Assignment 7

1) Consider the following languages

\[ L_1 = \{ <M> \mid M \text{ is a single tape TM that on any input } x \text{ does not change the input portion of the tape} \} \]

\[ L_2 = \{ <M> \mid M \text{ is a single tape TM that on any input } x \text{ does not change any portion of the tape} \} \]

Which of the following is correct?

- \( L_1 \) is decidable but \( L_2 \) is undecidable
- \( L_1 \) is undecidable but \( L_2 \) is decidable
- Both \( L_1 \) and \( L_2 \) are decidable
- Both \( L_1 \) and \( L_2 \) are undecidable

**Accepted Answers:**

\( L_1 \) is undecidable but \( L_2 \) is decidable

2) Which of the following are correct?

- If \( L_1 \leq_m L_2 \) and \( L_2 \leq_m L_3 \), then \( L_1 \leq_m L_3 \)
- If \( L_1 \leq_m L_2 \) then \( L_2 \leq_m L_1 \)
- If \( L_1 \leq_m L_2 \) then \( \overline{L_1} \leq_m \overline{L_2} \)
- \( L \leq_m \overline{L} \)

**Accepted Answers:**

- If \( L_1 \leq_m L_2 \) and \( L_2 \leq_m L_3 \), then \( L_1 \leq_m L_3 \)
- If \( L_1 \leq_m L_2 \) then \( \overline{L_1} \leq_m \overline{L_2} \)

3) Consider the following language

\[ L = \{ <M> \mid M \text{ is a TM such that if } M \text{ accepts } w \text{ then } M \text{ accepts } \overline{w} \text{ also} \} \]
Which of the following is correct?

- $L$ is decidable
- $L$ is recognizable but undecidable
- $L$ is unrecognizable and $\overline{L}$ is decidable
- $L$ is unrecognizable and $\overline{L}$ is undecidable

**Accepted Answers:**
$L$ is unrecognizable and $\overline{L}$ is undecidable

4) Let $L_1$ and $L_2$ be two languages such that $L_1 \leq_{\text{redu}} L_2$. Consider the following statements:

- $S_1$: If $L_1$ is non-regular then $L_2$ is also non-regular
- $S_2$: If $L_1$ is non-CFL then $L_2$ is also non-CFL

Which of the following is correct?

- $S_1$ is true but $S_2$ is false
- $S_2$ is true but $S_1$ is false
- Both $S_1$ and $S_2$ are true
- Both $S_1$ and $S_2$ are false

**Accepted Answers:**
Both $S_1$ and $S_2$ are false

5) Consider the following languages:

- $EQ(P_1, P_2) = \{ <P_1, P_2> | P_1$ and $P_2$ are pushdown automata such that $L(P_1) = L(P_2) \}$
- $EQN(D_1, D_2) = \{ <D_1, D_2> | D_1$ and $D_2$ are DFAs such that $|L(D_1)| = |L(D_2)| \}$

Which of the following is correct?

- $EQ(P_1, P_2)$ is decidable but $EQN(D_1, D_2)$ is not
- $EQN(D_1, D_2)$ is decidable but $EQ(P_1, P_2)$ is not
- Both $EQ(P_1, P_2)$ and $EQN(D_1, D_2)$ are decidable
- Both $EQ(P_1, P_2)$ and $EQN(D_1, D_2)$ are not decidable

**Accepted Answers:**
$EQN(D_1, D_2)$ is decidable but $EQ(P_1, P_2)$ is not

6) Consider the following languages:

$E(Q(P_1, P_2)) = \{ <P_1, P_2> | P_1$ and $P_2$ are pushdown automata such that $L(P_1) = L(P_2) \}$

$E(QN(D_1, D_2)) = \{ <D_1, D_2> | D_1$ and $D_2$ are DFAs such that $|L(D_1)| = |L(D_2)| \}$

Which of the following is correct?

- $E(Q(P_1, P_2))$ is decidable but $E(QN(D_1, D_2))$ is not
- $E(QN(D_1, D_2))$ is decidable but $E(Q(P_1, P_2))$ is not
- Both $E(Q(P_1, P_2))$ and $E(QN(D_1, D_2))$ are decidable
- Both $E(Q(P_1, P_2))$ and $E(QN(D_1, D_2))$ are not decidable

**Accepted Answers:**
$E(QN(D_1, D_2))$ is decidable but $E(Q(P_1, P_2))$ is not
Which of the following is correct?

- $L_1$ and $L_2$ both are decidable
- $L_1$ and $L_2$ both are recognizable but not decidable
- $L_1$ is recognizable but not decidable and $L_2$ is not recognizable
- $L_1$ is decidable and $L_2$ is recognizable but not decidable

**Accepted Answers:**
$L_1$ and $L_2$ both are decidable

7) Let $L_1$ and $L_2$ be two languages. If $L_1 \leq_m L_2$ and $L_1$ is Turing recognizable but not decidable, then which of the following are necessarily true?  

- $L_2$ is Turing recognizable
- $\overline{L}_2$ is Turing recognizable
- $L_2$ is not Turing recognizable
- $\overline{L}_2$ is not Turing recognizable

**Accepted Answers:**
$\overline{L}_2$ is not Turing recognizable

8) Which of the following statements are false?  

- Every subset of a decidable language is decidable
- Every superset of a decidable language is decidable
- Every subset of an undecidable language is undecidable
- Every superset of an undecidable language is undecidable

**Accepted Answers:**
Every subset of a decidable language is decidable
Every superset of a decidable language is decidable
Every subset of an undecidable language is undecidable
Every superset of an undecidable language is undecidable