac{-1}{3 \sqrt{3V}} [\text{Im}(I_{ul}) - 2\text{Im}(I_{2l}) - \beta \text{Re}(I_{ul})]
\]

\[
B_{\gamma}^{ca} = \frac{-1}{3 \sqrt{3V}} [\text{Im}(I_{ul}) + \text{Im}(I_{2l}) + \sqrt{3} \text{Re}(I_{2l}) - \beta \text{Re}(I_{ul})]
\]

In the above equation, the susceptances of the compensator are expressed in terms of real and imaginary parts of symmetrical components of load currents. It is however advantageous to express these susceptances in terms of instantaneous values of voltages and currents from implementation point of view. The first step to achieve this is to express these susceptances in terms of instantaneous values of voltages and currents from implementation point of view. The first step to achieve this is to express these susceptances in terms of load currents, i.e., \(I_{al}, I_{bl}\) and \(I_{cl}\), which is described below. Using equation (3.71), the sequence components of the load currents are expressed as,

\[
I_{0l} = \frac{1}{\sqrt{3}} [I_{al} + I_{bl} + I_{cl}]
\]

\[
I_{1l} = \frac{1}{\sqrt{3}} [I_{al} + \alpha I_{bl} + \alpha^2 I_{cl}]
\]

\[
I_{2l} = \frac{1}{\sqrt{3}} [I_{al} + \alpha^2 I_{bl} + \alpha I_{cl}].
\]

Substituting these values of sequence components of load currents, in (3.93), we can obtain compensator susceptances in terms of real and imaginary components of the load currents. Let us start from the \(B_{\gamma}^{bc}\), as obtained following.

\[
B_{\gamma}^{bc} = \frac{-1}{3 \sqrt{3V}} [\text{Im}(I_{ul}) - 2\text{Im}(I_{2l}) - \beta \text{Re}(I_{ul})]
\]

\[
= \frac{-1}{3 \sqrt{3V}} \left[ \text{Im} \left( \frac{I_{al} + \alpha I_{bl} + \alpha^2 I_{cl}}{\sqrt{3}} \right) - 2 \text{Im} \left( \frac{I_{al} + \alpha^2 I_{bl} + \alpha I_{cl}}{\sqrt{3}} \right) - \beta \text{Re} \left( \frac{I_{al} + \alpha I_{bl} + \alpha^2 I_{cl}}{\sqrt{3}} \right) \right]
\]

\[
= \frac{-1}{9V} \left[ \text{Im} \left\{ (-I_{al} + (2 + 3\alpha) I_{bl} + (2 + 3\alpha^2) I_{cl}) - \beta \text{Re} \left( I_{al} + \alpha I_{bl} + \alpha^2 I_{cl} \right) \right\} \right]
\]

\[
= \frac{-1}{9V} \left[ \text{Im} \left\{ -I_{al} + 2I_{bl} + 2I_{cl} + 3\alpha I_{bl} + 3\alpha^2 I_{cl} \right\} - \beta \text{Re} \left( I_{al} + \alpha I_{bl} + \alpha^2 I_{cl} \right) \right]
\]

By adding and subtracting \(I_{bl}\) and \(I_{cl}\) in the above equation we get,

\[
B_{\gamma}^{bc} = \frac{-1}{9V} \left[ \text{Im} \left\{ (-I_{al} - I_{bl} - I_{cl}) + 3I_{bl} + 3I_{cl} + 3\alpha I_{bl} + \alpha^2 I_{cl} \right\} - \beta \text{Re} \left( I_{al} + \alpha I_{bl} + \alpha^2 I_{cl} \right) \right]
\]

We know that \(I_{al} + I_{bl} + I_{cl} = 0\), therefore \(I_{al} + I_{bl} = -I_{cl}\).

\[
B_{\gamma}^{bc} = \frac{-1}{3V} \left[ -\text{Im} \left( I_{al} \right) + \text{Im} \left( \alpha I_{bl} \right) + \text{Im} \left( \alpha^2 I_{cl} \right) - \frac{\beta}{3} \text{Re} \left( I_{al} + \alpha I_{bl} + \alpha^2 I_{cl} \right) \right] \] \hspace{1cm} (3.95)
Similarly, it can be proved that,

\[
B_{\gamma}^{ca} = -\frac{1}{3V} \left[ \text{Im}\left(\mathcal{I}_{al}\right) - \text{Im}\left(\alpha \mathcal{I}_{bl}\right) + \text{Im}\left(\alpha^2 \mathcal{I}_{cl}\right) - \frac{\beta}{3} \text{Re}\left(\mathcal{I}_{al} + \alpha \mathcal{I}_{bl} + \alpha^2 \mathcal{I}_{cl}\right) \right] \quad (3.96)
\]

\[
B_{\gamma}^{ab} = -\frac{1}{3V} \left[ \text{Im}\left(\mathcal{I}_{al}\right) + \text{Im}\left(\alpha \mathcal{I}_{bl}\right) - \text{Im}\left(\alpha^2 \mathcal{I}_{cl}\right) - \frac{\beta}{3} \text{Re}\left(\mathcal{I}_{al} + \alpha \mathcal{I}_{bl} + \alpha^2 \mathcal{I}_{cl}\right) \right] \quad (3.97)
\]

The above expressions for \(B_{\gamma}^{ca}\) and \(B_{\gamma}^{ab}\) are proved below. For convenience, the last term associated with \(\beta\) is not considered. For the sake simplicity in equations (3.96) and (3.97) are proved to those given in equations (3.93).

\[
B_{\gamma}^{ca} = -\frac{1}{3V} \left[ \text{Im}\left(\mathcal{I}_{al}\right) - \text{Im}\left(\alpha \mathcal{I}_{bl}\right) + \text{Im}\left(\alpha^2 \mathcal{I}_{cl}\right) \right]
= -\frac{1}{3V} \left[ \text{Im}\left\{\frac{(\mathcal{I}_{0l} + \mathcal{I}_{1l} + \mathcal{I}_{2l})}{\sqrt{3}} - \alpha \frac{(\mathcal{I}_{0l} + \alpha^2 \mathcal{I}_{1l} + \alpha \mathcal{I}_{2l})}{\sqrt{3}} + \alpha^2 \frac{(\mathcal{I}_{0l} + \alpha \mathcal{I}_{1l} + \alpha^2 \mathcal{I}_{2l})}{\sqrt{3}} \right\} \right]
\]

Since \(\mathcal{I}_{0l}=0\)

\[
B_{\gamma}^{ca} = -\frac{1}{3\sqrt{3}V} \text{Im}\left[\mathcal{I}_{1l} - 2\alpha^2 \mathcal{I}_{2l}\right]
= -\frac{1}{3\sqrt{3}V} \text{Im}\left[\mathcal{I}_{1l} - 2\left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right) \mathcal{I}_{2l}\right]
= -\frac{1}{3\sqrt{3}V} \text{Im}\left[\mathcal{I}_{1l} + \mathcal{I}_{2l} + j\sqrt{3} \mathcal{I}_{2l}\right]
= -\frac{1}{3\sqrt{3}V} \left[ \text{Im}\left(\mathcal{I}_{1l}\right) + \text{Im}\left(\mathcal{I}_{2l}\right) + \sqrt{3} \text{Re}\left(\mathcal{I}_{2l}\right) \right]
\]

