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Advanced Foundation Engineering

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CHAPTER 3: COMBINED FOOTING AND RAFT FOUNDATION

Chapter 3

Combined Footing and Raft Foundation

3.1 Introduction

In chapter 2 we considered the common methods of transmitting loads to subsoil through spread footings carrying single column loads. In this chapter we consider the following types of foundations:

1. Cantilever footings

2. Combined footings

3. Mat foundations

When a column is near or right next to a property limit, a square or rectangular footing concentrically loaded under the column would extend into the adjoining property. If the adjoining property is a public side walk or alley, local building codes may permit such footings to project into public property. But when the adjoining property is privately owned, the footings must be constructed within the property. In such cases, there are three alternatives which are illustrated in Fig. 3.1(a). These are

(1) Cantilever footing:

A cantilever or strap footing normally comprises two footings connected by a beam called a strap. A strap footing is a special case of a combined footing.

(2) Combined footing:

A combined footing is a long footing supporting two or more columns in one row.

(3) Mat or raft foundations:

A mat or raft foundation is a large footing, usually supporting several columns in two or more rows.

The choice between these types depends primarily upon the relative cost. In the majority of cases, mat foundations are normally used where the soil has low bearing capacity and where the

total area occupied by individual footings is not less than 50 per cent of the loaded area of the building. When the distances between the columns and the loads carried by each column are not equal, there will be eccentric loading. The effect of eccentricity is to increase the base pressure on the side of eccentricity and decrease it on the opposite side. The effect of eccentricity on the base pressure of rigid footings is also considered here.

3.2 Mat Foundation in Sand

Experience indicates that the ultimate bearing capacity of a mat foundation on cohesionless soil is much higher than that of individual footings of lesser width. With the increasing width of the mat, or increasing relative density of the sand, the ultimate bearing capacity increases rapidly. Hence, the danger that a large mat may break into a sand foundation is too remote to require consideration. On account of the large size of mats the stresses in the underlying soil are likely to be relatively high to a considerable depth. Therefore, the influence of local loose pockets distributed at random throughout the sand is likely to be about the same beneath all parts of the mat and differential settlements are likely to be smaller than those of a spread foundation designed for the same soil pressure. The methods of calculating the ultimate bearing capacity dealt with in Chapter 2 are also applicable to mat foundations.
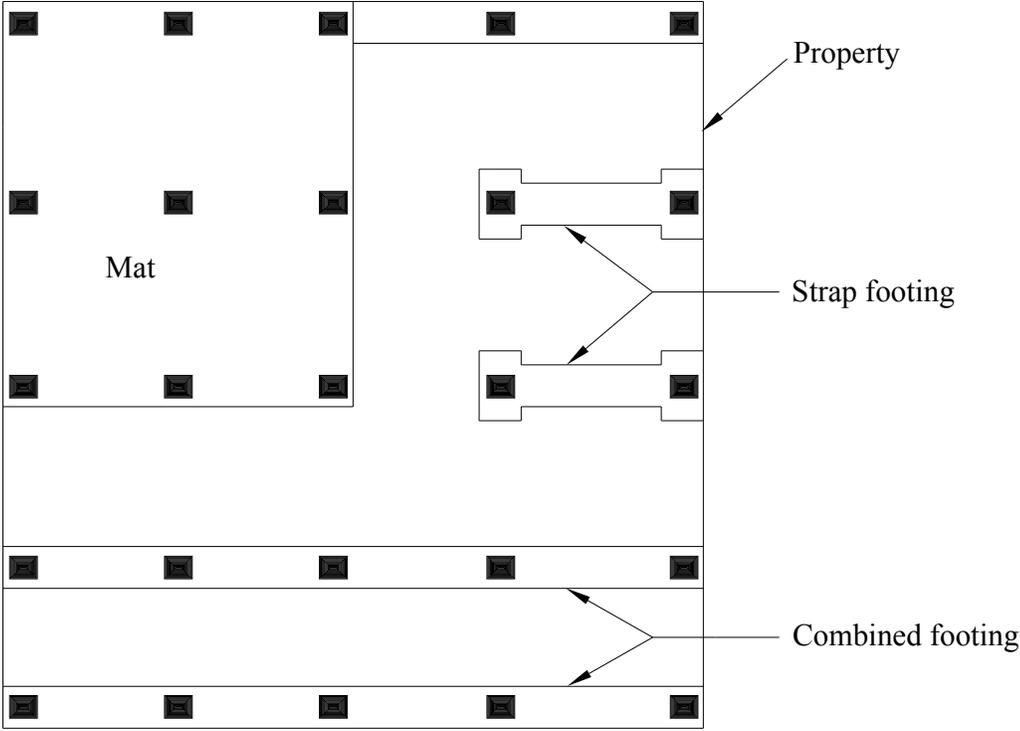
3.3 Mat Foundation in Clay

