Lesson 16

Power Factor Improvement, Harmonic Reduction, Filter
Instructional Objectives

Study of the following:

- Schemes for the improvement of power factor in AC-DC converters.
- Methods for harmonic reduction in the current waveforms of the converters.
- Types of filters used to obtain ripple free (dc) output voltage and currents, reducing the harmonics.

2.8.1 Introduction

After the discussion of various types of ac to dc converters (rectifiers), both single- and three-phase, in the lessons (#2.1-2.6) of this module (#2), the drop in the output voltage due to the commutation overlap in the converter, was presented, the inductance on the source (ac) side being taken into account, in the previous lesson (#2.7).

In this (last) lesson (#2.8), three important points – power factor improvement, harmonic reduction, and filters, as applicable to converters, are described. The three schemes for power factor improvement are discussed. Then, the use of various filters to reduce the harmonics in the output voltage and current waveforms, are presented. Lastly, the harmonic reduction techniques are taken up, in brief. In all these cases, the circuit of a single phase full wave half (semi) controlled bridge converter (ac-dc) is used mostly as an example.

Power Factor Improvement

For phase-controlled operation in both single phase full wave half and full controlled bridge converters as discussed in this module (#2), the displacement factor (or power factor, which is lagging) decreases, as the average value of output voltage (V_{dc}) decreases, with the increase in firing angle delay, \( \alpha \). This is also applicable for both three phase half wave and full wave (bridge) converters. The three schemes used for power factor (pf) improvement are:

- Extinction angle control
- Symmetrical angle control
- Pulse width modulation (PWM) control

Extinction Angle Control

The circuit diagram of a single phase full wave half-controlled (semi) force-commutated bridge converter is shown in Fig. 16.1(a). The thyristors, T_1 & T_2, are replaced by the switches, self-commutated devices, such as power transistor or equivalent. The power transistor is turned on by applying a signal at the base, and turned off by withdrawing the signal at the base. A gate turn-off thyristor (GTO) also may be used, in which case, it may be turned off by applying a short negative pulse to its gate, but is turned on by a short positive pulse, like a thyristor.

In extinction angle control, switch, S_1 is turned on at \( \omega t = 0 \), and then turned off by forced commutation at \( \omega t = (\pi - \beta) \). The switch, S_2 is turned on at \( \omega t = \pi \), and then turned off at \( \omega t = (2\pi - \beta) \). The output voltage is controlled by varying the extinction angle, \( \beta \). Fig. 16.1(b) shows the waveforms for input voltage, output voltage, input current, and the current through thyristor switches. The fundamental component of input current leads the input voltage, and the
displacement factor (and power factor) is leading. This feature may be desirable to simulate a capacitive load, thus compensating the line voltage drops.
The average output voltage is
\[ V_{dc} = \frac{2}{2\pi} \int_0^{\pi-\beta} \sqrt{2} \sin \omega t \, d(\omega t) = \frac{\sqrt{2}}{\pi} \cdot V(1 + \cos \beta) \]

The value of \( V_{dc} \) is varied from \((2\sqrt{2}/\pi)\)V to 0, as \( \beta \) varies from 0 to \( \pi \).

The rms value of output voltage is
\[ V_o = \left[ \frac{2}{2\pi} \int_0^{\pi-\beta} 2V^2 \sin^2 \omega t \, d(\omega t) \right]^{\frac{1}{2}} = \sqrt{\frac{1}{\pi} \left( (\pi-\beta) + \frac{1}{2} \sin 2\beta \right)} \]

Here also, \( V_o \) varies from \( V \) to 0.

