Integer Programming

Mixed Integer Linear Programming
Objectives

- To discuss about the Mixed Integer Programming (MIP)
- To discuss about the Mixed Integer Linear Programming (MILP)
- To discuss the generation of Gomory constraints
- To describe the procedure for solving MILP
Introduction

- **Mixed Integer Programming:**
  - Some of the decision variables are real valued and some are integer valued

- **Mixed Integer Linear Programming:**
  - MIP with linear objective function and constraints
  - The procedure for solving an MIP is similar to that of All Integer LP with some exceptions in the generation of Gomory constraints.
Generation of Gomory Constraints

- Consider the final tableau of an LP problem consisting of $n$ basic variables (original variables) and $m$ non basic variables (slack variables).
- Let $x_i$ be the basic variable which has integer restrictions.
From the $i^{th}$ equation,

$$x_i = b_i - \sum_{j=1}^{m} c_{ij} y_j$$

Expressing $b_j$ as an integer part plus a fractional part,

$$b_i = \bar{b}_i + \beta_i$$

Expressing $c_{ij}$ as $c_{ij} = \bar{c}_{ij}^+ + \bar{c}_{ij}^-$ where

$$\bar{c}_{ij}^+ = \begin{cases} c_{ij} & \text{if } c_{ij} \geq 0 \\ 0 & \text{if } c_{ij} < 0 \end{cases}$$

$$\bar{c}_{ij}^- = \begin{cases} 0 & \text{if } c_{ij} \geq 0 \\ c_{ij} & \text{if } c_{ij} < 0 \end{cases}$$
Thus,

\[ \sum_{j=1}^{m} (c_{ij}^+ + c_{ij}^-) y_j = \beta_i + (\overline{b_i} - x_i) \]

Since \( x_i \) and \( \overline{b_i} \) are restricted to take integer values and also \( 0 < \beta_i < 1 \) the value of \( \beta_i + (\overline{b_i} - x_i) \) can be \( \geq 0 \) or \( < 0 \)

Thus we have to consider two cases.
Case I:  $\beta_i + (\bar{b}_i - x_i) \geq 0$

- For $x_i$ to be an integer,
  
  $\beta_i + (\bar{b}_i - x_i) = \beta_i \text{ or } \beta_i + 1 \text{ or } \beta_i + 2, \ldots$

- Therefore,
  
  $$\sum_{j=1}^{m} \left( c_{ij}^+ + c_{ij}^- \right) y_j \geq \beta_i$$

- Finally it takes the form,
  
  $$\sum_{j=1}^{m} \bar{c}_{ij}^+ y_j \geq \beta_i$$
Generation of Gomory Constraints …contd.

Case II: $\beta_i + (\overline{b}_i - x_i) < 0$

- For $x_i$ to be an integer,
  \[
  \beta_i + (\overline{b}_i - x_i) = -1 + \beta_i \text{ or } -2 + \beta_i \text{ or } -3 + \beta_i, \ldots
  \]

- Therefore,
  \[
  \sum_{j=1}^{m} (\overline{c}_{ij}^+ + \overline{c}_{ij}^-) y_j \leq \beta_i - 1
  \]

- Finally it takes the form,
  \[
  \sum_{j=1}^{m} \overline{c}_{ij}^- y_j \leq \beta_i - 1
  \]
Generation of Gomory Constraints …contd.

- Dividing this inequality by \((\beta_i - 1)\) and multiplying with \(\beta_i\), we have

\[
\frac{\beta_i}{\beta_i - 1} \sum_{j=1}^{m} \tilde{c}_{ij} y_j \geq \beta_i
\]

- Now considering both cases I and II, the final form of the Gomory constraint after adding one slack variable \(s_i\) is,

\[
s_i - \sum_{j=1}^{m} \tilde{c}_{ij}^+ y_j - \frac{\beta_i}{\beta_i - 1} \sum_{j=1}^{m} \tilde{c}_{ij}^- y_j = -\beta_i
\]
Procedure for solving Mixed-Integer LP

- Solve the problem as an ordinary LP problem neglecting the integrality constraints.
- Generate Gomory constraint for the fractional valued variable that has integer restrictions.
- Insert a new row with the coefficients of this constraint, to the final tableau of the ordinary LP problem.
- Solve this by applying the dual simplex method
- The process is continued for all variables that have integrality constraints
Thank You