Note that \(\text{Im}(j \mathcal{I}_{2l}) = \text{Re}(\mathcal{I}_{2l})\). Adding \(\beta\) term, we get the following.

\[
B_{\gamma}^{ca} = -\frac{1}{3\sqrt{3}V} \left[ \text{Im}(\mathcal{I}_{1l}) + \text{Im}(\mathcal{I}_{2l}) + \sqrt{3} \text{Re}(\mathcal{I}_{2l}) - \beta \text{Re}(\mathcal{I}_{1l}) \right]
\]

Similarly,

\[
B_{\gamma}^{ab} = -\frac{1}{3V} \left[ \text{Im}\left(\mathcal{I}_{al}\right) + \text{Im}\left(\alpha \mathcal{I}_{bl}\right) - \text{Im}\left(\alpha^2 \mathcal{I}_{cl}\right) \right]
= -\frac{1}{3V} \left[ \text{Im}\left\{\frac{(\mathcal{I}_{0l} + \mathcal{I}_{1l} + \mathcal{I}_{2l})}{\sqrt{3}} + \alpha \frac{(\mathcal{I}_{0l} + \alpha^2 \mathcal{I}_{1l} + \alpha \mathcal{I}_{2l})}{\sqrt{3}} - \alpha^2 \frac{(\mathcal{I}_{0l} + \alpha \mathcal{I}_{1l} + \alpha^2 \mathcal{I}_{2l})}{\sqrt{3}} \right\} \right]
\]

101
\[
B_{\gamma}^{ab} = -\frac{1}{3\sqrt{3}V} \text{Im} \left[ \bar{T}_{1l} - 2\alpha \bar{T}_{2l} \right] \\
\quad = -\frac{1}{3\sqrt{3}V} \text{Im} \left[ \bar{T}_{1l} - 2 \left( -\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \bar{T}_{2l} \right] \\
\quad = -\frac{1}{3\sqrt{3}V} \text{Im} \left[ \bar{T}_{1l} + \bar{T}_{2l} - j\sqrt{3}\bar{T}_{2l} \right] \\
\quad = -\frac{1}{3\sqrt{3}V} \left[ \text{Im} (\bar{T}_{1l}) + \text{Im} (\bar{T}_{2l}) - \sqrt{3} \text{Re} (\bar{T}_{2l}) \right]
\]

Thus, Compensator susceptances are expressed as following.

\[
B_{\gamma}^{ab} = -\frac{1}{3V} \left[ \text{Im} (\bar{T}_{1l}) + \text{Im} (\alpha\bar{T}_{bl}) - \text{Im} (\alpha^2\bar{T}_{cl}) - \frac{\beta}{3} \text{Re} (\bar{T}_{al} + \alpha\bar{T}_{bl} + \alpha^2\bar{T}_{cl}) \right] \\
B_{\gamma}^{bc} = -\frac{1}{3V} \left[ -\text{Im} (\bar{T}_{al}) + \text{Im} (\alpha\bar{T}_{bl}) + \text{Im} (\alpha^2\bar{T}_{cl}) - \frac{\beta}{3} \text{Re} (\bar{T}_{al} + \alpha\bar{T}_{bl} + \alpha^2\bar{T}_{cl}) \right] \\
B_{\gamma}^{ca} = -\frac{1}{3V} \left[ \text{Im} (\bar{T}_{al}) - \text{Im} (\alpha\bar{T}_{bl}) + \text{Im} (\alpha^2\bar{T}_{cl}) - \frac{\beta}{3} \text{Re} (\bar{T}_{al} + \alpha\bar{T}_{bl} + \alpha^2\bar{T}_{cl}) \right]
\]

(3.98)

An unity power factor is desired from the source. For this \( \cos \phi_l = 1 \), implying \( \tan \phi_l = 0 \) hence \( \beta = 0 \). Thus we have,

\[
B_{\gamma}^{ab} = -\frac{1}{3V} \left[ \text{Im} (\bar{T}_{1l}) + \text{Im} (\alpha\bar{T}_{bl}) - \text{Im} (\alpha^2\bar{T}_{cl}) \right] \\
B_{\gamma}^{bc} = -\frac{1}{3V} \left[ -\text{Im} (\bar{T}_{al}) + \text{Im} (\alpha\bar{T}_{bl}) + \text{Im} (\alpha^2\bar{T}_{cl}) \right] \\
B_{\gamma}^{ca} = -\frac{1}{3V} \left[ \text{Im} (\bar{T}_{al}) - \text{Im} (\alpha\bar{T}_{bl}) + \text{Im} (\alpha^2\bar{T}_{cl}) \right]
\]

(3.99)

The above equations are easy to realize in order to find compensator susceptances. As mentioned above, sampling and averaging techniques will be used to convert above equation into their time equivalents. These are described below.

### 3.5.1 Sampling Method

Each current phasor in above equation can be expressed as,

\[
\bar{T}_{al} = \text{Re} (\bar{T}_{al}) + j \text{Im} (\bar{T}_{al}) = I_{al,R} + j I_{al,X}
\]

(3.100)

An instantaneous phase current is written as follows.

\[
i_{al}(t) = \sqrt{2} \text{Im} (\bar{T}_{al} e^{j\omega t}) \\
\quad = \sqrt{2} \text{Im} \left[ (I_{al,R} + j I_{al,X}) e^{j\omega t} \right] \\
\quad = \sqrt{2} \text{Im} \left[ (I_{al,R} + j I_{al,X})(\cos \omega t + j \sin \omega t) \right] \\
\quad = \sqrt{2} \text{Im} \left[ (I_{al,R} \cos \omega t - I_{al,X} \sin \omega t) + j(I_{al,R} \sin \omega t + I_{al,X} \cos \omega t) \right] \\
\quad = \sqrt{2} \left[(I_{al,R} \sin \omega t + I_{al,X} \cos \omega t)\right]
\]

(3.101)
\[
\text{Im} (\bar{I}_{at}) = \bar{T}_{at,X} = \frac{i_{at}(t)}{\sqrt{2}} \text{ at } \sin \omega t = 0, \cos \omega t = 1
\] (3.102)

From equation (3.63), the phase voltages can be expressed as below.

\[
\begin{align*}
v_a(t) &= \sqrt{2}V \sin \omega t \\
v_b(t) &= \sqrt{2}V \sin(\omega t - 120^\circ) \\
v_c(t) &= \sqrt{2}V \sin(\omega t + 120^\circ)
\end{align*}
\] (3.103)

