Linear Programming

Graphical method
Objectives

- To visualize the optimization procedure explicitly
- To understand the different terminologies associated with the solution of LPP
- To discuss an example with two decision variables
Example

Maximize \[ Z = 6x + 5y \]
subject to \[
2x - 3y \leq 5 \quad (c-1) \\
x + 3y \leq 11 \quad (c-2) \\
4x + y \leq 15 \quad (c-3) \\
x, y \geq 0 \quad (c-4 & c-5)
\]
Graphical method: Step - 1

Plot all the constraints one by one on a graph paper.
Graphical method: Step - 2

Identify the common region of all the constraints.

This is known as ‘feasible region’
Graphical method: Step - 3

Plot the objective function assuming any constant, $k$, i.e.

$$6x + 5y = k$$

This is known as ‘Z line’, which can be shifted perpendicularly by changing the value of $k$. 
Graphical method: Step - 4

Notice that value of the objective function will be maximum when it passes through the intersection of \(x + 3y = 11\) and \(4x + y = 15\) (straight lines associated with 2\textsuperscript{nd} and 3\textsuperscript{rd} constraints).

This is known as ‘Optimal Point’
Graphical method: Step - 5

Thus the **optimal point** of the present problem is

\[ x^* = 3.091 \]
\[ y^* = 2.636 \]

And the optimal solution is

\[ 6x^* + 5y^* = 31.726 \]
Different cases of optimal solution

A linear programming problem may have

1. A unique, finite solution (example already discussed)
2. An unbounded solution,
3. Multiple (or infinite) number of optimal solution,
4. Infeasible solution, and
5. A unique feasible point.
Unbounded solution: Graphical representation

Situation: If the feasible region is not bounded

Solution: It is possible that the value of the objective function goes on increasing without leaving the feasible region, i.e., unbounded solution
Multiple solutions: Graphical representation

Situation: $Z$ line is parallel to any side of the feasible region

Solution: All the points lying on that side constitute optimal solutions
Infeasible solution: Graphical representation

**Situation:** Set of constraints does not form a feasible region at all due to inconsistency in the constraints

**Solution:** Optimal solution is not feasible
Unique feasible point: Graphical representation

**Situation:** Feasible region consist of a single point. Number of constraints should be at least equal to the number of decision variables.

**Solution:** There is no need for optimization as there is only one feasible point.
Thank You