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An infinitesimally small current element is called the Hertz Dipole (Length \( L < \lambda/50 \))

Assume an infinitesimal current element of length \( dl \) carrying an alternating current \( I_o \). The instantaneous current is

\[
i(t) = I_o e^{jw t} i_z
\]

\[
A = A_z \hat{Z} = \frac{\mu}{4\pi} I_o dl \frac{e^{-jkr}}{r} e^{jw t} \hat{Z}
\]

where, \( k = \frac{2\pi}{\lambda} \)

Dipole and its field components in spherical polar co-ordinate.
Uniform Current – Magnetic Vector Potential

\[ A = \hat{a}_z A_z = \hat{a}_z \frac{\mu I_o \ell}{4\pi r} e^{-jkr} \]

\[ A_r = A_z \cos \theta = \frac{\mu I_o \ell}{4\pi r} \cos \theta \ e^{-jkr} \]

\[ A_\theta = -A_z \sin \theta = -\frac{\mu I_o \ell}{4\pi r} \sin \theta \ e^{-jkr} \]

\[ A_\phi = 0 \]
E and H Fields from Magnetic Vector Potential

\[ H = \frac{1}{\mu} \nabla \times A = \hat{a}_\phi \frac{1}{\mu r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \]

\[ E = -j \omega A - j \frac{1}{\omega \mu \varepsilon} \nabla (\nabla \cdot A) \]

\[ P = \frac{1}{2} \int_\mathcal{S} E \times H^* \cdot ds \]
Uniform Current – E and H Fields

\[
E_r = \eta \frac{I_o \ell}{2\pi r^2} \cos \theta \left[1 + \frac{1}{jkr}\right] e^{-jkr}
\]

\[
E_\theta = j \eta \frac{kI_o \ell}{4\pi r} \sin \theta \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right] e^{-jkr}
\]

\[
E_\phi = H_r = H_\theta = 0
\]

\[
H_\phi = j \frac{kI_o \ell}{4\pi r} \left[1 + \frac{1}{jkr}\right] \sin \theta \, e^{-jkr}
\]
Uniform Current – Near and Far Fields

Near Field Region

\[ r \ll \frac{\lambda}{2\pi} \]

\[ \frac{\lambda}{6} < r < \frac{2d^2}{\lambda} \]

where \( d \) is the maximum dimension of the antenna

Far Field Region

\[ r \gg \frac{\lambda}{2\pi} \]

\[ r > \frac{2d^2}{\lambda} \]
Uniform Current - Radiation Pattern

Far Field Region (kr>>1)

\[
E_\theta = j\eta \frac{kI_ol}{4\pi r} \sin\theta \\
H_\phi = j \frac{kI_ol}{4\pi r} \sin\theta \\
\frac{E_\theta}{H_\phi} = \eta = 120\pi \\
R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2
\]

\[E_r \approx E_\phi = H_r = H_\theta = 0\]

Directivity

\[D_0 = 4\pi \frac{U_{\text{max}}}{P_{\text{rad}}} = \frac{3}{2}\]

Note: Infinitesimal antenna is not an efficient radiator

3-D radiation pattern

E-plane radiation pattern

H-plane radiation pattern
A current element whose length is $\lambda/50 < l \leq \lambda/10$ is called small dipole antenna.
Small Dipole – Radiation Resistance

Small dipole current distribution

\[ I_e(x',y',z') = \hat{a}_z I_0 \left( 1 - \frac{2}{l} z' \right), \quad 0 \leq z' \leq l/2 \]

\[ \hat{a}_z I_0 \left( 1 + \frac{2}{l} z' \right), \quad -l/2 \leq z' \leq 0 \]

Far Field Region (kr >> 1)

\[ E_\theta \approx j\eta \frac{k I_0 e^{-jkr}}{8\pi r} \sin \theta \]

\[ E_r \approx E_\phi = H_r = H_\theta = 0 \]

\[ H_\phi \approx j \frac{k I_0 e^{-jkr}}{8\pi r} \sin \theta \]

\[ R_r = \frac{2P_{rad}}{|I_0|^2} = 20\pi^2 \left( \frac{l}{\lambda} \right)^2 \]

For \( l = \lambda / 10 \), \( R_r = 2 \Omega \)

For \( l = \lambda / 4 \), \( R_r = 12.3 \Omega \)

Dipoles also have reactive impedance
Input Impedance of Transmission Line

\[ Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] \]

Case 1: \( Z_L = 0 \), \( \rightarrow Z_{in} = jZ_0 \tan \beta l \)

Case 2: \( Z_L = \infty \), \( \rightarrow Z_{in} = \frac{Z_0}{j \tan \beta l} \)

Case 3: \( Z_L = Z_0 \), \( \rightarrow Z_{in} = Z_0 \)