This scheme of extinction angle control can also be used for single phase full wave full controlled bridge converter with four switches, instead of two needed in the earlier case. The students are requested to study this matter from textbooks, but details are not included here.
Symmetrical Angle Control

This control can be applied for the same half-controlled force commutated bridge converter with two switches, $S_1$ and $S_2$ as shown in Fig. 16.1(a). The switch, $S_1$ is turned on at $\omega t = (\pi - \beta)/2$ and then turned off at $\omega t = (\pi + \beta)/2$. The other switch, $S_2$ is turned on at $\omega t = (3\pi - \beta)/2$ and then turned off at $\omega t = (3\pi + \beta)/2$. The output voltage is varied by varying conduction angle, $\beta$. The gate signals are generated by comparing half-sine waves with a dc signal as shown in Fig. 16.2(b). The half-sine waves can be obtained using a full wave diode (uncontrolled) bridge converter. The gate signals can also be generated by comparing triangular waves with a dc signal as shown in Fig. 16.2(c). In the second case, the conduction angle varies linearly with the dc signals, but in inverse ratio, i.e., when the dc signal is zero, full conduction ($\beta = \pi$) takes place, and the dc signal being same as the peak of the triangular reference signal, no conduction ($\beta = 0$) takes place. Fig. 16.2(a) shows the waveforms for input voltage, output voltage, input current and the current through the switches. The fundamental component of input current is in phase with input voltage, and the displacement factor is unity (1.0). Therefore, the power factor is improved.
\( v_s = V_m \sin \omega t \)

Fig. 16.2 Symmetrical angle control.
The average output voltage is

\[ V_{dc} = \frac{2}{\pi} \int_{(\pi-\beta)/2}^{(\pi+\beta)/2} \sqrt{2} \sin \omega t \, d(\omega t) = \left( \frac{2\sqrt{2}}{\pi} \sin \left( \frac{\beta}{2} \right) \right) \]

The value of \( V_{dc} \) varies from \( (2\sqrt{2}\pi) \) to 0 as \( \beta \) varies from \( \pi \) to 0.

The rms value of output voltage is

\[ V_o = \left[ \frac{2}{2\pi} \int_{(\pi-\beta)/2}^{(\pi+\beta)/2} 2V^2 \sin^2 (\omega t) \, d(\omega t) \right]^{1/2} = V \left[ \frac{1}{\pi} (\beta + \sin \beta) \right]^{1/2} \]

**Pulse Width Modulation (PWM) Control**

If the output voltage of single phase half-controlled converter is controlled by delay angle, extinction angle or symmetrical, there is only one pulse per half cycle in the input current of the converter, and as a result, the lowest order harmonic is third. It is difficult to filter out the lower order harmonic current. In Pulse Width Modulation (PWM) control, the converter switches are turned on and off several times during a half cycle, and the output voltage is controlled by varying the width of pulses. The gate signals are generated by comparing a triangular wave with a dc signal as shown in Fig. 16.3c. In this case, all the pulse widths obtained are equal. Fig. 16.3a shows the input voltage, output voltage, and input current. The lowest order harmonic can be eliminated or reduced by selecting the number of pulses per half cycle. However, increasing the number of pulses would also increase the magnitude of higher order harmonics, which could easily be filtered out. The earlier case of symmetrical angle control can be considered as single pulse PWM. For more details of PWM methods used, the students are requested to study the two lessons (#5.4-5.5) in module 5 (DC-AC converter, or inverter).
The details of output voltage and current waveforms of the converter are given. The output voltage (i.e., performance parameters) can be obtained in two steps: (i) by considering only one pair of pulses such that, if one pulse starts at $\omega t = \alpha$, and ends at $\omega t = \alpha + \delta$, the other pulse starts at $\omega t = \pi + \alpha$, and ends at $\omega t = (\pi + \alpha + \delta)$, and (2) then by combining the effects of all pairs of pulse.

