5.3 Design for Shear (Part II)

This section covers the following topics.

- Design of Transverse Reinforcement
- Detailing Requirements
- Design Steps

5.3.1 Design of Transverse Reinforcement

When the shear demand \(V_u\) exceeds the shear capacity of concrete \(V_c\), transverse reinforcements in the form of stirrups are required. The stirrups resist the propagation of diagonal cracks, thus checking diagonal tension failure and shear tension failure.

The stirrups resist a failure due to shear by several ways. The functions of stirrups are listed below.

1) Stirrups resist part of the applied shear.
2) They restrict the growth of diagonal cracks.
3) The stirrups counteract widening of the diagonal cracks, thus maintaining aggregate interlock to a certain extent.
4) The splitting of concrete cover is restrained by the stirrups, by reducing dowel forces in the longitudinal bars.

After cracking, the beam is viewed as a plane truss. The top chord and the diagonals are made of concrete struts. The bottom chord and the verticals are made of steel reinforcement ties. Based on this truss analogy, for the ultimate limit state, the total area of the legs of the stirrups \(A_{sv}\) is given as follows.

\[
\frac{A_{sv}}{s} = \frac{V_u - V_c}{0.87f_y d_t}
\]

The notations in the above equation are explained.

- \(s\) = spacing of the stirrups
- \(d_t\) = greater of \(d_p\) or \(d_s\)
- \(d_p\) = depth of CGS from the extreme compression fiber
- \(d_s\) = depth of centroid of non-prestressed steel
- \(f_y\) = yield stress of the stirrups
The grade of steel for stirrups should be restricted to Fe 415 or lower.

**Design of Stirrups for Flanges**

For flanged sections, although the web carries the vertical shear stress, there is shear stress in the flanges due to the effect of shear lag. Horizontal reinforcement in the form of single leg or closed stirrups is provided in the flanges. The following figure shows the shear stress in the flange at the face of the web.

![Figure 5-3.1 Shear stress in flange due to shear lag effect](image)

The horizontal reinforcement is calculated based on the shear force in the flange. The relevant quantities for the calculation based on an elastic analysis are as follows.

1) Shear flow (shear stress × width)
2) Variation of shear stress in a flange ($\tau_f$)
3) Shear forces in flanges ($V_f$).
4) Ultimate vertical shear force ($V_u$)

The following sketch shows the above quantities for an I-section (with flanges of constant widths).

![Figure 5-3.2 Shear flow and shear forces in an I-section](image)
The design shear force in a flange is given as follows.

\[ V_f = \frac{\tau_{f,\text{max}} b_f D_f}{2} \]  \hspace{1cm} (5-3.2)

Here,

- \( b_f \) = breadth of the flange
- \( D_f \) = depth of the flange
- \( \tau_{f,\text{max}} \) = maximum shear stress in the flange.

The maximum shear stress in the flange is given by an expression similar to that for the shear stress in web.

\[ \tau_{f,\text{max}} = \frac{V_u A_1 \bar{y}}{I D_f} \]  \hspace{1cm} (5-3.3)

Here,

- \( V_u \) = ultimate vertical shear force
- \( I \) = moment of inertia of the section.
- \( A_1 \) = area of half of the flange
- \( \bar{y} \) = distance of centroid of half of the flange from the neutral axis at CGC.

**Figure 5-3.3** Cross-section of a beam showing the variables for calculating shear stress in the flange

The amount of horizontal reinforcement in the flange (\( A_{svf} \)) is calculated from \( V_f \).

\[ A_{svf} = \frac{V_f}{0.87 f_y} \]  \hspace{1cm} (5-3.4)

The yield stress of the reinforcement is denoted as \( f_y \).
5.3.2 Detailing Requirements

The detailing requirements for the stirrups in IS:1343 - 1980 are briefly mentioned.

Maximum Spacing of Stirrups

The spacing of stirrups \( s_v \) is restricted so that a diagonal crack is intercepted by at least one stirrup. This is explained by the following sketch.

\[ \begin{align*}
& \text{Cross-section} \\
& \text{Elevation} \\
\end{align*} \]

**Figure 5-3.4  Cross-section and elevation of a beam showing stirrups**

As per **Clause 22.4.3.2**, the maximum spacing is \( 0.75d_t \) or \( 4b_w \), whichever is smaller.

