1) Let \( L \) be a regular language and

\[
\begin{align*}
L_1 &= \{ x \mid \text{there exist } y \text{ so that } xy \in L \text{ and } |x| = 2|y| \} \\
L_2 &= \{ x \mid \text{there exist } y \text{ so that } yx \in L \text{ and } |x| = 2|y| \}
\end{align*}
\]

Then which of the following is necessarily correct?

- \( L_1 \) is regular but \( L_2 \) is not.
- \( L_2 \) is regular but \( L_1 \) is not.
- Both \( L_1 \) and \( L_2 \) are regular.
- Both \( L_1 \) and \( L_2 \) are not regular.

2) Let \( L \) be a regular language over \( \Sigma \). Define

\[
\begin{align*}
\text{shift}(L) &= \{ w \mid w \in L \text{ or } w = a_1 a_2 \ldots a_n \text{ and for some } l \in L \} \\
\text{incrpad}(L) &= \{ w \mid w = a_1 a_2 a_2 a_2 \ldots a_n a_n a_n \} \text{ where } a_1, a_2, \ldots, a_n \in \Sigma \text{ and } a_1 a_2 \ldots a_n \in L \\
\text{unipad}(L) &= \{ w \mid w = a_1 a_2 a_2 \ldots a_n \} \text{ where } a_1, a_2, \ldots, a_n \in \Sigma \text{ and } a_1 a_2 \ldots a_n \in L \\
\end{align*}
\]

Then which of the following is correct?

- Only \( \text{shift}(L) \) is regular.
- Only \( \text{shift}(L) \) and \( \text{unipad}(L) \) are regular.
- Only \( \text{unipad}(L) \) and \( \text{incrpad}(L) \) are regular.
- All of \( \text{shift}(L) \), \( \text{incrpad}(L) \) and \( \text{unipad}(L) \) are regular.

3) Let \( L \) be the language of binary strings where every 0 is followed by a 1. What is the minimum number of states in a DFA and an NFA respectively that accepts the complement of \( L \)?

- 3 and 2.
- 3 and 3.
- 2 and 2.
- 2 and 3.
4) Consider the following grammar G

\[ S \rightarrow aA | bA | a | b \]
\[ A \rightarrow aB | bB | a | b \]
\[ B \rightarrow aS | bS \]

Which of the following statements are true about G?

- \[ L(G) = \{a,b\}^* \]
- \[ L(G) \] is the set of all strings \( w \) such that \( |w| \) is not divisible by 6
- \[ L(G) \] is the set of all strings \( w \) such that \( |w| \) is not divisible by 3
- \[ L(G) \] is regular.

5) Consider the three languages \( A \), \( B \) and \( C \).

\[ A = \{a^n b^n \mid n \geq 0\} \]
\[ B = \{a^n \mid n \geq 0\} \]
\[ C = \{b^n \mid n \geq 0\} \]

Which of the following languages are regular? (Note that \( . \) is the concatenation operator)

- \( B \) and \( C \)
- \( B A \) and \( C A \)
- \( A B C \)
- \( B A C \)
6) Consider the following grammar

\[
S \to aA \mid bB \mid \epsilon \\
A \to aS \mid a \\
B \to bS \mid b
\]

What is the regular expression corresponding to \( L(G) \)?

- \((a + b)^*\)
- \((aa + bb)^*\)
- \((ab + ba)^*\)
- \(a^* + b^*\)

7) Consider the following languages over the alphabet \( \Sigma = \{a, b\} \)

\[
L_1 = \{a^m b^n \mid m, n \geq 0\} \\
L_2 = \{a^p \mid p \text{ is a prime}\}
\]

Which of the following is correct?

- \(L_1 - L_2\) is a regular language.
- \(L_1 \cup L_2\) is a regular language.
- \(L_1 \cap L_2\) is a regular language.
- \(L_1 \cap \overline{L_2}\) is a regular language.

8) Consider the following language

\[
A = \{a^{2^n} \mid n \geq 0\}
\]

Which of the following languages are regular?

- \(A\)
- \(A \cap \overline{A}\)
- \(A^*\)
- \(A \cdot \overline{A}\)