From above voltage expressions, it is to be noted that, \( \sin \omega t = 0, \cos \omega t = 1 \) implies that the phase-\( a \) voltage, \( v_a(t) \) is going through a positive zero crossing, hence, \( v_a(t) = 0 \) and \( \frac{dv_a}{dt} = 0 \). Therefore, equation (3.102), can be expressed as following.

\[
\bar{T}_{at} = \frac{i_{at}(t)}{\sqrt{2}} \text{ when, } v_a(t) = 0, \frac{dv_a}{dt} > 0
\] (3.104)

Similarly,

\[
\bar{T}_{b,l} = I_{bl,R} + jI_{bl,X}
\] (3.105)

Therefore,

\[
\alpha (\bar{T}_{b,l}) = \alpha (I_{bl,R} + jI_{bl,X}) \\
= \left( -\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) (I_{bl,R} + jI_{bl,X}) \\
= \left( -\frac{1}{2}I_{bl,R} - \frac{\sqrt{3}}{2}I_{bl,X} \right) + j \left( \frac{\sqrt{3}}{2}I_{bl,R} - \frac{1}{2}I_{bl,X} \right)
\] (3.106)

From the above,

\[
\text{Im} \{ \alpha (I_{bl}) \} = \frac{\sqrt{3}}{2}I_{bl,R} - \frac{1}{2}I_{bl,X}
\] (3.107)
Similar to equation (3.101), we can express phase-\(b\) current in terms \(\text{Im}(\alpha I_{bl})\), as given below.

\[
i_{bl}(t) = \sqrt{2} \text{Im}(\alpha I_{bl} e^{j\omega t})
\]

\[
= \sqrt{2} \text{Im}(\alpha I_{bl} e^{j\omega t} \alpha^{-1})
\]

\[
= \sqrt{2} \text{Im}(\alpha I_{bl} e^{j(\omega t - 120^\circ)})
\]

\[
= \sqrt{2} \text{Im}\left[\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(I_{bl,R} + jI_{bl,X}) e^{j(\omega t - 120^\circ)}\right]
\]

\[
= \sqrt{2} \text{Im}\left[\left(-\frac{1}{2} I_{bl,R} - \frac{\sqrt{3}}{2} I_{bl,X}\right) + j\left(\frac{\sqrt{3}}{2} I_{bl,R} - \frac{1}{2} I_{bl,X}\right)\right]
\]

\[
\left\{(\cos(\omega t - 120^\circ) + j\sin(\omega t - 120^\circ))\right\}
\]

\[
= \sqrt{2}\left[\left(-\frac{1}{2} I_{bl,R} - \frac{\sqrt{3}}{2} I_{bl,X}\right)\sin(\omega t - 120^\circ) + \left(\frac{\sqrt{3}}{2} I_{bl,R} - \frac{1}{2} I_{bl,X}\right)\cos(\omega t - 120^\circ)\right]
\]

From the above equation, we get the following.

\[
\text{Im}(\alpha I_{bl}) = \frac{i_{bl}(t)}{\sqrt{2}} \quad \text{when, } v_{b}(t) = 0, \frac{dv_{b}}{dt} > 0 \quad (3.109)
\]

Similarly for phase-\(c\), it can proved that,

\[
\text{Im}(\alpha^2 I_{cl}) = \frac{i_{cl}(t)}{\sqrt{2}} \quad \text{when, } v_{c}(t) = 0, \frac{dv_{c}}{dt} > 0 \quad (3.110)
\]

Substituting \(\text{Im}(I_{al})\), \(\text{Im}(\alpha I_{bl})\) and \(\text{Im}(\alpha^2 I_{cl})\) from (3.104), (3.109) and (3.110) respectively, in (3.99), we get the following.

\[
B_{abr} = -\frac{1}{3\sqrt{2}V} \left[ i_{a}|_{(v_{a}=0, \frac{dv_{a}}{dt} > 0)} + i_{b}|_{(v_{b}=0, \frac{dv_{b}}{dt} > 0)} - i_{c}|_{(v_{c}=0, \frac{dv_{c}}{dt} > 0)} \right]
\]

\[
B_{brc} = -\frac{1}{3\sqrt{2}V} \left[ -i_{a}|_{(v_{a}=0, \frac{dv_{a}}{dt} > 0)} + i_{b}|_{(v_{b}=0, \frac{dv_{b}}{dt} > 0)} + i_{c}|_{(v_{c}=0, \frac{dv_{c}}{dt} > 0)} \right] \quad (3.111)
\]

\[
B_{car} = -\frac{1}{3\sqrt{2}V} \left[ i_{a}|_{(v_{a}=0, \frac{dv_{a}}{dt} > 0)} - i_{b}|_{(v_{b}=0, \frac{dv_{b}}{dt} > 0)} + i_{c}|_{(v_{c}=0, \frac{dv_{c}}{dt} > 0)} \right]
\]

Thus the desired compensating susceptances are expressed in terms of the three line currents sampled at instants defined by positive-going zero crossings of the line-neutral voltages \(v_{a}, v_{b}, v_{c}\). An artificial neutral at ground potential may be created measuring voltages \(v_{a}, v_{b}\) and \(v_{c}\) to implement above algorithm.

### 3.5.2 Averaging Method

In this method, we express the compensator susceptances in terms of real and reactive power terms and finally expressed them in time domain through averaging process. The method is described
From equation (3.99), susceptance, $B_{ab}$, can be re-written as following.

$$B_{ab} = -\frac{1}{3V} \left[ \text{Im} (\mathcal{I}_{al}) + \text{Im} (\alpha \mathcal{I}_{bl}) - \text{Im} (\alpha^2 \mathcal{I}_{cl}) \right]$$

$$= -\frac{1}{3V^2} \left[ V \text{Im} (\mathcal{I}_{al}) + V \text{Im} (\alpha \mathcal{I}_{bl}) - V \text{Im} (\alpha^2 \mathcal{I}_{cl}) \right]$$

$$= -\frac{1}{3V^2} \left[ \text{Im} (V \mathcal{I}_{al}) + \text{Im} (V \alpha \mathcal{I}_{bl}) - \text{Im} (V \alpha^2 \mathcal{I}_{cl}) \right]$$

(3.112)

Note the following property of phasors and applying it for the simplification of the above expression.

$$\text{Im} (V \mathcal{I}) = -\text{Im} (V^* \mathcal{I}^*)$$

(3.113)

Using above equation (3.112) can be written as,

$$B_{ab} = \frac{1}{3V^2} \left[ \text{Im} (V \mathcal{I}_{al})^* + \text{Im} (\alpha \mathcal{I}_{bl})^* - \text{Im} (\alpha^2 \mathcal{I}_{cl})^* \right]$$

$$= \frac{1}{3V^2} \left[ \text{Im} (V^* \mathcal{I}_{al}) + \text{Im} (\alpha^* \mathcal{I}_{bl})^* - \text{Im} ((\alpha^2)^* \mathcal{I}_{cl})^* \right]$$

$$= \frac{1}{3V^2} \left[ \text{Im} (V^* \mathcal{I}_{al}) + \text{Im} (\alpha^2 V^* \mathcal{I}_{bl}) - \text{Im} (\alpha V^* \mathcal{I}_{cl}) \right]$$

Since $\mathcal{V}_a = V \angle 0^\circ$ is a reference phasor, therefore $\mathcal{V}_a = \mathcal{V}_a^* = V$, $\alpha^2 \mathcal{V}_a = \mathcal{V}_b^*$ and $\alpha \mathcal{V}_a = \mathcal{V}_c^*$.