When \( V_u \) is larger than \( 1.8V_c \), the maximum spacing is \( 0.5d_t \).

The variables are as follows.

- \( b_w \) = breadth of web
- \( d_t \) = greater of \( d_p \) or \( d_s \)
- \( d_p \) = depth of CGS from the extreme compression fiber
- \( d_s \) = depth of centroid of non-prestressed steel
- \( V_u \) = shear force at a section due to ultimate loads
- \( V_c \) = shear capacity of concrete.

Minimum Amount of Stirrups

A minimum amount of stirrups is necessary to restrict the growth of diagonal cracks and subsequent shear failure. For \( V_u < V_c \), minimum amount of transverse reinforcement is provided based on the following equation.

\[
\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87f_y} \tag{5-3.5}
\]
\[ b = \text{breadth of the section} \]
\[ = b_w, \text{breadth of the web for flanged sections.} \]

If \( V_u < 0.5V_c \) and the member is of minor importance, stirrups may not be provided.

Another provision for minimum amount of stirrups \( (A_{sv,\text{min}}) \) is given by \textbf{Clause 18.6.3.2} for beams with thin webs. The minimum amount of stirrups is given in terms of \( A_{wh}, \) the horizontal sectional area of the web in plan. The area is shown in the following sketch.

![Elevation and horizontal section of a beam showing stirrups](image)

**Figure 5-3.5** Elevation and horizontal section of a beam showing stirrups

In presence of dynamic load,
\[
A_{sv,\text{min}} = 0.3\% \ A_{wh} \\
= 0.2\% \ A_{wh}, \text{ when } h \leq 4b_w
\]

With high strength bars,
\[
A_{sv,\text{min}} = 0.2\% \ A_{wh} \\
= 0.15\% \ A_{wh}, \text{ when } h \leq 4b_w
\]

In absence of dynamic load, when \( h > 4b_w \)
\[
A_{sv,\text{min}} = 0.1\% \ A_{wh}
\]

There is no specification for \( A_{sv,\text{min}} \) when \( h \leq 4b_w \).
Anchorage of Stirrups
The stirrups should be anchored to develop the yield stress in the vertical legs.

1) The stirrups should be bent close to the compression and tension surfaces, satisfying the minimum cover.
2) Each bend of the stirrups should be around a longitudinal bar. The diameter of the longitudinal bar should not be less than the diameter of stirrups.
3) The ends of the stirrups should be anchored by standard hooks.
4) There should not be any bend in a re-entrant corner. In a re-entrant corner, the stirrup under tension has the possibility to straighten, thus breaking the cover concrete.

The following sketches explain the requirement of avoiding the bend of a stirrup at a re-entrant corner.

![Incorrect detailing](image1)

![Correct detailing](image2)

*Figure 5-3.6  Cross-section of the bottom flange of a beam showing stirrups*

Minimum Thickness (Breadth) of Web
To check web crushing failure, The **Indian Roads Congress Code IRC:18 - 2000** specifies a minimum thickness of the web for T-sections ([Clause 9.3.1.1](#)). The minimum thickness is 200 mm plus diameter of the duct hole.

5.3.3 Design Steps

The following quantities are known.

- $V_u$ = factored shear at ultimate loads. For gravity loads, this is calculated from $V_{DL}$ and $V_{LL}$.
- $V_{DL}$ = shear due to dead load
- $V_{LL}$ = shear due to live load.
After a member is designed for flexure, the self-weight is known. It is included as dead load.

The grade of concrete is known from flexure design. The grade of steel for stirrups is selected before the design for shear. As per **IS:1343 - 1980**, the grade of steel is limited to Fe 415.

The following quantities are unknown.
- \( V_c \) = shear carried by concrete
- \( A_{sv} \) = total area of the legs of stirrups within a distance \( s_v \)
- \( s_v \) = spacing of stirrups.

The steps for designing stirrups along the length of a beam are given below.

1) Calculate the shear demand \( V_u \) at the critical location.

2) Check \( \frac{V_u}{bd_t} < \tau_{c,\text{max}} \). If it is not satisfied, increase the depth or breadth of the section. Here, \( b \) is the breadth of the web \( (b_w) \) and \( d_t \) is larger of \( d_p \) and \( d_s \).

