6.4 Binary Choice Models

6.4.1 Binary Logit Model

These are the simplest type of mode choice models. These models compare the travel choices between two modes. Say $C^m_{ij}$ is the generalized Cost of travel between zone ‘i’ and ‘j’ using a mode m, then:

- If $C^2_{ij} - C^1_{ij} = +ve$ then mode 1 would be chosen
- If $C^2_{ij} - C^1_{ij} = -ve$ then mode 2 would be chosen
- If $C^2_{ij} - C^1_{ij} = 0$ then both the model have been equal probability of being chosen.

The probability of choosing mode for a trip between zones $i$ & $j$ is given by:

$$P^1_{ij} = \frac{T_{ij}}{\sum T_{ij}} = \frac{e^{-\alpha c^1_{ij}}}{e^{-\alpha c^1_{ij}} + e^{-\alpha c^2_{ij}}} \quad (6.5)$$

$$P^2_{ij} = \frac{e^{\alpha c^2_{ij}}}{e^{-\alpha c^1_{ij}} + e^{-\alpha c^2_{ij}}} \quad (6.6)$$

Where, $\alpha$ = coefficients related to variables, the coefficient related to generic variables are negative indicating the decrease in the utility of a mode with the increase in the related attributes (here travel time & travel Cost)

6.4.2 Discriminant Analysis

The basic principal of discriminant analysis is that the choice of trip makers in an urban area may be classified into two groups according to the modes of transport used. The objective is to find the liner combination of explanatory variables for the two groups of trip maker that possesses little overlap.

Figure 6.3 explains the essence of this method graphically. The diagram shows the frequency distribution of the values of disutility measure ($z$) for the users of the two transport modes being observed. The best estimate of the probability of trip maker with a given magnitude, say $z'$, of choosing mode 2, is the ratio of the ordinate of the mode 2 distribution at $z'$ to sum of the ordinates of the mode 1 and 2 ordinates at $z'$.
Figure 6.3: Discriminant Function of Estimation Modal Choice

The Best discriminant function is the one minimizes the number of misclassification of trip makers to their observed transport modes.

Quarmby had used discriminant analysis method to develop relationship for estimating car-bus model split for work trips to central London. The disutility measure was developed as a function of differences in total travel time, costs and income related variables.

\[
p(c|z) = \frac{2.26e^{1.04(z-0.431)}}{1 + 2.26e^{1.04(z-0.431)}}
\]

\( p(c|z) \) = The probability of choosing the car mode – given that the travel disutility is \( z \)

Talvitie assumed that \( z \) is normally distributed and developed for estimating the probability of model patronage in binary choice situation.

\[
P(m = 1|ij) = \frac{e^{x+\ln(x/y)}}{1 + e^{x+\ln(x/y)}}
\]

\[
P(m = 2|ij) = \frac{1}{1 + e^{x+\ln(x/y)}}
\]

Where \( P(m|ij) \) = the probability that an individual will use mode \( m \) given that the trip is between zones \( i \) and \( j \)

\( x, y \) = The priori probability of membership in groups \( m = 1 \) and \( m = 2 \) respectively
6.4.3 Probit Analysis

The probit analysis is based on the principle that as choice tripmakers are subjected to changing magnitudes of relative trip costs, the proportion of the tripmakers that respond by choosing a particular mode of transport will follow a relationship shown in the following figure. Figure 6.4 is a theoretical representation of the proportion of tripmakers that would choose private cars as the difference in the generalized cost of using public transport and car transport varied.

![Figure 6.4: Probit Type Model Function](image)

(i) Lave has developed the following equation for estimating the probability of bas-car model using Probit analysis

\[
Y = -2.08 + 0.00759 \times KW\Delta T + 0.0186\Delta C -0.0254 \times IDC_C + 0.0255A
\]

\[R^2 = 0.379\]

Where,

- \(Y\) = Binary variable with positive magnitude denoting transit riders and negative magnitude car riders.
- \(KW\Delta T\) = the difference the modes multiply by the trip maker’s wage rate and this marginal preference for leisure time.
- \(IDC_C\) = a binary valued comfort variables multiplied income and trip distance.
A = age of trip maker

(ii) Stopher has used a technique known as logit analysis to construct a stochastic model split function to logit model has the following form:

\[
P(m = 1/ij) = \frac{e^{Z_{ij}^*}}{1 + e^{Z_{ij}^*}}
\]

\[
P(m = 2/ij) = \frac{1}{1 + e^{Z_{ij}^*}}
\]

Where,

\[
Z_{ij}^* = \text{some function of the generalized costs of travel by models } m = 1 \text{ & } m = 2
\]

Stopher has proposed a modal split model of the following form:

\[
P(m = c) = a (c_1 - c_2) + b (t_2 - t_1) + d
\]

Where,

\[
P(m = c) = \text{the probability of using a car for trip making.}
\]

\[
c_1, c_2 = \text{the out of Pocket costs of travel by car and public transport, respectively.}
\]

\[
t_1, t_2 = \text{the travel time by a car and public transport respectively.}
\]

\[
a, b = \text{Parameters determined empirically with the ratio } b/a \text{ representing the implied value of travel time.}
\]

**Comparison between Talvitie & Lave, Stopher equation**

Talvitie concluded from his study that: “The methods of estimation, commonly used in probabilistic model choice model, probit, logit and discriminant analysis, all yielded comparable result, any of them can be used with equal success. On the other hand Stopher and Lave concluded from their study that. It was found that discriminant analysis was clearly inferior to either Probit or logit analysis. Since probit analysis requires a more time consuming calibration procedure that logit analysis, and yields a more cumbersome model, another recommended that logit analysis be considered as the preferred technique for these model.