The net ultimate bearing capacity that can be sustained by the soil at the base of a mat on a deep deposit of clay or plastic silt may be obtained in the same manner as for footings on clay discussed in Chapter 2. However, by using the principle of flotation, the pressure on the base of the mat that induces settlement can be reduced by increasing the depth of the foundation. A brief discussion on the principle of flotation is dealt with in this chapter.

3.4 Rigid and Elastic Foundation

The conventional method of design of combined footings and mat foundations is to assume the foundation as infinitely rigid and the contact pressure is assumed to have a planar distribution. In the case of an elastic foundation, the soil is assumed to be a truly elastic solid obeying Hooke's law in all directions. The design of an elastic foundation requires knowledge of the subgrade reaction which is briefly discussed here. However, the elastic method does not

readily lend itself to engineering applications because it is extremely difficult and solutions are available for only a few extremely simple cases.

(a) Schematic plan showing
mat, strap and combined footings
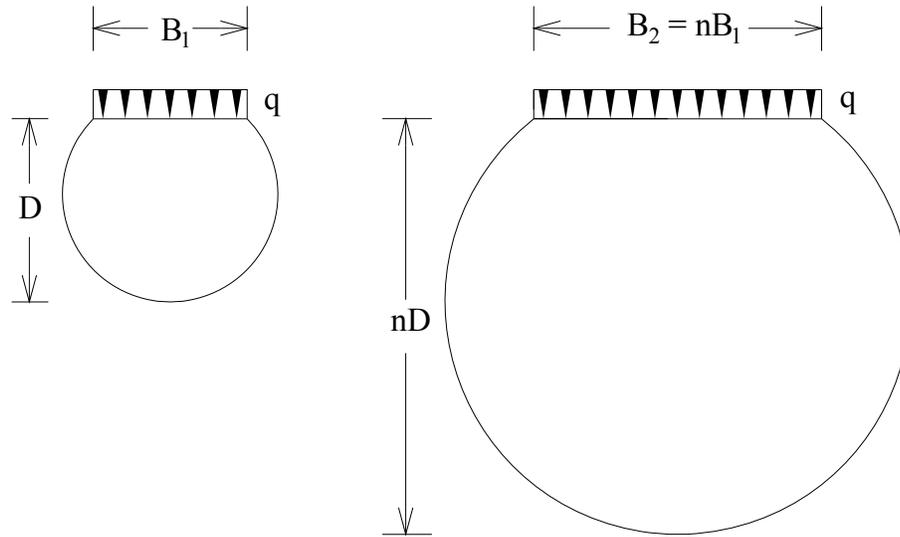
(b) Bulb of pressure for vertical stress for different beams

Fig 3.1 (a) Types of footings; (b) Beams on compressible subgrade

3.5 Safe Bearing Pressures for Mat Foundations on Sand and Clay

3.5.1 Mats on Sand

Because the differential settlements of a mat foundation are less than those of a spread foundation designed for the same soil pressure, it is reasonable to permit larger safe soil pressures on a raft foundation. Experience has shown that a pressure approximately twice as great as that allowed for individual footings may be used because it does not lead to detrimental differential settlements. The maximum settlement of a mat may be about 50 mm (2 in) instead of 25 mm as for a spread foundation.

The shape of the curve in Fig 3.1(b) shows that the net soil pressure corresponding to a given settlement is practically independent of the width of the footing or mat when the width becomes large. The safe soil pressure for design may with sufficient accuracy be taken as twice the pressure. Peck et al., (1974) recommend the following equation for computing net safe pressure,

$$q_s = 21 \, N_{corr} \, kPa \qquad\qquad ……Eq. 3.1$$

$$\text{for } 5 < N_{corr} < 50$$

where, N = SPT value corrected for energy, overburden pressure and field procedures.

Peck et al., (1974) also recommend that the q_s values as given by Eq. 3.1 may be increased somewhat if bedrock is encountered at a depth less than about one half the width of the raft. The value of N to be considered is the average of the values obtained up to a depth equal to the least width of the raft. If the average value of N after correction for the influence of overburden pressure and dilatancy is less than about 5, Peck et al., say that the sand is generally considered to be too loose for the successful use of a raft foundation. Either the sand should be compacted or else the foundation should be established on piles or piers.