Using this, the above equation can be written as following.

$$B_{ab} = \frac{1}{3V^2} \left[ \text{Im} (\mathcal{V}_a T_{al}^*) + \text{Im} (\mathcal{V}_b T_{bl}^*) - \text{Im} (\mathcal{V}_c T_{cl}^*) \right]$$

Similarly,

$$B_{bc} = \frac{1}{3V^2} \left[ -\text{Im} (\mathcal{V}_a T_{al}^*) + \text{Im} (\mathcal{V}_b T_{bl}^*) + \text{Im} (\mathcal{V}_c T_{cl}^*) \right]$$

$$B_{ca} = \frac{1}{3V^2} \left[ \text{Im} (\mathcal{V}_a T_{al}^*) - \text{Im} (\mathcal{V}_b T_{bl}^*) + \text{Im} (\mathcal{V}_c T_{cl}^*) \right]$$

(3.114)

It can be further proved that,

$$\text{Im} (\mathcal{V}_a T_{al}^*) = \frac{1}{T} \int_0^T v_a(t) \angle (-\pi/2) i_{al}(t) \, dt$$

$$\text{Im} (\mathcal{V}_b T_{bl}^*) = \frac{1}{T} \int_0^T v_b(t) \angle (-\pi/2) i_{bl}(t) \, dt$$

$$\text{Im} (\mathcal{V}_c T_{cl}^*) = \frac{1}{T} \int_0^T v_c(t) \angle (-\pi/2) i_{cl}(t) \, dt$$

(3.115)
In (3.115), the term \( v_a(t) \angle (-\pi/2) \) denotes the voltage \( v_a(t) \) shifted by \(-\pi/2\) radian in time domain. For balanced voltages, the following relationship between phase and line voltages are true.

\[
\begin{align*}
v_a(t) \angle (-\pi/2) &= v_{bc}(t)/\sqrt{3} \\
v_b(t) \angle (-\pi/2) &= v_{ca}(t)/\sqrt{3} \\
v_c(t) \angle (-\pi/2) &= v_{ca}(t)/\sqrt{3}
\end{align*}
\]

(3.116)

From (3.115) and (3.116), the following can be written.

\[
\begin{align*}
\text{Im}(V_a T_a^*) &= \frac{1}{\sqrt{3} T} \int_0^T v_{bc}(t) i_{al}(t) \, dt \\
\text{Im}(V_b T_b^*) &= \frac{1}{\sqrt{3} T} \int_0^T v_{ca}(t) i_{bl}(t) \, dt \\
\text{Im}(V_c T_c^*) &= \frac{1}{\sqrt{3} T} \int_0^T v_{ab}(t) i_{cl}(t) \, dt
\end{align*}
\]

(3.117)

Substituting above values of \( \text{Im}(V_a T_a^*) \), \( \text{Im}(V_b T_b^*) \) and \( \text{Im}(V_c T_c^*) \) into (3.114), we get the following.

\[
\begin{align*}
B_{ab}^\gamma &= -\frac{1}{3 \sqrt{3} V^2} \frac{1}{T} \int_0^T (v_{bc} i_{al} + v_{ca} i_{bl} - v_{ab} i_{cl}) \, dt \\
B_{bc}^\gamma &= -\frac{1}{3 \sqrt{3} V^2} \frac{1}{T} \int_0^T (-v_{bc} i_{al} + v_{ca} i_{bl} + v_{ab} i_{cl}) \, dt \\
B_{ca}^\gamma &= -\frac{1}{3 \sqrt{3} V^2} \frac{1}{T} \int_0^T (v_{bc} i_{al} - v_{ca} i_{bl} + v_{ab} i_{cl}) \, dt
\end{align*}
\]

(3.118)

The above equations can directly be used to know the the compensator susceptances by performing the averaging on line the product of the line to line voltages and phase load currents. The term \( \int_0^T = \int_{t_1}^{t_1+T} \) can be implemented using moving average of one cycle. This improves transient response by computing average value at each instant. But in this case the controller response which changes the susceptance value, should match to that of the above computing algorithm.

### 3.6 Compensator Admittance Represented as Positive and Negative Sequence Admittance Network

Recalling the following relations from equation (3.93) for unity power factor operation i.e. \( \beta = 0 \), we get the following.

\[
\begin{align*}
B_{ab}^\gamma &= -\frac{1}{3 \sqrt{3} V^2} [\text{Im}(\bar{T}_{11}) + \text{Im}(\bar{T}_{21}) - \sqrt{3} \text{Re}(\bar{T}_{21})] \\
B_{bc}^\gamma &= -\frac{1}{3 \sqrt{3} V^2} [\text{Im}(\bar{T}_{11}) - \beta \text{Re}(\bar{T}_{11}) - 2\text{Im}(\bar{T}_{21})] \\
B_{ca}^\gamma &= -\frac{1}{3 \sqrt{3} V^2} [\text{Im}(\bar{T}_{11}) + \text{Im}(\bar{T}_{21}) + \sqrt{3} \text{Re}(\bar{T}_{21})]
\end{align*}
\]

(3.119)
From these equations, it is evident the the first terms form the positive sequence suceptance as they involve $\bar{I}_{1l}$ terms. Similarly, the second and third terms in above equation form negative sequence susceptance of the compensator, as these involve $\bar{I}_{2l}$ terms. Thus, we can write,

\[
B^{ab}_{\gamma} = B^{ab}_{\gamma 1} + B^{ab}_{\gamma 2} \\
B^{ab}_{\gamma 1} = B^{ab}_{\gamma 1} + B^{ab}_{\gamma 2} \\
B^{ab}_{\gamma 2} = B^{ab}_{\gamma 1} + B^{ab}_{\gamma 2}
\]  

(3.120)

Therefore,

\[
B^{ab}_{\gamma 1} = B^{bc}_{\gamma 1} = B^{ca}_{\gamma 1} = -\frac{1}{3\sqrt{3}V} \left[ \text{Im}(\bar{I}_{1l}) \right]
\]  

(3.121)