If the $m$th pulse starts at $\omega t = \alpha_m$ and its width is $\delta_m$, the average output voltage due to $p$ number of pulses is found as

$$V_{dc} = \sum_{m=1}^{p} \left[ \frac{2}{\pi} \int_{\alpha_m}^{\alpha_m + \delta_m} \sqrt{2V} \sin \omega_t \text{d}(\omega t) \right] = \sqrt{2V} \sum_{m=1}^{p} \left[ \cos \alpha_m - \cos (\alpha_m + \delta_m) \right]$$

If the load current with an average value of $I_a$ is continuous and has negligible ripple, the instantaneous input current is expressed in a Fourier series as

$$i_s(t) = I_{dc} + \sum_{n=1,3,5,...}^{\infty} \left( a_n \cos nt + b_n \sin nt \right)$$

Due to symmetry of the input current waveform, even harmonics are absent, and $I_{dc}$ is zero. The Fourier coefficients are obtained as

$$a_n = \frac{1}{\pi} \int_{0}^{2\pi} i_s(t) \cos nt \text{d}(\omega t)$$

$$= \sum_{n=1}^{p} \left[ \frac{1}{\pi} \int_{\alpha_m}^{\alpha_m + \delta_m} I_a \cos nt \text{d}(\omega t) - \frac{1}{\pi} \int_{\pi + \alpha_m}^{\pi + \alpha_m + \delta_m} I_a \cos nt \text{d}(\omega t) \right] = 0$$

$$b_n = \frac{1}{\pi} \int_{0}^{2\pi} i_s(t) \sin nt \text{d}(\omega t)$$

$$= \sum_{n=1}^{p} \left[ \frac{1}{\pi} \int_{\alpha_m}^{\alpha_m + \delta_m} I_a \sin nt \text{d}(\omega t) - \frac{1}{\pi} \int_{\pi + \alpha_m}^{\pi + \alpha_m + \delta_m} I_a \sin nt \text{d}(\omega t) \right] - \frac{2I_a}{n\pi} \sum_{m=1}^{p} \left[ \cos n\alpha_m - \cos n(\alpha_m + \delta_m) \right]$$
So, the equation for $i_s(t)$ is written as

$$i_s(t) = \sum_{n=1,3,...}^{\alpha} \sqrt{2} I_n \sin (\alpha t + \phi_n)$$

where $\phi_n = \tan^{-1} \left( \frac{a_n}{b_n} \right) = 0$, and $I_n = (a_n^2 + b_n^2)^{\frac{1}{2}}/\sqrt{2} = b_n/\sqrt{2}$

**Sinusoidal Pulse Width Modulation (SPWM) Control**

Various types of modifications in PWM techniques have been proposed. One important method is sinusoidal pulse width modulation (SPWM) control, the pulse widths are generated by comparing a triangular reference voltage $v_r$ of amplitude $A_r$ and frequency $f_r$, with a carrier half sinusoidal voltage $v_c$ of variable amplitude $A_c$ and frequency $2f_s$. The sinusoidal voltage $v_c$ is in phase with the input phase voltage $v_s$ and has twice the supply frequency $f_s$. The widths of the pulses (and the output voltage) are varied by changing the amplitude $A_c$ or the modulation index $M$ from 0 to 1. The modulation index, $M$ is $A_c/A_r$. It may be noted that the width of the pulses obtained are variable. The width are smaller at the centre of the carrier signal (sinusoidal), and increases as one goes to the start and end of the above signal. Fig. 16.4 shows the various waveforms, including the currents through thyristors and the input current and load current (assumed to be continuous). It may be noted that, in the earlier case (multiple PWM control), the pulse widths are uniform (equal). In this type of control, the displacement factor is unity, and the power factor is improved. The lower order harmonics one eliminated or reduced. For example, width four pulses per half cycle, the lowest order harmonic is the fifth, and so on.

Different modifications have been suggested to take one such example, as the pulse width one small in the centre as shown in Fig. 16.4, the carrier signal is modified to care of this. The triangular waveforms are kept same, upto some point from the start and end of the cycle, and then the pulse widths can be made uniform. For more on the matter as given earlier, the students can, either study lesson #5.5 (module 5), or text books on various PWM methods applied for inverters (dc-ac converters).