The minimum depth of foundation recommended for a raft is about 2.5 m below the surrounding ground surface. Experience has shown that if the surcharge is less than this amount, the edges of the raft settle appreciably more than the interior because of a lack of confinement of the sand.

3.5.2 Mats on Clay

'q_{nu}' is the net bearing capacity at the elevation of the base of the raft in excess of that exerted by the surrounding surcharge. By increasing the depth of excavation, the pressure that can safely be exerted by the building is correspondingly increased. This aspect of the problem is considered further in Section 3.10 in floating foundation. As for footings on clay, the factor of safety against failure of the soil beneath a mat on clay should not be less than 3 under normal loads, or less than 2 under the most extreme loads. The net safe pressure should be decided on the basis of the permissible settlement.

3.6 Eccentric Loading

When the resultant of loads on a footing does not pass through the center of the footing, the footing is subjected to what is called eccentric loading. The loads on the footing may be vertical or inclined. If the loads are inclined it may be assumed that the horizontal component is resisted by the frictional resistance offered by the base of the footing. The vertical component in such a case is the only factor for the design of the footing.

3.6.1 Proportioning of Cantilever Footing

Strap or cantilever footings are designed on the basis of the following assumptions:

1. The strap is infinitely stiff. It serves to transfer the column loads to the soil with equal and uniform soil pressure under both the footings.
2. The strap is a pure flexural member and does not take soil reaction. To avoid bearing on the bottom of the strap a few centimeters of the underlying soil may be loosened prior to the placement of concrete.

A strap footing is used to connect an eccentrically loaded column footing close to the property line to an interior column as shown in Fig. 3.2. With the above assumptions, the design of a strap footing is a simple procedure. It starts with a trial value of e, Fig. 3.2. Then the reactions R_1 and R_2 are computed by the principle of statics. The tentative footing areas are equal to the reactions R_1 and R_2 divided by the safe bearing pressure q_s. With tentative footing sizes, the value of e is computed. These steps are repeated until the trial value of e is identical with the final one. The shears and moments in the strap are determined, and the straps designed to withstand the shear and moments. The footings are assumed to be subjected to uniform soil pressure and designed as simple spread footings. Under the assumptions given above the resultants of the column loads Q_1 and Q_2 would coincide with the center of gravity of the two footing areas. Theoretically, the bearing pressure would be uniform under both the footings. However, it is possible that sometimes the full design live load acts upon one of the columns while the other may be subjected to a small amount of live load. In such a case, the full reduction of column load from R_1 to R_2 may not be realized. It seems justified then that in designing the footing under column Q_2, only the dead load or dead load plus reduced live load should be used on column Q_1.

The equations for determining the position of the reactions (Fig 3.2) are

$$R_1 = Q_1 \left[1 + \frac{e}{L_R} \right]$$

$$R_2 = Q_2 - \frac{Q_1 e}{L_R}$$

……Eq 3.2

where R_1 and R_2 = reactions for the column loads Q_1 and Q_2 respectively

e = distance of R_1 from Q_1
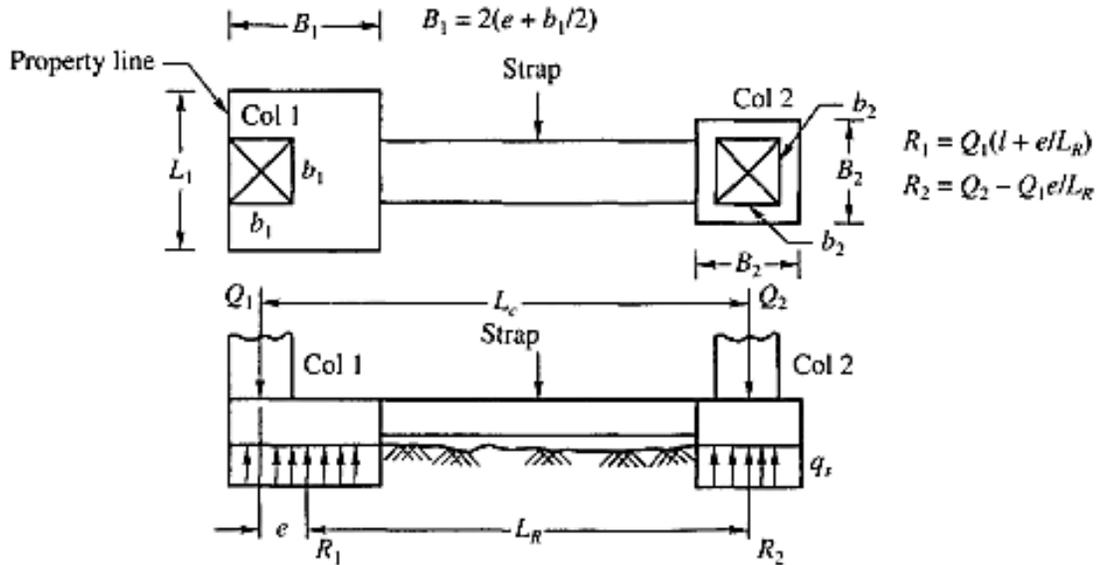
L_R = distance between R_1 and R_2.