And,

\[
B^{ab}_{\gamma 2} = -\frac{1}{3\sqrt{3}V} \left( \text{Im}(\bar{I}_{2l}) - \sqrt{3} \text{Re}(\bar{I}_{2l}) \right) \\
B^{bc}_{\gamma 2} = -\frac{1}{3\sqrt{3}V} \left( -2 \text{Im}(\bar{I}_{2l}) \right) \\
B^{ca}_{\gamma 2} = -\frac{1}{3\sqrt{3}V} \left( \text{Im}(\bar{I}_{2l}) + \sqrt{3} \text{Re}(\bar{I}_{2l}) \right)
\]  

(3.122)

Earlier in equation, (3.123), it was established that,

\[
\bar{I}_{0l} = 0 \\
\bar{I}_{1l} = (Y_{l}^{ab} + Y_{l}^{bc} + Y_{l}^{ca}) \sqrt{3}V \\
\bar{I}_{2l} = -(\alpha Y_{l}^{ab} + Y_{l}^{bc} + \alpha Y_{l}^{ca}) \sqrt{3}V
\]

Noting that,

\[
Y_{l}^{ab} = G_{l}^{ab} + jB_{l}^{ab} \\
Y_{l}^{bc} = G_{l}^{bc} + jB_{l}^{bc} \\
Y_{l}^{ca} = G_{l}^{ca} + jB_{l}^{ca}
\]

Therefore,

\[
\text{Im}(\bar{I}_{1l}) = \text{Im} \left( (Y_{l}^{ab} + Y_{l}^{bc} + Y_{l}^{ca}) \sqrt{3}V \right) = (B_{l}^{ab} + B_{l}^{bc} + B_{l}^{ca}) \sqrt{3}V
\]  

(3.123)

Thus equation (3.121) is re-written as following.

\[
B^{ab}_{\gamma 1} = B^{bc}_{\gamma 1} = B^{ca}_{\gamma 1} = -\frac{1}{3} \left( B_{l}^{ab} + B_{l}^{bc} + B_{l}^{ca} \right)
\]  

(3.124)
Now we shall compute $B_{\gamma_2}^{ab}, B_{\gamma_2}^{bc}$ and $B_{\gamma_2}^{ca}$ using equations (3.122) as following. Knowing that,

\[
\overline{I}_{2l} = - \left( \alpha^2 Y_i^{ab} + Y_i^{bc} + \alpha Y_i^{ca} \right) \sqrt{3} V
\]

\[
= - \left[ \left( -\frac{1}{2} - \frac{j\sqrt{3}}{2} \right) \left( G_i^{ab} + jB_i^{ab} \right) + \left( G_i^{bc} + jB_i^{bc} \right) + \left( \frac{1}{2} + \frac{j\sqrt{3}}{2} \right) \left( G_i^{ca} + jB_i^{ca} \right) \right] \sqrt{3} V
\]

\[
= - \left[ \left( -\frac{G_i^{ab}}{2} + \frac{\sqrt{3}}{2} B_i^{ab} + G_i^{bc} - \frac{G_i^{ca}}{2} - \frac{3}{2} B_i^{ca} - j \left( \frac{\sqrt{3}}{2} G_i^{ab} + B_i^{ab} - B_i^{bc} - \frac{\sqrt{3}}{2} G_i^{ca} + \frac{B_i^{ca}}{2} \right) \right] \sqrt{3} V
\]

\[
= \left[ \frac{G_i^{ab}}{2} - \frac{\sqrt{3}}{2} B_i^{ab} - G_i^{bc} + \frac{G_i^{ca}}{2} + \frac{\sqrt{3}}{2} B_i^{ca} + j \left( \frac{\sqrt{3}}{2} G_i^{ab} + \frac{B_i^{ab}}{2} - B_i^{bc} - \frac{3}{2} G_i^{ca} + \frac{B_i^{ca}}{2} \right) \right] \sqrt{3} V
\]

(3.125)

The above implies that,

\[
\text{Im} \left( \overline{I}_{2l} \right) = \left( \frac{\sqrt{3}}{2} G_i^{ab} + \frac{B_i^{ab}}{2} - B_i^{bc} - \frac{\sqrt{3}}{2} G_i^{ca} + \frac{B_i^{ca}}{2} \right) \sqrt{3} V
\]

\[
- \sqrt{3} \text{Re} \left( \overline{I}_{2l} \right) = \left( -\frac{\sqrt{3}}{2} G_i^{ab} + \frac{3}{2} B_i^{ab} + \sqrt{3} G_i^{bc} - \frac{\sqrt{3}}{2} G_i^{ca} - \frac{3}{2} B_i^{ca} \right) \sqrt{3} V
\]

Thus, $B_{\gamma_2}^{ab}$ can be given as,

\[
B_{\gamma_2}^{ab} = - \frac{1}{3 \sqrt{3} V} \left[ 2B_i^{ab} - B_i^{bc} - B_i^{ca} + \sqrt{3} G_i^{bc} - \sqrt{3} G_i^{ca} \right] \sqrt{3} V
\]

\[
= - \frac{1}{3} \left[ 2B_i^{ab} - B_i^{bc} - B_i^{ca} + \sqrt{3} \left( G_i^{bc} - G_i^{ca} \right) \right]
\]

\[
= \frac{1}{\sqrt{3}} \left( G_i^{ca} - G_i^{bc} \right) + \frac{1}{3} \left( B_i^{bc} + B_i^{ca} - 2B_i^{ab} \right) \quad (3.126)
\]

Similarly,

\[
B_{\gamma_2}^{ca} = - \frac{1}{3 \sqrt{3} V} \left[ \text{Im} \left( \overline{I}_{2l} \right) + \sqrt{3} \text{Re} \left( \overline{I}_{2l} \right) \right]
\]

\[
= - \frac{1}{3 \sqrt{3} V} \left[ \frac{\sqrt{3}}{2} G_i^{ab} + \frac{B_i^{ab}}{2} - B_i^{bc} - \sqrt{3} G_i^{bc} + \frac{B_i^{ca}}{2} + \sqrt{3} G_i^{ab} - \frac{3}{2} B_i^{ab} - \sqrt{3} G_i^{bc} + \frac{3}{2} G_i^{ca} + \frac{3}{2} B_i^{ca} \right] \sqrt{3} V
\]

\[
= - \frac{1}{3} \left[ \sqrt{3} G_i^{ab} - \sqrt{3} G_i^{bc} - B_i^{ab} - B_i^{bc} + 2B_i^{ca} \right]
\]

\[
= \frac{1}{3 \sqrt{3}} \left( G_i^{bc} - G_i^{ab} \right) + \frac{1}{3} \left( B_i^{ab} + B_i^{bc} - 2B_i^{ca} \right) \quad (3.127)
\]