So, the power factor is improved with various control methods discussed. For different PWM methods used, the harmonic components of the voltage waveforms are also decreased or eliminated.
Filters

It is known that the output voltage waveform of a single phase full wave diode (uncontrolled) bridge converter (rectifier) fed from \( f = 50 \text{ Hz} \) (fundamental) supply, contains harmonics of \( 2f = 100 \text{ Hz} \). So, it is necessary to filter out this and other harmonics from the output voltage to obtain dc component only. The harmonic frequency present in the output voltage waveforms of three-phase half-wave and full wave (bridge) diode converters, are \( 150 \text{ Hz} \) (3f) and \( 300 \text{ Hz} \) (6f) respectively. The higher the harmonic frequency, it is easier to filter it. For phase-controlled thyristor converters, the harmonic frequency remains same, but magnitudes vary, as the firing angle delay, \( \alpha \) is changed. It may also be noted that the harmonics present in the output current waveforms of the converters with resistive (R) load, remain same.

For simple filter, a capacitor (C) is connected in parallel across the output of the diode converters with resistive (R) load. The reactance of the capacitor should be low, such that harmonics currents pass through it. So, the harmonics in the output voltage decrease. The value of the capacitor chosen varies with the predominant harmonic frequency present. Thus, the capacitor of higher value is needed to filter lower harmonic frequency, say \( 100 \text{ Hz} \), whereas a lower value of C could be chosen for say, three phase converters. The function of the capacitor
may also be explained in the following way. The voltage across the capacitor changes as per the input voltage, which is the output voltage of the converter, fed to it, and the capacitor voltage tries to stabilize at the average value of the output voltage, as the capacitor voltage decreases, load resistance being connected across it.

Same is the case with the filter used to reduce the harmonic content of the output current waveform for the above converters with resistive (R) load. Instead of a capacitor in parallel, an inductor (L) is connected in series with the load. The reactance of the inductor increases, thus reducing the harmonic component in the current waveform. Here, a smaller value of the inductor is needed to filter higher harmonics, for example a three-phase bridge converter. These are all simple cases, known to those, who have studied the circuit (network) theory. Also, by Faraday’s laws, induced voltage (emf) appears across the inductor, L, when the current through it changes, and the sign of it opposes the cause, thus opposing the changes in current. So, the current is not allowed to change much, as an inductor is placed in series with the load. In actual practice, a combination of L, C & R is needed to get an optimum filter needed to reduce or eliminate the harmonics in both output voltage and current waveforms.

**Low Pass (L-C) Filter**

A passive low pass filter is the ideal choice. But two problems arise; one is the voltage level, the other is the power or current level. All the elements used, L C or R must be properly rated for the voltage or current level as needed. A single stage filter (L-C) is used to reduce the harmonic components in both voltage and current waveforms of a single phase full wave diode bridge converter with resistive (R_L) load as shown in Fig. 16.5(a). It may be noted that, for the lowest harmonic frequency of 100 Hz, the value of the inductor needed is high, needing an iron-cored coil. The size also may be large, if the power or current level is high. As stated earlier, such that n-th harmonic ripple content passes through the filter capacitor (C), the impedance of the series path must be much greater than that of the capacitance, i.e.,

\[
|Z_L| = \sqrt{(R_L)^2 + (n \omega L)^2} > \frac{1}{n \omega C}
\]

The condition to be satisfied is

\[
|Z_L| = \frac{10}{n \omega C}
\]

and the effect of load is negligible. As shown, the capacitive reactance chosen is total load impedance divided by a factor of 10

The advantages are small ripple factor with just a single stage (L-C) used, with higher dc output voltage. The main advantage is poor voltage regulation, also resulting in higher peak anode current and peak inverse voltage rating.
Two Stage Filter

A two-stage filter (Fig. 16.5(b)) may be used, instead of a single stage one given earlier. In this case, the first one is only capacitive \((C_1)\) to reduce the harmonic content in voltage waveform, followed by second stage (R-C), instead of L-C. The size may be reduced as the size of \(R\) is smaller than that of \(L\), as given earlier. This circuit offers satisfactory operation at light loads, but considerably poor voltage regulation due to drop in \(R\), resulting in higher ripple content, at heavy load.