Fig 3.2 Principles of cantilever or strap footing design

3.7 Design of Combined Footings by Rigid Method (Conventional Method)

The rigid method of design of combined footings assumes that

1. The footing or mat is infinitely rigid, hence, the deflection of the footing or mat does not influence the pressure distribution,

2. The soil pressure is distributed in a straight line or a plane surface such that the centroid of the soil pressure coincides with the line of action of the resultant force of all the loads acting on the foundation.

3.8 Design of Combined Footings

Two or more columns in a row joined together by a stiff continuous footing form a combined footing as shown in Fig. 3.3(a). The procedure of design for a combined footing is as follows:

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1. Determine the total column loads $\sum Q = Q_1 + Q_2 + Q_3$ ... and location of the line of action of the resultant $\sum Q$. If any column is subjected to bending moment, the effect of the moment should be taken into account.

2. Determine the pressure distribution q per lineal length of footing.

3. Determine the width B, of the footing.

4. Draw the shear diagram along the length of the footing. By definition, the shear at any section along the beam is equal to the summation of all vertical forces to the left or right of the section. For example, the shear at a section immediately to the left of Q_1 is equal to the area abed, and immediately to the right of Q_1 is equal to (abcd - Q_1) as shown in Fig. 3.3(a).

5. Draw the moment diagram along the length of the footing. By definition the bending moment at any section is equal to the summation of moment due to all the forces and reaction to the left (or right) of the section. It is also equal to the area under the shear diagram to the left (or right) of the section.

6. Design the footing as a continuous beam to resist the shear and moment.

7. Design the footing for transverse bending in the same manner as for spread footings.
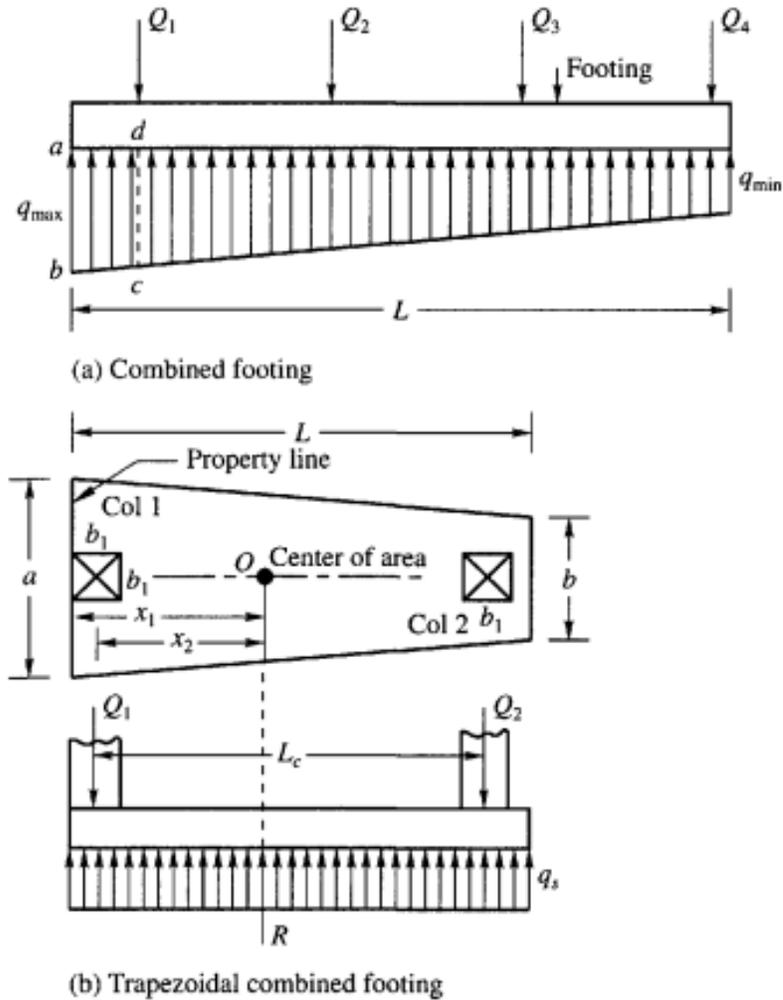
Fig 3.3 Combined or Trapezoidal footing design

It should be noted here that the end column along the property line may be connected to the interior column by a rectangular or trapezoidal footing. In such a case no strap is required and both the columns together will be a combined footing as shown in Fig. 3.3b. It is necessary that the center of area of the footing must coincide with the center of loading for the pressure to remain uniform.