And, $B_{\gamma_2}^{bc}$ is computed as below.
\[ B_{\gamma_2}^{bc} = -\frac{1}{3\sqrt{3}V} \left[ -2\text{Im}(\bar{I}_2) \right] \]
\[ = \frac{2}{3\sqrt{3}V} \left( \frac{\sqrt{3}}{2} G_i^{ab} + \frac{B_i^{ab}}{2} - B_i^{bc} - \frac{\sqrt{3}}{2} G_i^{ca} + \frac{B_i^{ca}}{2} \right) V \sqrt{3} \]
\[ = \frac{1}{\sqrt{3}} \left( G_i^{ab} - G_i^{ca} \right) + \frac{1}{3} \left( B_i^{ab} + B_i^{ca} - 2B_i^{bc} \right) \]

Using (3.121), (3.126)-(3.128), We therefore can find the overall compensator susceptances as following.

\[ B_{\gamma}^{ab} = B_{\gamma_1}^{ab} + B_{\gamma_2}^{ab} \]
\[ = -\frac{1}{3} \left( B_{\gamma}^{ab} + B_{\gamma}^{bc} + B_{\gamma}^{ca} \right) + \frac{1}{\sqrt{3}} \left( G_i^{ab} - G_i^{bc} \right) + \frac{1}{3} \left( B_i^{ab} + B_i^{bc} - 2B_i^{ab} \right) \]
\[ = -B_i^{ab} + \frac{1}{\sqrt{3}} \left( G_i^{ab} - G_i^{bc} \right) \]

Similarly,

\[ B_{\gamma}^{bc} = B_{\gamma_1}^{bc} + B_{\gamma_2}^{bc} \]
\[ = -\frac{1}{3} \left( B_{\gamma}^{ab} + B_{\gamma}^{bc} + B_{\gamma}^{ca} \right) + \frac{1}{\sqrt{3}} \left( G_i^{bc} - G_i^{ab} \right) + \frac{1}{3} \left( B_i^{bc} + B_i^{ca} - 2B_i^{bc} \right) \]
\[ = -B_i^{bc} + \frac{1}{\sqrt{3}} \left( G_i^{bc} - G_i^{ab} \right) \]

And,

\[ B_{\gamma}^{ca} = B_{\gamma_1}^{ca} + B_{\gamma_2}^{ca} \]
\[ = -\frac{1}{3} \left( B_{\gamma}^{ab} + B_{\gamma}^{bc} + B_{\gamma}^{ca} \right) + \frac{1}{\sqrt{3}} \left( G_i^{bc} - G_i^{ca} \right) + \frac{1}{3} \left( B_i^{bc} + B_i^{ab} - 2B_i^{ca} \right) \]
\[ = -B_i^{ca} + \frac{1}{\sqrt{3}} \left( G_i^{bc} - G_i^{ca} \right) \]

Thus, the compensator susceptances in terms of load parameters are given as in the following.

\[ B_{\gamma}^{ab} = -B_i^{ab} + \frac{1}{\sqrt{3}} \left( G_i^{ca} - G_i^{bc} \right) \]
\[ B_{\gamma}^{bc} = -B_i^{bc} + \frac{1}{\sqrt{3}} \left( G_i^{ab} - G_i^{ca} \right) \]
\[ B_{\gamma}^{ca} = -B_i^{ca} + \frac{1}{\sqrt{3}} \left( G_i^{bc} - G_i^{ab} \right) \]

(3.129)
It is interesting to observe above equations. The first parts of the equation nullifies the effect of the load susceptances and the second parts of the equations correspond to the unbalance in resistive load. The two terms together make source current balanced and in phase with the supply voltages. The compensator’s positive and negative sequence networks are shown in Fig. 3.13.

What happens if we just use the following values of the compensator susceptances as given below?

\[
\begin{align*}
B_{ab}^\gamma &= -B_{ab}^l \\
B_{be}^\gamma &= -B_{bc}^l \\
B_{ca}^\gamma &= -B_{ca}^l
\end{align*}
\] (3.130)

In above case, load suceptance parts of the admittance are fully compensated. However the source currents after compensation remain unbalanced due to unbalance conductance parts of the load.

![Sequence networks of the compensator](image)

**Example 3.5** For a delta connected load shown in 3.14, the load admittances are given as following,

\[
\begin{align*}
Y_{ab}^r &= G_{ab}^r + jB_{ab}^r \\
Y_{bc}^r &= G_{bc}^r + jB_{bc}^r \\
Y_{ca}^r &= G_{ca}^r + jB_{ca}^r
\end{align*}
\]

Given the load parameters:

\[
\begin{align*}
Z_{ab}^l &= 1/Y_{ab}^r = 5 + j12 \Omega \\
Z_{bc}^l &= 1/Y_{bc}^r = 3 + j4 \Omega \\
Z_{ca}^l &= 1/Y_{ca}^r = 9 - j12 \Omega
\end{align*}
\]

Determine compensator susceptances \((B_{ab}^\gamma, B_{be}^\gamma, B_{ca}^\gamma)\) so that the supply sees the load as balanced and unity power factor. Also find the line currents and source active and reactive powers before and after compensation.
Solution:

\( Z_{ab}^l = 5 + j12 \Omega \Rightarrow Y_{ab}^l = 0.03 - j0.0710 \Omega \)
\( Z_{bc}^l = 3 + j4 \Omega \Rightarrow Y_{bc}^l = 0.12 - j0.16 \Omega \)
\( Z_{ca}^l = 9 - j13 \Omega \Rightarrow Y_{ca}^l = 0.04 + j0.0533 \Omega \)

Once we know the admittances we know,

\[
\begin{align*}
G_{ab}^l &= 0.03, \quad B_{ab}^l = -0.0710 \\
G_{bc}^l &= 0.12, \quad B_{bc}^l = -0.16 \\
G_{ca}^l &= 0.04, \quad B_{ca}^l = 0.0533
\end{align*}
\]

\[
\begin{align*}
B_{\gamma}^{ab} &= -B_{ab}^l + \frac{1}{\sqrt{3}}(G_{ca}^l - G_{bc}^l) = 0.0248 \Omega \\
B_{\gamma}^{bc} &= -B_{bc}^l + \frac{1}{\sqrt{3}}(G_{ab}^l - G_{ca}^l) = 0.1540 \Omega \\
B_{\gamma}^{ca} &= -B_{ca}^l + \frac{1}{\sqrt{3}}(G_{bc}^l - G_{ab}^l) = -0.0011 \Omega
\end{align*}
\]

Total admittances are:

\[
\begin{align*}
Y_{\gamma}^{ab} &= Y_{\gamma}^{ab} + Y_{\gamma}^{ab} = 0.03 - j0.0462 \Omega \\
Y_{\gamma}^{bc} &= Y_{\gamma}^{bc} + Y_{\gamma}^{bc} = 0.12 - j0.006 \Omega \\
Y_{\gamma}^{ca} &= Y_{\gamma}^{ca} + Y_{\gamma}^{ca} = 0.04 + j0.0522 \Omega
\end{align*}
\]

Knowing these total admittances, we can find line currents using following expressions.