If a single capacitor \((C_1)\) is used as a single stage one, the ripple factor \((RF)\) is

\[
RF = 1\sqrt{2\left(4fR_iC_1-1\right)}
\]

For a chosen ripple factor, the value of \(C_1\) may be computed, if the values of rated output voltage and current for the converter are known or given.

These filters may also be used at the output of the dc-dc converter circuits described in module 3 (lessons §3.1-3.2). Only a single stage (L-C) filter may be used, as the frequency in the output voltage in this case is much higher than the harmonic frequency (100 Hz) for the single phase full wave ac-dc converter (rectifier) circuit described here.
Harmonic Reduction

The harmonic reduction schemes are presented in brief. The important point to be noted is that, recently due to increasing use of power electronic units, utility or electricity supply agencies (boards), have restricted that the power is drawn by the consumers, so as to decrease the harmonic content in the input current, or make it sinusoidal, and at the same time, improved load power factor is achieved. Two schemes – (a) passive (filter) circuits and (b) Active shaping of input line current, are presented, in brief.

Low pass (L-C) filter circuit on ac side

Before going into the aspect, let us take a rebook at the input current drawn in the circuit shown in Fig. 16.5a. Assuming that output (load) current is constant (dc) without any ripple, the ac input (source) current is square wave in nature (Fig. 16.6a), as this current changes sign, when the input voltage changes sign. If a Fourier analysis of the above current is done, there are harmonic components present in it. Just as filters have been used on the output (dc) side, a low pass (L-C) filter (Fig. 16.6b) is used on the input (source) side to reduce the harmonic components in the input current. The inductors used tend both to improve the power factor and also reduce harmonics as given earlier. The overall energy efficiency remains the same, though additional losses occur in the inductors, but conduction losses in the diodes are reduced.

![Figure 16.6(a) Output and input currents.](image-url)
Active Shaping of Input (line) Current

By using a power electronic converter for current shaping, as shown in Fig. 16.7a, it is possible to shape the input current drawn by the single phase bridge converter (rectifier) to be sinusoidal and also in phase with the input voltage. The choice of the power electronic converter is based on the following considerations:

- No need for electrical isolation between the input (dc) and output (dc) sides
- The power flow is always unidirectional from the utility side to the equipment
- The cost, power losses and size of the circuit used should be small.

Based on the above, a step-up (boost) dc-dc converter as described in next module (#3), lesson 3.1, is used as the current shaping circuit. The basic principle of operation is as follows. At the input side, the current, $i_s$, is desired to be sinusoidal, and also in phase with the voltage, $v_s$, as shown in Fig. 16.7b. Therefore, at the full wave bridge converter output, $i_L$ and $|v_s|$ have the same waveform as shown in Fig. 16.7c.
The control used is constant tolerance-band one. Here, the current, $i_L$, is controlled, such that peak-to-peak ripple $I_{\text{rip}}$ in $i_L$ remains constant. The reference input, $i_L^*$, is made sinusoidal having same (line) frequency. With a pre-selected value of $I_{\text{rip}}$, $i_L$ is forced to be in tolerance band ($i_L + I_{\text{rip}}/2$) and ($i_L - I_{\text{rip}}/2$) by controlling the status of the switch, $S$. So, the input current, $i_L$, follows the reference input, $i_L^*$, which is sinusoidal. As described later (module #3), the switch, $S$ may be a self-commutated switching device, power transistor or MOSFET. For detail, any text book may be used by the student, as only a brief discussion is presented here.

In this lesson, last one in this module, three important points – power factor (pf) improvement, harmonic reduction and filters, are presented. Firstly, three methods, viz extinction angle control, symmetrical angle control and pulse width modulation (PWM) control, are described in detail with relevant waveforms. Then, various types of filters (C, L-C & R-C) used for the reduction in harmonic content of output voltage and current waveforms of the ac-dc
converters, are discussed, with the equations for the value of the filter components needed. Lastly, in brief, harmonic reduction aspect is taken up. In this module of ac-dc converter consisting of eight lessons, all types of single-phase and three-phase converters, with other relevant points, have been thoroughly discussed.