Assignment 8

1) Which of the following statements are false?

- If $P \neq NP$ then $Clique \notin P$  
- If $P = NP$ then $Clique \in P$  
- $4SAT \leq_p 3SAT$  
- $L_1 \leq_p L_2$ and $L_2 \leq_p L_3$ then $L_3 \leq_p L_1$

**Accepted Answers:**
If $L_1 \leq_p L_2$ and $L_2 \leq_p L_3$ then $L_3 \leq_p L_1$

2) Which of the following statements are implied by $Clique \leq_p Clique$?

- $NP = coNP$  
- $NP \neq coNP$  
- $SAT \leq_p Clique$  
- $Clique \leq_p SAT$

**Accepted Answers:**
$NP = coNP$  
$SAT \leq_p Clique$  
$Clique \leq_p SAT$

3) Let $L_1$ and $L_2$ be two NP languages. Which of the following are known to be true?

- $L_1 \cup L_2 \in NP$  
- $L_1 \cap L_2 \in NP$  
- $L_1 \cdot L_2 \in NP$

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2 points

4) Consider the following statements

\( S_1 : \text{If } L_1 \leq_p L_2 \text{ and } L_1 \text{ is an NP complete problem then } L_2 \text{ is also an NP complete problem} \)

\( S_2 : \text{If } P = NP \text{ then } NP = coNP \)

Which of the following are correct?

- \( S_1 \) is true but \( S_2 \) is false
- \( S_2 \) is true but \( S_1 \) is false
- Both \( S_1 \) and \( S_2 \) are true
- Both \( S_1 \) and \( S_2 \) are false

Accepted Answers:

\( S_2 \) is true but \( S_1 \) is false

2 points

5) Consider the following languages

\( L_1 = \{ x \mid x \text{ is a palindromic binary string} \} \)

\( L_2 = \{ x \mid x \text{ is a binary string whose decimal representation is an even number} \} \)

We define \( DTIME(2^n), DTIME(n) \) and \( DTIME(c) \) to be the class of languages which can be decided in exponential, linear and constant time respectively. Which of the following are true?

- \( L_1 \in DTIME(c) \)
- \( L_1 \in DTIME(n) \) and \( L_1 \notin DTIME(c) \)
- \( L_2 \in DTIME(n) \)
- \( L_2 \in DTIME(2^n) \) and \( L_2 \notin DTIME(n) \)

Accepted Answers:

\( L_1 \in DTIME(n) \) and \( L_1 \notin DTIME(c) \)

\( L_2 \in DTIME(n) \)

6) Which of the following languages about encodings of TMs are decidable?

- \( \{ < M > \mid L(M) \text{ has odd number of strings} \} \)
- \( \{ < M > \mid L(M) \text{ has odd length strings} \} \)
- \( \{ < M > \mid \overline{A_{TM}} \leq_{m} L(M) \} \)

Accepted Answers:

- \( \{ < M > \mid L(M) \text{ has odd number of strings} \} \)
- \( \{ < M > \mid L(M) \text{ has odd length strings} \} \)
- \( \{ < M > \mid \overline{A_{TM}} \leq_{m} L(M) \} \)
7) Consider the following languages
\[ L_1 = \{ <M, x> | M \text{ is a single tape TM whose input head reaches the } k \text{th cell on input } x \} \]
\[ L_2 = \{ <M> | M \text{ is a single tape TM whose input head reaches the } k \text{th cell on every input} \} \]
Which of the following is true?

- \( L_1 \) is decidable but \( L_2 \) is undecidable
- \( L_1 \) is undecidable but \( L_2 \) is decidable
- Both \( L_1 \) and \( L_2 \) are decidable
- Both \( L_1 \) and \( L_2 \) are undecidable

\text{Accepted Answers:}
\text{Both } L_1 \text{ and } L_2 \text{ are decidable}

8) Consider the following languages
\[ L_1 = \{ <M_1, M_2, x> | \text{at least one out of } M_1 \text{ and } M_2 \text{ halts on } x \} \]
\[ L_2 = \{ <M_1, M_2, x> | \text{exactly one out of } M_1 \text{ and } M_2 \text{ halts on } x \} \]
Which of the following is true?

- \( L_1 \) is recognizable but \( L_2 \) is unrecognizable
- \( L_2 \) is recognizable but \( L_1 \) is unrecognizable
- Both \( L_1 \) and \( L_2 \) are recognizable
- Both \( L_1 \) and \( L_2 \) are unrecognizable

\text{Accepted Answers:}
\text{\( L_1 \) is recognizable but \( L_2 \) is unrecognizable}

9) Consider the following language about encoding of TMs
\[ L = \{ <M> | M \text{ does not halt on every input} \} \]
Which of the following is correct?

- \( L \) is decidable
- \( L \) is recognizable but undecidable
- \( L \) is unrecognizable and \( \overline{L} \) is also unrecognizable
- \( L \) is unrecognizable but \( \overline{L} \) is recognizable
Accepted Answers:

$L$ is unrecognizable but $L'$ is recognizable