**Current Before Compensation**
\( T_a = \overline{T}_{ab} - \overline{T}_{ca} = [(1 - \alpha^2)Y_{ab} - (\alpha - 1)Y_{ca}] \quad V = 0.2150 V \angle -9.51^\circ A \)
\( T_b = \overline{T}_{bc} - \overline{T}_{ab} = [(\alpha^2 - \alpha)Y_{bc} - (1 - \alpha^2)Y_{ab}] \quad V = 0.4035 V \angle -161.66^\circ A \)
\( T_c = \overline{T}_{ca} - \overline{T}_{bc} = [(\alpha - 1)Y_{ca} - (\alpha^2 - \alpha)Y_{bc}] \quad V = 0.2358 V \angle 43.54^\circ A \)

**Powers Before Compensation**

\[
\overline{S}_a = \overline{V}_a (\overline{T}_{al})^* = P_a + jQ_a = V (0.2121 + j0.0355)
\]
\[
\overline{S}_b = \overline{V}_b (\overline{T}_{bl})^* = P_b + jQ_b = V (0.3014 + j0.2682)
\]
\[
\overline{S}_c = \overline{V}_c (\overline{T}_{cl})^* = P_c + jQ_c = V (0.0552 + j0.2293)
\]

Total real power, \( P = P_a + P_b + P_c = V 0.5688 W \)

Total reactive power, \( Q = Q_a + Q_b + Q_c = V 0.5330 VAr \)

Power factor in phase-a, \( \rho_{fa} = \cos \phi_a = \cos(9.51^\circ) = 0.9863 \) lag

Power factor in phase-b, \( \rho_{fb} = \cos \phi_b = \cos(41.63^\circ) = 0.7471 \) lag

Power factor in phase-c, \( \rho_{fc} = \cos \phi_c = \cos(76.45^\circ) = 0.2334 \) lag

Thus we observe that the phases draw reactive power from the lines and currents are unbalanced in magnitude and phase angles.

**After Compensation**

\( T_a = \overline{T}_{ab} - \overline{T}_{ca} = [(1 - \alpha^2)Y_{ab} - (\alpha - 1)Y_{ca}] \quad V = 0.1896 V \angle 0^\circ A \)
\( T_b = \overline{T}_{bc} - \overline{T}_{ab} = [(\alpha^2 - \alpha)Y_{bc} - (1 - \alpha^2)Y_{ab}] \quad V = 0.1896 V \angle -120^\circ A \)
\( T_c = \overline{T}_{ca} - \overline{T}_{bc} = [(\alpha - 1)Y_{ca} - (\alpha^2 - \alpha)Y_{bc}] \quad V = 0.1896 V \angle 120^\circ A \)

**Powers After Compensation**

\[
\overline{S}_a = \overline{V}_a (\overline{T}_{al})^* = P_a + jQ_a = V (0.1986 + j0.0)
\]
\[
\overline{S}_b = \overline{V}_b (\overline{T}_{bl})^* = P_b + jQ_b = V (0.1986 + j0.0)
\]
\[
\overline{S}_c = \overline{V}_c (\overline{T}_{cl})^* = P_c + jQ_c = V (0.1986 + j0.0)
\]

Total real power, \( P = P_a + P_b + P_c = (V \times 0.5688) W \)

Total reactive power, \( Q = Q_a + Q_b + Q_c = 0 VAr \)

Power factor in phase-a, \( \rho_{fa} = \cos \phi_a = \cos(0^\circ) = 1.0 \)

Power factor in phase-b, \( \rho_{fb} = \cos \phi_b = \cos(0^\circ) = 1.0 \)

Power factor in phase-c, \( \rho_{fc} = \cos \phi_c = \cos(0^\circ) = 1.0 \)

From above results we observe that after placing compensator of suitable values as calculated above, the line currents become balanced and have unity power factor relationship with their voltages.

112
Example 3.6 Consider the following 3-phase, 3-wire system. The 3-phase voltages are balanced sinusoids with RMS value of 230 V at 50 Hz. The load impedances are $Z_a = 3 + j 4 \, \Omega$, $Z_b = 5 + j 12 \, \Omega$, $Z_c = 12 - j 5 \, \Omega$. Compute the following.

1. The line currents $I_{la}$, $I_{lb}$, $I_{lc}$.

2. The active ($P$) and reactive ($Q$) powers of each phase.

3. The compensator susceptance ($B_{\gamma a}$, $B_{\gamma b}$, $B_{\gamma c}$), so that the supply sees the load balanced and unity power factor.

4. For case (3), compute the source, load, compensator active and reactive powers (after compensation).

![Diagram](Fig. 3.15 An unbalanced three-phase three-wire star connected load)

**Solution:**

Given that $Z_a = 3 + j 4 \, \Omega$, $Z_b = 5 + j 12 \, \Omega$, $Z_c = 12 - j 5 \, \Omega$.

1. **Line currents** $I_{la}$, $I_{lb}$, and $I_{lc}$ are found by first computing neutral voltage as given below.

   \[
   V_{nN} = \frac{1}{\left(\frac{1}{Z_a} + \frac{1}{Z_b} + \frac{1}{Z_c}\right)}(\frac{V_a}{Z_a} + \frac{V_b}{Z_b} + \frac{V_c}{Z_c})
   \]

   \[
   = \left(\frac{Z_aZ_bZ_c}{Z_aZ_b + Z_bZ_c + Z_cZ_a}\right)(\frac{V_a}{Z_a} + \frac{V_b}{Z_b} + \frac{V_c}{Z_c})
   \]

   \[
   = Z_{abc}\left(\frac{V_a}{Z_a} + \frac{V_b}{Z_b} + \frac{V_c}{Z_c}\right)
   \]

   \[
   = \frac{50 \angle 97.82^\circ}{252.41 \angle 55.5^\circ}(24.12 \angle -99.55^\circ)
   \]

   \[
   = 43.79 - j 67.04 \, V
   \]

   \[
   = 80.75 \angle -57.15^\circ \, V
   \]

Now the line currents are computed as below.