3) Calculate the shear capacity of concrete \( V_c \) from the lower of \( V_{co} \) and \( V_{cr} \). In presence of inclined tendons or vertical prestress, the vertical component of the prestressing force \( (V_p) \) can be added to \( V_{c0} \).

4) Calculate the requirement of shear reinforcement through \( A_{sv} / s_v \). Compare the value with the minimum requirement.

5) Calculate the maximum spacing and round it off to a multiple of 5 mm.

6) Calculate the size and number of legs of the stirrups based on the amount required, type of section and space to accommodate.

Repeat the calculations for other locations of the beam, if the spacing of stirrups needs to be varied.
Example 5-3.1

Design the stirrups for the Type 1 prestressed beam with the following section (location of tendons shown at mid span).

Longitudinal reinforcement of 12 mm diameter is provided to hold the stirrups.

The properties of the sections are as follows.
\[ A = 159,000 \text{ mm}^2, \]
\[ I = 1.7808 \times 10^{10} \text{ mm}^4 \]
\[ A_p = 960 \text{ mm}^2 \]

The grade of concrete is M 35 and the characteristic strength of the prestressing steel \( f_{pk} \) is 1470 N/mm². The effective prestress \( f_{pe} \) is 860 N/mm².

The uniformly distributed load including self weight, is \( w_T = 30.2 \text{ kN/m} \).

The span of the beam (\( L \)) is 10.7 m. The width of the bearings is 400 mm. The clear cover to longitudinal reinforcement is 30 mm.

Solution

1) Calculate \( V_u \) at the face of the support (neglecting the effect of compression in concrete).

\[
V_u = 1.5 \times w_T \left( \frac{L}{2} - x \right)
\]
\[
= 1.5 \times 30.2 \times \left( \frac{10.7}{2} - 0.2 \right)
\]
\[
= 233.3 \text{ kN}
\]
Here, $x$ denotes half of the width of bearing. $x = 200$ mm.

2) Check $(V_u / bd_t) < \tau_{c,max}$.

Effective depth $d_t = \text{total depth} - \text{cover} - \text{diameter of stirrups} - \frac{1}{2} \text{diameter of longitudinal bar}$.

Assume the diameter of stirrups to be 8 mm.

\[d_t = \left(920 - 30 - 8 - \frac{1}{2} \times 12\right)\]
\[= 876 \text{ mm}\]

\[\frac{V_u}{b_u d_t} = \frac{233.3 \times 10^3}{100 \times 876}\]
\[= 2.7 \text{ N/mm}^2\]

$\tau_{c,max}$ for M 35 is 3.7 N/mm². Hence, $(V_u / bd_t) < \tau_{c,max}$.

3) Calculate $V_c$ from the lower of $V_{c0}$ and $V_{cr}$.

\[V_{c0} = 0.67bD \sqrt{f_t^2 + 0.8f_{cp}f_t}\]

Here,

\[f_t = 0.24 \sqrt{35}\]
\[= 1.42 \text{ N/mm}^2\]

\[f_{cp} = \frac{P_e}{A}\]
\[= \frac{826 \times 10^3}{159,000}\]
\[= 5.19 \text{ N/mm}^2\]

\[V_{c0} = 0.67 \times 100 \times 920 \sqrt{1.42^2 + 0.8 \times 5.19 \times 1.42}\]
\[= 173.4 \text{ kN}\]

The vertical component of the prestressing force can be found out from the equation of the parabolic tendon.

\[y = \frac{4y_m}{L^2} x (L - x)\]
The following is the expression of the slope of the parabolic tendon.

\[
\tan \theta = \frac{dy}{dx} = \frac{4y_m}{L^2} (L - 2x)
\]

At \(x = 0.2\) m, \(y = 20\) mm, \(dy/dx = 0.105\) and \(\theta = 6.0^\circ\).

\[
V_p = P_e \sin \theta = 826 \times 0.104 = 86.0 \text{kN}
\]

\[
V_{co} + V_p = 173.4 + 86.0 = 259.4 \text{kN}
\]

\[
V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pk}}\right) \tau_{cd} bd + M_0 \frac{V_u}{M_u}
\]