3.9 Design of Mat Foundation by Rigid Method

In the conventional rigid method the mat is assumed to be infinitely rigid and the bearing pressure against the bottom of the mat follows a planar distribution where the centroid of

the bearing pressure coincides with the line of action of the resultant force of all loads acting on the mat. The procedure of design is as follows:

1. The column loads of all the columns coming from the superstructure are calculated as per standard practice. The loads include live and dead loads.
2. The line of action of the resultant of all the loads is found. However, the weight of the mat is not included in the structural design of the mat because every point of the mat is supported by the soil under it, causing no flexural stresses.
3. Calculate the soil pressure at desired locations by the use of Eq. 3.3

$$q = \frac{Q_r}{A} \pm \frac{Q_r e_x}{I_y} x \pm \frac{Q_r e_y}{I_x} y$$

……Eq 3.3

where $Q_r = \Sigma Q =$ total load on the mat

A = total area of the mat

x, y = coordinates of any given point on the mat with respect to the x andy axes passing through the centroid of the area of the mat

e_x , e_y = eccentricities of the resultant force

I_x , I_y = moments of inertia of the mat with respect to the x and y axes

respectively.

4. The mat is treated as a whole in each of two perpendicular directions. Thus the total shear force acting on any section cutting across the entire mat is equal to the arithmetic sum of all forces and reactions (bearing pressure) to the left (or right) of the section. The total bending moment acting on such a section is equal to the sum of all the moments to the left (or right) of the section.

3.10 Design of Combined Footings by Elastic Line Method

The relationship between deflection, y, at any point on an elastic beam and the corresponding bending moment M may be expressed by the equation

$$EI\frac{d^2y}{dx^2} = M$$

……Eq 3.4

The equations for shear V and reaction q at the same point may be expressed as

$$EI\frac{d^3y}{dx^3} = V$$

……Eq 3.5

$$EI\frac{d^4y}{dx^4} = q$$

……Eq 3.6

where, x is the coordinate along the length of the beam.

From basic assumption of an elastic foundation

$$q = -yBk_s$$

where, B = width of the footing,

k_s = coefficient of subgrade reaction

Substituting for q, Eq 3.6 may be written as

$$EI\frac{d^4y}{dx^4} = -yBk_s$$

……Eq 3.7

The classical solutions of Eq. (3.7) being of closed form, are not general in their application. Hetenyi (1946) developed equations for a load at any point along a beam. The development of solutions is based on the concept that the beam lies on a bed of

elastic springs and is based on Winkler's hypothesis. As per this hypothesis, the reaction at any point on the beam depends only on the deflection at that point. Methods are also available for solving the beam-problem on an elastic foundation by the method of finite differences (Malter, 1958). The finite element method has been found to be the most efficient of the methods for solving beam-elastic foundation problem. Computer programs are available for solving the problem. Since all the methods mentioned above are quite involved, they are not dealt with here. Interested readers may refer to Bowles (1996).