113
\[ I_{la} = \frac{V_a - V_{nN}}{Z_a} \]
\[ = \frac{230\angle 0^\circ - 80.75\angle -57.15^\circ}{3 + j 4} \]
\[ = 33.2 - j 21.65 \]
\[ = 39.63\angle -33.11^\circ A \]

\[ I_{lb} = \frac{V_b - V_{nN}}{Z_b} \]
\[ = \frac{230\angle -120^\circ - 80.75\angle -57.15^\circ}{5 + j 12} \]
\[ = 15.85\angle 152.21^\circ A \]

\[ I_{lc} = \frac{V_c - V_{nN}}{Z_c} \]
\[ = \frac{230\angle 120^\circ - 80.75\angle -57.15^\circ}{12 - j 5} \]
\[ = 23.89\angle 143.35^\circ A \]

2. **Active and Reactive Powers**

For phase a

\[ P_a = V_a I_a \cos \phi_a = 230 \times 39.63 \times \cos(33.11^\circ) = 7635.9 \text{ W} \]
\[ Q_a = V_a I_a \sin \phi_a = 230 \times 39.63 \times \sin(33.11^\circ) = 4980 \text{ VAR} \]

For phase b

\[ P_b = V_b I_b \cos \phi_b = 230 \times 15.85 \times \cos(-152.21^\circ - 120^\circ) = 140.92 \text{ W} \]
\[ Q_b = V_b I_b \sin \phi_b = 230 \times 15.85 \times \sin(-152.21^\circ - 120^\circ) = 3643.2 \text{ VAR} \]

For phase c

\[ P_c = V_c I_c \cos \phi_c = 230 \times 23.89 \times \cos(-143.35^\circ + 120^\circ) = 5046.7 \text{ W} \]
\[ Q_c = V_c I_c \sin \phi_c = 230 \times 23.89 \times \sin(-143.35^\circ + 120^\circ) = -2179.3 \text{ VAr} \]

**Total three phase powers**

\[ P = P_a + P_b + P_c = 12823 \text{ W} \]

\[ Q = Q_a + Q_b + Q_c = 6443.8 \text{ VAr} \]

**3. Compensator Susceptance**

First we convert star connected load to a delta load as given below.

\[ Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c} = 4 + j \, 19 = 19.42 \angle 77.11^\circ \Omega \]

\[ Z_{bc} = \frac{\Delta Z}{Z_a} = 50.44 + j \, 2.08 = 50.42 \angle 2.36^\circ \Omega \]

\[ Z_{ca} = \frac{\Delta Z}{Z_b} = 19.0 - j \, 4.0 = 19.42 \angle -11.89^\circ \Omega \]

The above implies that,

\[ Y_{ab} = 1/Z_{ab} = G_{ab}^o + j \, B_{ab} = 0.0106 - j \, 0.050 \Omega \]

\[ Y_{bc} = 1/Z_{bc} = G_{bc}^o + j \, B_{bc} = 0.0198 - j \, 0.0008 \Omega \]

\[ Y_{ca} = 1/Z_{ca} = G_{ca}^o + j \, B_{ca} = 0.0504 + j \, 0.0106 \Omega \]

From the above, the compensator susceptances are computed as following.

\[ B_{ab} = -B_{ab}^o + \frac{(G_{ca} - G_{bc})}{\sqrt{3}} = 0.0681 \Omega \]

\[ B_{bc} = -B_{bc}^o + \frac{(G_{ab} - G_{ca})}{\sqrt{3}} = -0.0222 \Omega \]

\[ B_{ca} = -B_{ca}^o + \frac{(G_{bc} - G_{ab})}{\sqrt{3}} = -0.0053 \Omega \]

\[ Z_{ab}^o = -j \, 14.69 \Omega \text{ (capacitance)} \]

\[ Z_{bc} = j \, 45.13 \Omega \text{ (inductance)} \]

\[ Z_{ca} = j \, 188.36 \Omega \text{ (inductance)} \]
4. After Compensation

\[ Z_{i}^{ab} = Z_{i}^{ab} \parallel Z_{i}^{ab} = 24.97 - j 41.59 = 48.52\angle -59.01^{\circ} \Omega \]

\[ Z_{i}^{bc} = Z_{i}^{bc} \parallel Z_{i}^{bc} = 21.52 - j 24.98 = 32.97\angle 49.25^{\circ} \Omega \]

\[ Z_{i}^{ca} = Z_{i}^{ca} \parallel Z_{i}^{ca} = 24.97 - j 41.59 = 19.7332\angle -6.0^{\circ} \Omega \]

Let us convert delta connected impedances to star connected.

\[ Z_{a} = \frac{Z_{ab} \times Z_{ca'}}{Z_{ab} + Z_{bc} + Z_{ca'}} \]
\[ = \frac{9.0947 - j 10.55}{13.93\angle -49.25^{\circ} \Omega} \]
\[ = 9.47\angle 59.01^{\circ} \Omega \]

\[ Z_{b} = \frac{Z_{bc} \times Z_{ab'}}{Z_{ab} + Z_{bc} + Z_{ca'}} \]
\[ = \frac{23.15 + j 2.43}{23.28\angle 6.06^{\circ} \Omega} \]
\[ = 4.8755 + j 8.12 \]

\[ Z_{c} = \frac{Z_{ca} \times Z_{bc'}}{Z_{ab} + Z_{bc} + Z_{ca'}} \]
\[ = \frac{18.58\angle 0^{\circ} A}{18.58\angle 120^{\circ} A} \]

The new voltage between the load and system neutral after compensation is given by,

\[ V_{nN}' = \frac{1}{1/Z_{a} + \frac{1}{Z_{b}} + \frac{1}{Z_{c}}} (V_{a}/Z_{a} + V_{b}/Z_{b} + V_{c}/Z_{c}) \]
\[ = 205.42\angle -57.15^{\circ} \text{V} \]

Based on the above, the line currents are computed as following.

\[ I_{a} = \frac{V_{a} - V_{nN}'}{Z_{a}} \]
\[ = 18.58\angle 0^{\circ} \text{A} \]

\[ I_{b} = \frac{V_{b} - V_{nN}'}{Z_{b}} \]
\[ = 18.58\angle -120^{\circ} \text{A} \]

\[ I_{c} = \frac{V_{c} - V_{nN}'}{Z_{c}} \]
\[ = 18.58\angle 120^{\circ} \text{A} \]
Thus, it is seen that after compensation, the source currents are balanced and have unity power factor with respective supply voltages.

Source powers after compensation

\[ P_a = P_b = P_c = 230 \times 18.58 = 4272.14 \text{ W} \]

\[ P = 3P_a = 12820.2 \text{ W} \]

\[ Q_a = Q_b = Q_c = 0 \]

\[ Q = 0 \text{ VAr} \]

Compensator powers

\[ S_{\gamma}^{ab} = V_{ab} (T_{\gamma}^{ab*}) \]
\[ = V_{ab} (V_{ab}^* Y_{\gamma}^{ab*}) \]
\[ = V_{ab}^2 Y_{\gamma}^{ab*} \]
\[ = (230 \times \sqrt{3})^2 \times (-j 0.0068) \]
\[ = -j 10802 \text{ VA} \]

\[ S_{\gamma}^{bc} = V_{bc}^2 Y_{\gamma}^{bc*} \]
\[ = (230 \times \sqrt{3})^2 \times (j 0.0222) \]
\[ = j 3516 \text{ VA} \]

\[ S_{\gamma}^{ca} = V_{ca}^2 Y_{\gamma}^{ca*} \]
\[ = (230 \times \sqrt{3})^2 \times (j 0.0053) \]
\[ = j 842 \text{ VA} \]

References


