Dynamic Programming

Computational Procedure in Dynamic Programming
Objectives

- To explain the computational procedure of solving the multistage decision process using recursive equations for backward approach
Computational Procedure

Consider a serial multistage problem and the recursive equations developed for backward recursion.

The objective function is

\[ f = \sum_{t=1}^{T} NB_t = \sum_{t=1}^{T} h_t(X_t, S_t) \]

Considering first sub-problem (last stage), the objective function is

\[ f^*_T(S_T) = \text{opt}_{X_T}[h_T(X_T, S_T)] \]
Computational Procedure ...contd.

- The input variable is $S_T$ and the decision variable is $X_T$.
- Optimal value of the objective function $f^*_T$ depend on the input $S_T$.
- But at this stage, the value of $S_T$ is not known.
- Value of $S_T$ depends upon the values taken by the upstream components.
- Therefore, $S_T$ is solved for all possible range of values.
- The results are entered in a graph or table which contains the calculated optimal values of $X^*_T$, $S_{T+1}$ and also $f^*_T$. 
The results are entered in a graph or table which contains the calculated optimal values of $X_T^*$, $S_{T+1}$ and also $f_T^*$.

Typical table showing the results from the sub-optimization of stage 1

<table>
<thead>
<tr>
<th>Sl no</th>
<th>$S_T$</th>
<th>$X_T^*$</th>
<th>$f_T^*$</th>
<th>$S_{T+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Computational Procedure …contd.

Consider the second sub-problem by grouping the last two components.

The objective function is

\[
f_{T-1}^*(S_{T-1}) = \text{opt}_{X_{T-1},X_T} \left[h_{T-1}(X_{T-1}, S_{T-1}) + h_T(X_T, S_T)\right]
\]
Computational Procedure …contd.

From the earlier lecture,

$$f^*_T(S_{T-1}) = \text{opt}_{X_{T-1}} \left[ h_{T-1}(X_{T-1}, S_{T-1}) + f^*_T(S_T) \right]$$

The information of first sub-problem is obtained from the previous table.

A range of values are considered for $S_{T-1}$.

The optimal values of $X^*_{T-1}$ and $f^*_{T-1}$ are found for these range of values.
In general, consider the sub-optimization of \( i+1 \text{th} \) sub-problem (\( T-i \text{th} \) stage)

\[
f^*_T(S_{T-i}) = \text{opt} \left[ h_{T-i}(X_{T-i}, S_{T-i}) + \ldots + h_{T-1}(X_{T-1}, S_{T-1}) + h_T(X_T, S_T) \right]
\]

\[
= \text{opt} \left[ h_{T-i}(X_{T-i}, S_{T-i}) + f^*_T(S_{T-i}) \right]
\]  

\[\text{...}(7)\]
Computational Procedure …contd.

At this stage, the sub-optimization has been carried out for all last $i$ components.

The information regarding the optimal values of $i^{th}$ sub-problem will be available in the form of a table.

Substituting this information in the objective function and considering a range of values, the optimal values of $f_{T-i}^*$ and $X_{T-i}^*$ can be calculated.
**Computational Procedure ...contd.**

The table showing the sub-optimization of \( i+1^{th} \) sub-problem is shown

<table>
<thead>
<tr>
<th>Sl no</th>
<th>( S_{T-1} )</th>
<th>( X_{T-1}^* )</th>
<th>( S_{T-(i-1)} )</th>
<th>( f_{T-(i-1)}^* \left( S_{T-(i-1)} \right) )</th>
<th>( f_{T-i}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

This procedure is repeated until stage 1 is reached

For initial value problems, only one value \( S_1 \) need to be analyzed for stage 1
Computational Procedure …contd.

After completing the sub-optimization of all the stages, retrace the steps through the tables generated to find the optimal values of $X$

The $T^{th}$ sub-problem gives the values of $X_1^*$ and $f_1^*$ for a given value of $S_1$ (since the value of $S_1$ is known for an initial value problem)

Calculate the value of $S_2^*$ using the transformation equation $S_2 = g(X_1, S_1)$, which is the input to the $2^{nd}$ stage ($T-1^{th}$ sub-problem)

From the tabulated results for the $2^{nd}$ stage, the values of $X_2^*$ and $f_2^*$ are found out

Again use the transformation equation to find out $S_3^*$ and the process is repeated until the $1^{st}$ sub-problem or $T^{th}$ stage is reached

The final optimum solution vector is given by $X_1^*, X_2^*, ..., X_T^*$
Thank You