Winkler's (1867) hypothesis assumes that a soil medium may be approximated by a series of infinitely closely spaced independent elastic springs.Fig.3.4 (a) shows a loaded beam resting on an elastic foundation. The reaction at any point on the base of the beam is actually a function of every point along the beam since soil material exhibit varying degrees of continuity. The beam shown in Fig. 3.4 (a) can be replaced by a beam in Fig. 3.4 (c). In this figure the beam rests on a bed of elastic springs wherein each spring is independent of the other. According to Winkler's hypothesis the reaction at any point on the base of the beam in Fig. 3.4 (c)depends only on the deflection at that point.
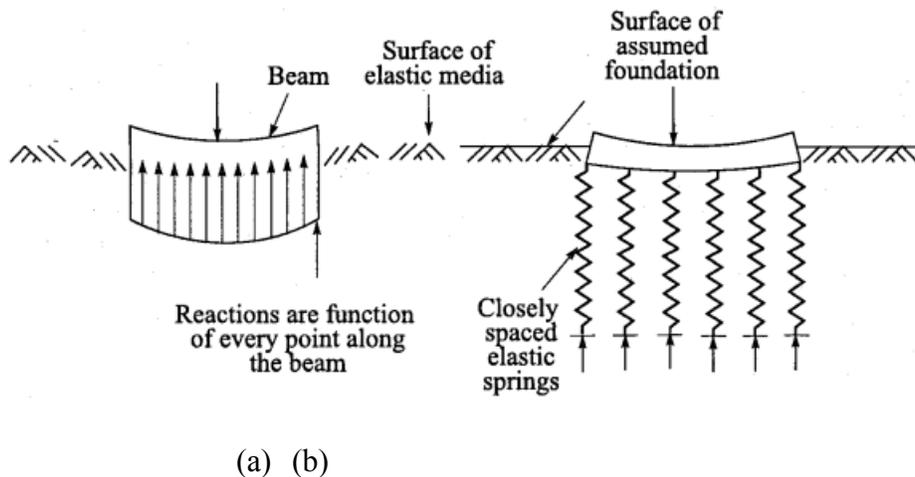

(a) (b)

Fig.3.4 Winkler hypothesis

3.11 Design of Mat Foundations by Elastic Plate Method

Of the many methods that are available for the design of mat-foundations, the one that is very much in use is the finite difference method. This method is based on the assumption that the subgrade can be substituted by a bed of uniformly distributed coil springs with a spring constant k_s called the coefficient of subgrade reaction. The finite difference method uses the fourth order differential equation.

$$\nabla^4 w = \frac{q - k_s w}{D} \ldots \qquad\qquad \text{Eq 3.8}$$

where, $\nabla^4 w = \dfrac{\partial^4 w}{\partial x^4} + 2\dfrac{\partial^4 w}{\partial x^2 \partial y^2} + \dfrac{\partial^4 w}{\partial y^4}$

q = subgrade reaction per unit area

k_s = coefficient of subgrade reaction

w = deflection

D = rigidity of the mat $= \dfrac{Et^3}{12(1 - \mu^2)}$

E = modulus of elasticity of the material of the footing

t = thickness of mat

μ = Poisson's ratio.

Eq. 3.8 may be solved by dividing the mat into suitable square grid elements, and writing difference equations for each of the grid points. By solving the simultaneous equations so obtained the deflections at all the grid points are obtained. The equations can be solved rapidly with an electronic computer. After the deflections are known, the bending moments are calculated using the relevant difference equations. Interested readers may refer to Teng (1969) or Bowles (1996) for a detailed discussion of the method.

3.12 Floating Foundation

3.12.1 General Consideration

A floating foundation for a building can be defined as a foundation in which the weight of the building is approximately equal to the full weight which includes water of the soil removed from the site of the building. With reference to Fig 3.5 this principle of flotation may be explained. Fig. 3.5(a) shows a horizontal ground surface with water table at a depth dw below

the ground surface. Fig. 3.5(b) shows an excavation made in the ground to a depth D where, $D > d_w$ and Fig. 3.5(c) shows a structure built in the excavation and completely filling it. If the weight of the building is equal to the weight of the soil and water removed from the excavation, then it is clear that the total vertical pressure in the soil below depth D in Fig. 3.5(c) is the same as in Fig. 3.5(a) before excavation. Since there is no change in the water level, the neutral pressure and the effective pressure remain unchanged. If we could move from Fig. 3.5(a) to Fig. 3.5(c) without coming across the intermediate case of 3.5(b), the building in Fig. 3.5(c) would not settle at all, since an increase in effective vertical pressure cause settlements. *Principle of a floating foundation: An exact balance of weight removed against weight imposed. The obtained result is zero settlement of the building*.

However, it may be noted, that we cannot jump from the stage shown in Fig. 3.5(a) to the stage in Fig. 3.5(c) without passing stage 3.5(b). This is because the excavation stage of the building is the critical stage.

There may occur cases where we cannot have a fully floating foundation. The foundations of this type are called "partly compensated foundations" (as against fully compensated or fully floating foundations).

While dealing with floating foundations, it is important that we consider the following two types of soils.

Type 1: The foundation soils are of such strength that shear failure of soil will not occur under the building load. But the settlements, differential settlements in particular are so large that they constitute the failure of the structure. In such cases a floating foundation is used to reduce settlements to an acceptable value.