Where, \( \beta = \frac{2\pi}{\lambda} \)

if \( l < \frac{\lambda}{4} \) \( \rightarrow \tan \beta l = +ve \)

For Short-circuit, \( Z_L = 0 \), \( Z_{in} \) is inductive, so T-Line represents inductance

Open-circuit, \( Z_L \infty \), \( Z_{in} \) is capacitive, so T-Line represents capacitance

if \( \frac{\lambda}{4} < l < \frac{\lambda}{2} \) \( \rightarrow \tan \beta l = -ve \)
Half wavelength Dipole

Electric and magnetic fields of a half-wavelength dipole

\[
E_\theta \approx j \eta \frac{I_0 e^{-jkr}}{2\pi r} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right]
\]

\[
H_\phi \approx j \frac{I_0 e^{-jkr}}{2\pi r} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right]
\]

\(\lambda/2\) Dipole Radiation Resistance

\[
R_r = \frac{2P_{rad}}{|I_0|^2} \approx 73
\]

Directivity of half-wavelength dipole

\[
D_0 = 4\pi \frac{U_{max}}{P_{rad}} \approx 1.643
\]

D = 2.1 dB

Note: Input impedance for \(\lambda/2\) dipole is \(73+j42.5\Omega\). To make imaginary part equal to zero, the antenna length is reduced until the input impedance becomes real.

Real Input impedance is \(\leq 68\Omega\).
Current Distribution of Dipole Antenna for Different Lengths
Radiation Pattern of Dipole Antenna for Different Lengths

\[ E_\theta \approx j\eta I_0 e^{-jkr} \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{2\pi r \sin \theta} \]

- \( l = \lambda/50 \) 3-dB beamwidth = 90°
- \( l = \lambda/4 \) 3-dB beamwidth = 87°
- \( l = \lambda/2 \) 3-dB beamwidth = 78°
- \( l = 3\lambda/4 \) 3-dB beamwidth = 64°
- \( l = \lambda \) 3-dB beamwidth = 47.8°
Directivity is maximum for a thin dipole of length $l = 1.25\lambda$. 

**Two Dimensional**

**Three Dimensional**
Dipole Antenna Resistance and Directivity

\[ D_0 = 3.25 \]
Flat Dipole Antenna

Length of each segment = 50 mm
Width = 4 mm, Gap = 2 mm

BW for $|S_{11}| \leq 10$ dB is from 1.39 to 1.54 GHz (150 MHz, 10.2%)
Flat Dipole Antenna Pattern and Directivity

Directivity of 4.8 dB is maximum at 3.75 GHz where length of dipole is approx. 1.25 λ
Printed Dipole Antenna

Length of each segment = 50 mm
Width = 4mm, Gap = 2mm
FR4 substrate: $\varepsilon_r = 4.4$, $\tan\delta = 0.02$, $h = 1.6mm$

$BW = 1.14 \text{ to } 1.28 \text{ GHz (140 MHz, 11.6\%)}$
Broadband Dipole Antenna

Bandwidth of dipole antenna is directly proportional to its diameter

Cylindrical dipole antenna  
(can use hollow pipe also)

Biconical dipole antenna  
(can use wire grid also)
Balun Design

Devices that can be used to balance inherently unbalanced systems by cancelling or choking the outside current, are known as *baluns* (*balance to unbalance)*.

\[ \text{Coaxial line} \]

Shorted together

\[ \frac{\lambda}{4} \text{ Coaxial Balun (1:1)- Narrow Bandwidth} \]
Ferrite core maintains high impedance levels over a wide frequency range. A good design can provide bandwidths of 10 to 1 whereas coil coaxial baluns can provide bandwidths of 2 or 3 to 1.
Microstrip Balun Dipole Antenna for GSM900

Microstrip Balun Dipole Antenna

- \( L = 127 \text{ mm} \), \( w = 4 \text{ mm} \)
- FR4 substrate: \( \varepsilon_r = 4.4 \), \( \tan\delta = 0.02 \), \( h = 1.6\text{mm} \)

BW for \( |S_{11}| \leq 10 \text{ dB} \) is from 881 to 967 MHz
(covers GSM900 band of 890 to 960 MHz)
Folded Dipole Antenna

The impedance of the N fold folded dipole is $N^2$ times greater than that of an isolated dipole of the same length as one of its side.

$$Z_{in} = \frac{V}{I_1} \simeq N^2 Z_{11} = N^2 Z_r$$

Impedance for 2-fold dipole antenna is

$$Z_{in} = 2^2 Z_r \quad Z_{in} = 4Z_r$$

2-fold dipole antennas are used in Yagi-Uda Antennas for TV reception using balanced line of $Z_0 = 300 \, \Omega$
Dipole Antenna Applications

Compact Dipole Antenna for RFID

Folded Broadband Dipole Antenna for RF Harvesting

(Triangular shape for broadband and multi-fold gave $Z_{in} = 750 \, \Omega$)