### Assignment 4

1) Let \( L_1 \) and \( L_2 \) be languages and \( L_1 \subseteq L_2 \). Which of the following is true?  

- [ ] If \( L_1 \) is regular then \( L_2 \) is also regular.  
- [ ] If \( L_1 \) is CFL then \( L_2 \) is also CFL.  
- [ ] \( L_2 \) can be regular even if \( L_1 \) is CFL but not regular.  
- [ ] None of the other option.

**Accepted Answers:**  
\( L_2 \) can be regular even if \( L_1 \) is CFL but not regular.

2) Consider the following language,  
\[ L = \{ w \in \{0, 1\}^* \mid w \text{ is palindrome} \} \]  
Which of the following grammar generates the above language.

- [ ] \( S \rightarrow 0S0 \mid 1S1 \mid e \)
- [ ] \( S \rightarrow 0S0S \mid 1S1S \mid e \)
- [ ] \( S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \)
- [ ] \( S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid e \)

**Accepted Answers:**  
\( S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid e \)

3) Consider the following grammar \( G \),  
\[ S \rightarrow aSaS \mid e \]  
Which of the following is true?

- [ ] \( G \) is ambiguous.  
- [ ] \( G \) is unambiguous.

**Accepted Answers:**  
\( G \) is ambiguous.
4) Consider the following grammar $G$, 

\[ S \rightarrow AB \\
A \rightarrow XAX \\
B \rightarrow XBX \\
X \rightarrow 0 | 1 \]

Consider a language 

\[ L = \{xy \mid x, y \in \{0, 1\}^*, lx = ly \text{ and } x \neq y\} \]

Which of the following production should be added to above grammar $G$ so that $L(G) = L$?

- $A \rightarrow 0, B \rightarrow 1$ only.
- $A \rightarrow 1, B \rightarrow 0$ only.
- $S \rightarrow BA, A \rightarrow 1, B \rightarrow 0$ only.
- None of the other option.

**Accepted Answers:**

$S \rightarrow BA, A \rightarrow 1, B \rightarrow 0$ only.

5) Consider the following language, 

\[ L_1 = \{ww^r \#ww^r \mid w \in \{0, 1\}^*\} \]
\[ L_2 = \{a^ib^jc^k \mid i = j \text{ or } j = k\} \]

Which of the above is CFL?

- Only $L_1$
- Only $L_2$
- Both $L_1$ and $L_2$.
- None.

**Accepted Answers:**

Only $L_2$

6) Consider the following language, 

\[ L_1 = \{uvw^r \mid u, v, w \in \{0, 1\}^*, lw = lv = 2\} \]
\[ L_2 = \{utu^r \mid u, t \in \{0, 1\}^*, lu = lt\} \]

Which of the following is true?

- $L_1$ and $L_2$ both are CFL.
- $L_1$ is CFL but $L_2$ is not a CFL.
- $L_2$ is CFL but $L_1$ is not a CFL.
- Both are not CFL.

**Accepted Answers:**

Only $L_2$
7) Let $L$ be a language, we define $PERMUTE(L)$ as,

$$PERMUTE(L) = \{ w \mid w \text{ is a permutation of some string } x \in L \}.$$ 

Note that for a string $x \in L$, all its permutation will be in $PERMUTE(L)$. Which of the following is true.

- If $L$ is regular then $PERMUTE(L)$ is also regular.
- If $L$ is regular then $PERMUTE(L)$ may or may not be regular.
- If $L$ is CFL then $PERMUTE(L)$ is also CFL.
- If $L$ is CFL then $PERMUTE(L)$ may or may not be CFL.

8) Let $L$ be a language, we define $\frac{L}{2}$ as,

$$\frac{L}{2} = \{ x \mid xw \in L \text{ for some } w \in \Sigma^* \text{ and } |x| = |w| \},$$

consider the following languages

- $L_1 = \{ a^n b^n c^n \mid n \geq 0 \}$
- $L_2 = \{ a^n b^m c^m \mid n, m \geq 0 \}$

Match the following languages with the class they belong to.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
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<tbody>
<tr>
<td>(1) $L_1$</td>
<td>(a) Regular</td>
</tr>
<tr>
<td>(2) $L_2$</td>
<td>(b) CFL But not regular</td>
</tr>
<tr>
<td>(3) $\frac{L_1}{2}$</td>
<td>(c) Not CFL</td>
</tr>
<tr>
<td>(4) $\frac{L_2}{2}$</td>
<td></td>
</tr>
</tbody>
</table>
Accepted Answers:
(1, c)(2, b)(3, b)(4, a)