Type 2: The shear strength of the foundation soil is so low that rupture of the soil would occur if the building were to be founded at ground level. At a reasonable depth, in the absence of a strong layer the building can only be built on a floating foundation. This reduces the shear stresses to an acceptable value. Solving this problem helps solve the settlement problem. In the above mentioned cases, a rigid raft or box type of foundation is required for the floating foundation as shown in Fig. 3.4(d)
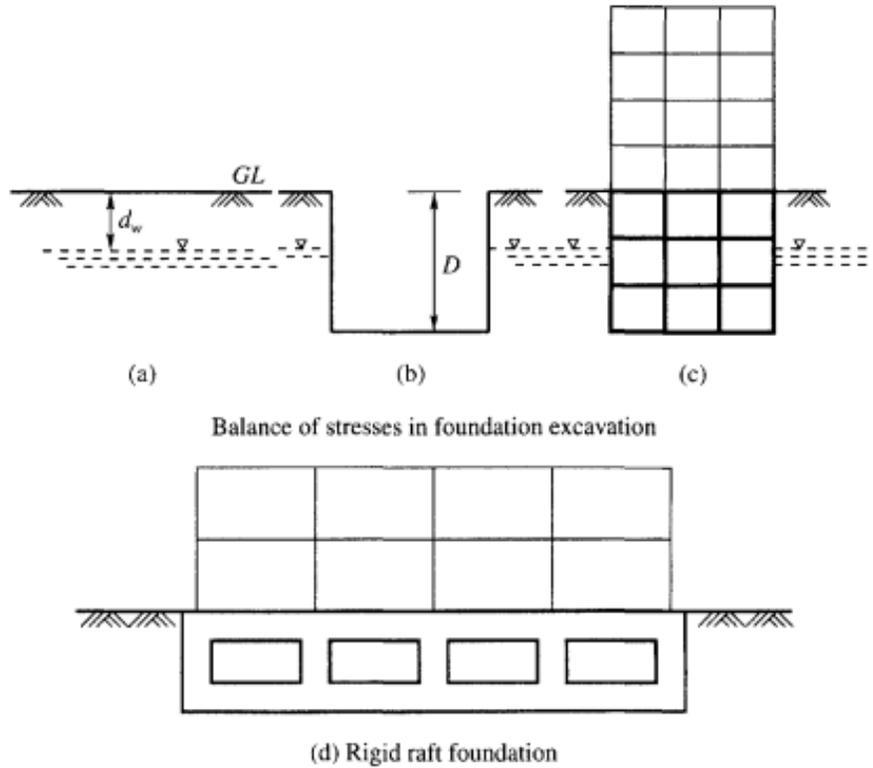
Fig 3.5 Principles of floating foundation; and a typical rigid raft foundation

3.12.2 Problems to be considered in the Design of a Floating Foundation

The following problems are to be considered during the design and construction stage of a floating foundation.

1. Excavation

   The excavation for the foundation has to be done with care. The sides of the excavation should suitably be supported by sheet piling, soldier piles and timber lagging or some other standard method.

2. Dewatering

   Dewatering will be necessary when excavation has to be taken below the water table level. Care has to be taken to see that the adjoining structures are not affected due to the lowering of the water table.

3. Critical depth

In Type 2 foundations the shear strength of the soil is low and there is a theoretical limit to the depth to which an excavation can be made. Terzaghi (1943) has proposed the following equation for computing the critical depth D_c,

$$D_c = \frac{5.7s}{\gamma - \left(\frac{S}{B}\right)\sqrt{2}}$$

……Eq 3.9

For an excavation which is long compared to its width

where, γ = unit weight of soil

s = shear strength of soil $= \dfrac{qu}{2}$

B = width of foundation

L = length of foundation

Skempton (1951) proposes the following equation for D_c, which is based on actual failures in excavations.

$$D_c = N_c \frac{s}{\gamma}$$

……Eq 3.10

or the factor of safety F_s against bottom failure for an excavation of depth D is

$$F_s = N_c \frac{s}{\gamma D + p}$$

where, N_c is the bearing capacity factor given by Skempton,

p is the surcharge load.

The above equations may be used to determine the maximum depth of excavation.

4. Bottom heave

Excavation for foundations reduces the pressure in the soil below the founding depth which results in the heaving of the bottom of the excavation. Any heave which occurs

will be reversed and appear as settlement during the construction of the foundation and the building. Though heaving of the bottom of the excavation cannot be avoided it can be minimized to a certain extent. There are three possible causes of heave:

1. Elastic movement of the soil as the existing overburden pressure is removed.

2. A gradual swelling of soil due to the intake of water if there is some delay in placing the foundation on the excavated bottom of the foundation.

