Project Planning & Control

Lesson 5
PERT Example Problem, Summary

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Determine

(i) Project Duration = ?

(ii) Critical path ?

(iii) \( P(PD \geq 95d) = ? \)

(iv) \( P(PD \leq 85d) = ? \)

(iv) \( P(85d \leq PD \leq 95d) = ? \)

(v) \( P(PD \leq ? ) = 0.90 \)
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\[
(t_e) = \frac{(a + 4m + b)}{6}
\]

\[
\sigma (t_e) = \frac{(b-a)}{6}
\]

\[
\sigma^2 (t_e) = \frac{(b-a)^2}{6}
\]

\[
(t_{eA}) = \frac{(10 + 4 \times 16 + 20)}{6} = 15.667 = 15.7\text{ days}
\]

\[
(t_{eE}) = \frac{(25 + 4 \times 30 + 32)}{6} = 29.5\text{ days}
\]

\[
\sigma (t_{eA}) = \frac{(20-10)}{6} = 1.6667 = 1.67\text{ days}
\]

\[
\sigma (t_{eE}) = \frac{(32-25)}{6} = 1.1667 = 1.17\text{ days}
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Var = 10.44
STDEV = 3.2 days

μ=89.0d
σ=3.2d
(iii) \( P(PD \geq 95) = ? \)

\[
z = \frac{95 - 89}{3.2} = 1.88
\]

\[
P( PD \geq 95) = (1 - 0.9699) = 0.0301
\]
(iv) \( P(PD \leq \{ \} ) = \) ?

\[
z = \frac{x - \mu}{\sigma} = \frac{85 - 89}{3.2} = -1.25
\]

\[P(P \leq 85) = 0.1056\]
(iv) \( P(85d \leq PD \leq 95d) = ? \)

\[ P(P<85) = 0.1056 \]

\[ P( PD > 95) = 0.0301 \]

\[ z = \frac{x-\mu}{\sigma} \]

\[ P(85d \leq PD \leq 95d) = (1 - 0.0301 - 0.1056) = 0.8643 \]
\( (v) \quad P(PD < ?) = 0.90 \)

\[
? = 89 + 1.281667 \times 3.2 = 93.10 \text{ days}
\]

Interpolation.

\[
1.28 + (0.09 - 0.08) \times (0.90 - 0.8997)/(0.9015 - 0.8997) = 1.281667 \quad (SD \text{ From Mean})
\]
EXAMPLE Results

(i) Project Duration = **89 days**  
\[ \sigma = 3.2 \text{d} \]

(ii) Critical path- **ABEGH**

(iii) \( P(PD \geq 95\text{d}) = 0.0301 \)

(iv) \( P(PD \leq 85\text{d}) = 0.1056 \)

(iv) \( P(85\text{d} \leq PD \leq 95\text{d}) = 0.8643 \)

(v) \( P(PD \leq 93.10\text{ days}) = 0.90 \)

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**FLOATS ETC** can also be calculated as in CPM AOA/AON

**Numbers rounded off**

\[ \mu = 89.0\text{d} \  \  \  \ \sigma = 3.2\text{d} \]
Summary

• Probabilistic scheduling – Data Driven or Expert Opinion for distribution mapping

• Simulation or PERT type approach

• PERT is based several assumptions – value is in enabling the project team to discuss and quantify risks

• PERT can be used to generate several interesting results
QUESTIONS & DISCUSSIONS