Here,

\[
\frac{f_{pe}}{f_{pk}} = \frac{860}{1470} = 0.58
\]

\[
\frac{100A_p}{bd} = \frac{100 \times 960}{100 \times 480} = 2.0
\]

\[
d = 460 + y = 460 + 20 = 480 \text{ mm}
\]
From Table 6, for M 35 concrete, \( \tau_c = 0.86 \text{ N/mm}^2 \).

\[
M_0 = 0.8 f_{pt} \frac{l}{y}
\]

Here,

\[
f_{pt} = \frac{P_e}{A} \frac{P_x y}{l} = \frac{826 \times 10^3}{159,000} \times \frac{826 \times 10^3 \times 20}{1.7808 \times 10^{10} \times 20} = -5.19 - 0.02 = -5.21 \text{ N/mm}^2
\]

\[
M_0 = 0.8 \times 5.21 \times \frac{1.7808 \times 10^{10}}{20} = 3711.2 \times 10^6 \text{ Nmm} = 3711.2 \text{ kNm}
\]

At the critical section,

\[
M_u = 1.5 w \frac{x}{2} (L - x)
\]

\[
= 1.5 \times 30.2 \times \frac{0.2}{2} (10.7 - 0.2) = 47.6 \text{ kNm}
\]

Therefore,

\[
V_{cr} = \left(1 - 0.55 \frac{f_{pt}}{f_{pk}} \right) \tau_c b d + M_0 \frac{V_u}{M_u}
\]

\[
V_{cr} = (1 - 0.55 \times 0.58) \times \frac{0.86}{10^3} \times 100 \times 480 + 3711.2 \times \frac{233.3}{47.6} = 28.1 + 18204.8 = 18232.9 \text{ kN}
\]

The governing value of \( V_c \) is 259.4 kN.

\( \Rightarrow V_u < V_c \).

4) Calculate \( A_{sv} / s_v \).

Provide minimum steel.

\[
\frac{A_{sv}}{b w s_v} = \frac{0.4}{0.87 f_y}
\]
5) Calculate maximum spacing

\[ s_v = 0.75 \, d_t = 0.75 \times 876 = 656 \, mm \]

\[ s_v = 4b_w = 4 \times 100 = 400 \, mm \]

Select \( s_v = 400 \, mm \).

6) Calculate the size and number of legs of the stirrups

Select \( f_y = 250 \, N/mm^2 \).

\[
A_{sv} = b_w s_v \frac{0.4}{0.87f_y}
\]

\[
= 100 \times 400 \times \frac{0.4}{0.87 \times 250}
\]

\[ = 73.6 \, mm^2 \]

Provide 2 legged stirrups of diameter 8 mm.

\[
A_{sv,provided} = 2 \times 50.3
\]

\[ = 100.6 \, mm^2 \]

Check minimum amount of stirrups.

\[
A_{sv,\text{min}} = 0.1\% A_{wh}
\]

\[ = \frac{0.1}{100} \times 100 \times 400 \]

\[ = 40 \, mm^2 \]

Provided amount of stirrups is larger. OK.

Provide same spacing of stirrups throughout the span.

Design of stirrups for flange

\[
A_i = \frac{1}{2} \times b_f \times D_f
\]

\[ = \frac{1}{2} \times 435 \times 100 \]

\[ = 21750 \, mm^2 \]
\[
\bar{y} = 410 \text{ mm}
\]

\[
\tau_{f_{\text{max}}} = \frac{V_u A_h \bar{y}}{I D_f}
\]

\[
= \frac{233.3 \times 10^3 \times 21750 \times 410}{1.7808 \times 10^{10} \times 100}
\]

\[
= 1.17 \text{ N/mm}^2
\]

\[
V_f = \frac{\tau_{f_{\text{max}}} b_i D_f}{2}
\]

\[
= \frac{1.17 \times 435}{2} \times 100
\]

\[
= 12724 \text{ N}
\]

\[
A_{svf} = \frac{V_f}{0.87f_y}
\]

\[
= \frac{12724}{0.87 \times 250}
\]

\[
= 59.0 \text{ mm}^2
\]

For minimum steel \[
A_{svf} = D_s f_v \frac{0.4}{0.87f_y}
\]

\[
= 100 \times 400 \times \frac{0.4}{0.87 \times 250}
\]

\[
= 73.6 \text{ mm}^2
\]

Provide 2 legged stirrups of diameter 8 mm.

Designed section

\[
\text{8 mm diameter stirrups @ 400 mm c/c}
\]