3. Plastic inward movement of the surrounding soil.

The last movement of the soil can be avoided by providing proper lateral support to the excavated sides of the trench. Heaving can be minimized by phasing out excavation in narrow trenches and placing the foundation soon after excavation. It can be minimized by lowering the water table during the excavation process. Friction piles can also be used to minimize the heave. The piles are driven either before excavation commences or when the excavation is at half depth and the pile tops are pushed down to below foundation level. As excavation proceeds, the soil starts to expand but this movement is resisted by the upper part of the piles which go into tension. The heave is prevented or very much reduced. It is only a practical and pragmatic approach that would lead to a safe and sound settlement free floating (or partly floating) foundation.

3.13 Approximate Design of Raft Foundations

The design of raft foundations from a soil mechanics point of view is relatively simple and is similar to that of isolated footings. However the following points are to be noted.

1) Rafts on Cohesionless soil

The bearing capacity of raft foundations on sand is always very high due to size effects. Differential settlement governs the design but this is determined by the strength and stiffness of the raft structure and is very difficult to assess. The structural design of a raft is quite a difficult procedure due to the interaction between contact pressure distribution and deflected shape of the structure.

However the flexural action of a raft serves to reduce the differential settlements considerably i.e., if the raft is rigid (as for a grain silo) there will be zero differential

settlement, though tilting or uneven settlement of the structure as a whole may occur. The normal design rule is therefore to allow twice the total settlement allowed for isolated footings. This procedure, is, however quite empirical. The pressure which produces settlement is the net building pressure since the excavated soil is not replaced.

2) Rafts on Saturated Cohesive soil

It was noted earlier that, contrary to sands, the allowable bearing pressure for structures on clay does not increase with foundation size. Consequently, rafts founded on saturated clay have to be checked for both bearing capacity and settlement since either may govern the design.

3.14 Rectangular Combined Footings

Space restrictions or architectural features may require one footing to carry at least two column loads as shown in Fig 3.6 Conventional design and analysis are based on the assumption that the footing is infinitely stiff and that stress distribution on the base of the footing is planar.
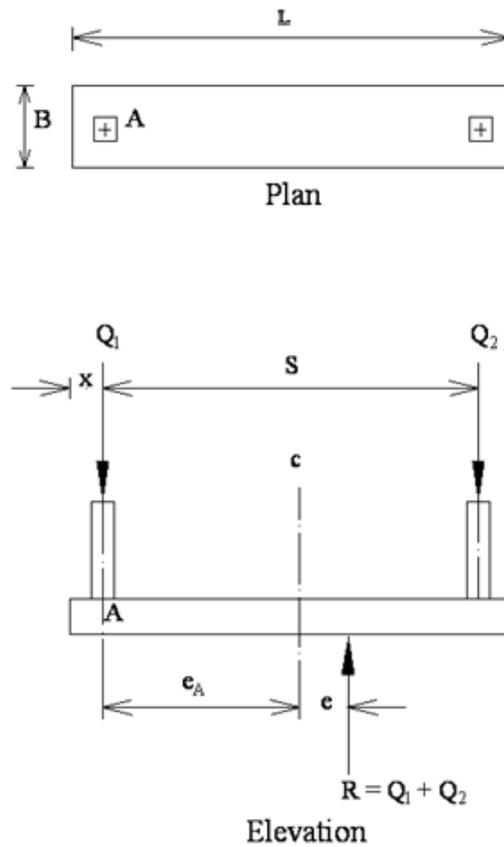
Fig 3.6 Combined Footing

1) Assume Q_1 , Q_2 and S are known; therefore $\Sigma Q = Q_1 + Q_2$

2) Since $e = \dfrac{\Sigma M}{\Sigma Q}$, the location of the resultant force ΣQ with respect to any point

   may be obtained by taking moments about that point. Hence, taking moments about A.

$$e_a = \frac{\Sigma M}{\Sigma Q} = \frac{Q_2 S}{Q_1 + Q_2}$$

……Eq 3.11

3) For uniform stress distribution, the required length L of the footing is:

$$L = 2(e_a + x)$$

……Eq 3.12

4) The required width B of the footing is:

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$$B = \frac{Q_1 + Q_2}{q_{allow} \times L}$$

<div align="right">……Eq 3.13</div>

The values of B and L are usually rounded to a convenient value and the actual values of q_{max} and q_{min} are computed from

$$q = \frac{\sum Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

<div align="right">……Eq 3.14</div>

where, e = eccentricity, with respect to the centre of gravity of the footing of the resultant column load.