What is a feedback control system?

Before we move on to study frequency control of a power system, we will study about another phenomena known as \textit{voltage instability}. To get a better understanding of frequency control and voltage instability, we will first briefly review the topic of feedback control systems.

What does one mean by a feedback control system and why/when is it necessary? To understand this let us take a simple example.

Let us suppose that you wish to buy 1 kg of rice from your grocer. If all rice grains were of the same weight, it would be possible (although by a very tedious process!) to calculate how many grains are required. However, not only will this be time consuming, but it is also true that there are uncertainties in the grain weight. Therefore, rather than follow this procedure the grocer will put some rice into his weighing pan (counter balanced by 1 kg standard weight) and depending on the extent of imbalance, he will incrementally add or reduce grain in the pan. A seasoned grocer may even hold some grain in his fist and gradually pour it by monitoring the position and rate at which the pan moves. By doing this, he can speed up the process of weighing the rice. This is a manual feedback control system. The major components are shown below:

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{feedback_control_system_diagram.png}
\caption{Components of a feedback control system}
\end{figure}

Components of a feedback control system

We saw in the previous slide that a feedback system can be used to quickly perform the task of weighing 1 kg of rice. It was a manual feedback which involved a human being as the controller and actuator. While a controller decides on what control actions are to be taken based on the feedback obtained by a sensor, an actuator is a device that converts the command of a controller into appropriate actions.
In an automatic feedback controller, the controller actions are decided by some mathematical function which is implemented by a computer or some electronic devices. The nature of mathematical function is decided based on the static and dynamic characteristics of the plant itself.

The main aim of the control system above is to ensure that the steady state value of the variable is equal to the reference value. This kind of a control system is referred to as a regulator. Note: some control systems do not necessarily have a steady state regulation objective, but use feedback to enhance the stability of a plant.

Note that an automatic controller (which is a mathematical function), if designed improperly may worsen the stability of the system. For example, in the rice weighing control system, if the grocer over-reacts to a small imbalance shown by the weighing machine, and pours in (removes) a lot of rice into (from) the weighing pan, then he may be caught in an endless cycle of removal and pouring in rice. This is an unstable situation. Therefore the controller actions should be designed with care.

Example: Suppose we wish to regulate the current in an R-L circuit by controlling the input voltage:

The equation describing the “plant” is:

\[
\frac{dI}{dt} = -\frac{R}{L} I + \frac{E}{L}
\]

If we wish to get a current \( I = I_{REF} \) in steady state, then we can set the input voltage to be \( E = R I_{REF} \). If we do this the transient response if such a voltage is applied to the circuit, assuming initial current is zero is:

\[
I = (1 - e^{-\frac{R}{L}}) \frac{E}{R} = (1 - e^{-\frac{R}{L}}) I_{REF}
\]

This kind of control is "open loop" and does not require continuous feedback of the current. However, the main problem is that one should know R accurately and also that the transient response is determined by R and L values -- it cannot be modified.

**Proportional Control**

An alternative to open loop control is feedback control wherein the input voltage \( E \) is adjusted continually based on the measured current (feedback). For example let us assume that the controller behaves as per the following law:

\[
E = K(I_{REF} - I)
\]

We assume that the current measurement is available without any delay. The resulting differential equation can be re-written as follows:

\[
\frac{dI}{dt} = -\frac{R}{L} I + \frac{K}{L} (I_{REF} - I) = -\left(\frac{R+K}{L}\right) I + \frac{K}{L} I_{REF}
\]
The response of the current if the controller comes into action at time $t=0$, and when initial current is zero is:

$$I = \left(1 - e^{-\frac{(R+K)}{L}}\right) \frac{Kt_{sp}}{R+K}$$

In steady state, $I = \frac{Kt_{sp}}{R+K}$. However, if $K$ is very large compared to $R$, then in steady state (approximately), $I = t_{sp}$. This will also ensure that response will be faster and practically independent of $R$.

In practice $K$ cannot be made too large, because it is possible that some component of transient response due to the unmodelled measurement (sensor) block may get destabilized.

The controller described above is called a proportional controller. An alternative to having large $K$ to obtain $I = t_{sp}$, is to modify the nature of the controller. We can also use an integral controller for performing regulation function.

### Integral Control

An alternative to using a proportional control law is to vary the input $E$ using the following rule:

$$\frac{dE}{dt} = K_i (I_{sp} - I)$$

For an integral controller, in steady state $I = I_{sp}$. If this were not true, $E$ would go on increasing, as a consequence of the above equation (therefore contradicting the statement that the system is in steady state).

Therefore integral control ensures perfect regulation in steady state.

However transient response can be poor with an integral controller. A step change in the reference value of current can lead to oscillatory behaviour. This is depicted for a integral controller for which the reference current is increased from 0 to 1 A at $t=1$s. The initial current in the circuit is zero and $R=1 \Omega$, $L=0.1$ H. It is clear that a very large value of integral controller gain gives an overshoot and oscillatory behaviour.

To obtain good regulation as well as good transient response, one can use a combination of a proportional and an integral controller. The following control law achieves this:

$$\frac{dE}{dt} = K_i (I_{sp} - I)$$

$$E_t = K_p (I_{sp} - I)$$

$$E = E_t + E_I$$

Alternatively, one could also have an additional component of $E$ which varies proportional to the rate of change of current. Such a controller is called a proportional-integral-derivative controller. Note that in practice one may only be able to obtain an approximate derivative (an exact derivative requires future information which is not possible to implement due to causality conditions).

While we have discussed the nature of control laws, we have not really discussed the design of the individual parameters of controllers. The choice of actual values of gains (e.g. $K$, $K_i$, $K_p$), are chosen based on the satisfaction of certain specifications like maximum steady state error, stability, overshoot etc. For a detailed account of design issues, you may refer to any book on basic control systems (see, for example, Nagrath I J & Gopal M: Control System Engineering – New age International Publishers).

We now discuss the transfer function representation which is commonly used to represent such systems.

### Transfer Function Representation of Systems

Many control systems are depicted as transfer function block diagrams. Transfer function representations are obtained by taking the Laplace
transformation of all the variables and obtaining relationships between them. For example, the circuit equation of the R-L circuit given by:

\[ \frac{dI}{dt} = -\frac{R}{L} I + \frac{E}{L}. \]

can be written down as:

\[ sI(s) = -\frac{R}{L} I(s) + \frac{E(s)}{L} \]

by taking the Laplace Transforms of the variables. The "transfer function" between \( I(s) \) and \( E(s) \) is given by:

\[ \frac{I(s)}{E(s)} = \frac{1}{R + sL}. \]

Note here that if \( X(s) \) is the laplace transform of \( x(t) \), then \( \frac{X(s)}{s} \) is the laplace transform of \( \int x(t)\,dt \).

The block diagram equivalent (this is not unique) using the transformed variables, of various transfer functions are shown in the figures below:

1. A first order transfer function. Steady state gain = 1

2. A "washout" transfer function. Steady state gain = 0

3. A lead (if \( T_1 > T_2 \)) or lag (if \( T_2 < T_1 \)) function. Steady state gain = 1

4. A Proportional-Integral Controller. Steady state gain = infinity

Steady state (or "DC") gain of a transfer function is obtained by substituting putting \( s = 0 \) in the transfer function.

Recap

In this lecture you have learnt the following

* What are Feedback Control Systems

* Proportional and Integral Controllers

Congratulations, you have finished Lecture 8a. To view the next lecture select it from the left hand side menu of the page.