**Solution of manning equation by Newton Raphson Method**

There is no general analytical solution to manning equation for determining the flow depth given the flow rate because the area $A$ and hydraulic radius $R$ may be complicated functions of the depth. Newton Raphson method can be applied iteratively to obtain a numerical solution. Suppose that at iteration $k$ the depth $y_k$ is selected and the flow rate $Q_n$, is computed using manning formula using the area and hydraulic radius corresponding to $y_k$. This $Q_k$ is compared with actual flow $Q_n$; then the objective is to chose $y$ such that the error.

\[ f(y_k) = Q_k - Q_n \] is within the tolerance limit. The gradient of $f$ with respect to $y$ is

\[
\frac{df(y_k)}{dy_k} = \frac{dQ_k}{dy_k}
\]

because $Q_n$ is constant. Hence, assuming manning roughness is constant,

\[
\left( \frac{df}{dy} \right)_k = \left( \frac{1}{n} S_0^{1/2} A_k R_k^{2/3} \right)
\]

\[
= \frac{1}{n} S_0^{1/2} \left( \frac{2A R^{-3} \frac{1}{3} \frac{dR}{dy} + R \frac{2}{3} \frac{dA}{dy} }{A} \right)_k
\]

\[
= \frac{1}{n} S_0^{1/2} A_k R_k^{2/3} \left( \frac{2}{3R} \frac{dR}{dy} + \frac{1}{A} \frac{dA}{dy} \right)_k
\]

\[
\left( \frac{df}{dy} \right)_k = Q_k \left( \frac{2}{3R} \frac{dR}{dy} + \frac{1}{A} \frac{dA}{dy} \right)_k
\]

in which the subscript $k$ out side the bracket indicates that the quantities in the bracket computed for $y = y_k$.

In Newton's method, given a choice of $y_k$, $y_{k+1}$ is chosen to satisfy

\[
\left( \frac{df}{dy} \right)_k = \frac{0 - f(y_k)}{y_k + y_{k+1}}
\]

This $y_{k+1}$ is the value of $y_k$,

\[
y_{k+1} = y_k - \frac{f(y_k)}{\left( \frac{df}{dy} \right)_k}
\]
Which is the fundamental equation of the Newton’s method. Iterations are continued
until there is no significant change in $y_n$; this will happen when the error is nearly zero or
an acceptable prescribed tolerance.

Thus for manning equation it may be written as

$$y_{k+1} = y_k - \frac{1 - Q/Q_k}{\left(\frac{2}{3R} \frac{dR}{dy} + \frac{1}{A} \frac{dA}{dy}\right)_k}$$

For rectangular channel $A = b_o y$ and $R = b_o y / (b_o + 2y)$ where $b_o$ is the
channel width; The quantity in denominator can be for rectangular channel

$$\frac{d}{dy} (R) = \frac{d}{dy} \left(\frac{A}{P}\right)$$

$$= \frac{1}{P} \frac{dA}{dy} - \frac{A}{P} \frac{dP}{dy}$$

$$= \left[ \frac{T}{P} - \frac{R \frac{dP}{dy}}{P} \right]$$

consider

$$\frac{2}{3R} \frac{dR}{dy} + \frac{1}{A} \frac{dA}{dy}$$

$$= \frac{2}{3} \frac{T}{A} - \frac{R}{P} \frac{dP}{dy} + \frac{T}{A}$$

$$= \frac{2}{3} \frac{T}{A} \frac{2}{3} \frac{dP}{dy} + \frac{T}{A}$$

$$= \left[ \frac{5}{3} \frac{T}{A} \frac{2}{3} \frac{dP}{dy} \right]$$

For rectangular channel

$$\frac{5}{3} \frac{b_o}{b_o y} \frac{2}{3} \frac{1}{(b_o + 2y)}$$

$$= \frac{5}{3} \frac{b_o}{y} \frac{4}{3} \frac{1}{(b_o + 2y)}$$

$$= \frac{5(b_o + 2y) - 12y}{3y(b_o + 2y)} = \frac{5b_o + 10y - 4y}{3y(b_o + 2y)}$$

$$= \frac{5b_o + 6y}{3y(b_o + 2y)}$$

$$y_{k+1} = y_k - \frac{1 - Q/Q_k}{\left(\frac{5b_o + 6y_k}{3y_k (b_o + 2y_k)}\right)}$$

Similarly the channel shape function $[\left(\frac{2}{3R} \frac{dR}{dy} + \frac{1}{A} \frac{dA}{dy}\right)]$
for other cross sections can be derived.
Let wide  \( b_o = 0.6m \)
Manning coefficient \( n = 0.015 \)
bed slope \( S_o = 0.025 \)
discharge \( Q = 0.25m^3s^{-1} \)

normal depth \( y = ? \)

Hydraulic mean radius \( R = \frac{A}{p} = \frac{b_o}{b_o + 2y} \)

\[
Q = \frac{1}{n} AR^\frac{3}{2} S_o^\frac{1}{2}
\]

\[
Q = \frac{1}{n} b_o y \left( \frac{by_k}{b + 2y_k} \right)^\frac{2}{3} S_o^\frac{1}{2}
\]
\[ Q = \frac{1}{n} \left[ \frac{(by_k)^{\frac{5}{3}}}{(b + 2y_k)^{\frac{2}{3}}} \right] S_o^{\frac{1}{2}} \]

\[ Q_k = \frac{1}{0.015} \times \left[ \frac{(0.6 \times y_k)^{\frac{5}{3}}}{(0.6 + 2y_k)^{\frac{2}{3}}} \right] \times (0.025)^{\frac{1}{2}} \]

\[ Q_k = 10.5409 \times \frac{0.6^{\frac{5}{3}}}{(0.6 + 2y_k)^{\frac{2}{3}}} = \frac{4.4993y_k^{\frac{5}{3}}}{(0.6 + 2y_k)^{\frac{2}{3}}} \quad (1) \]

Shape function = \frac{5b_o + 6y_k}{3y_k (b + 2y_k)}

\[ \frac{5(0.6) + 6y_k}{3y_k (0.6 + 2y_k)} = \frac{3 + 6y_k}{3y_k (0.6 + 2y_k)} = \frac{1 + 2y_k}{y_k (0.6 + 2y_k)} \]

\[ y_{k+1} = y_k - \frac{1 - 0.25}{Q_k} y_k \left( \frac{0.6 + 2y_k}{1 + 2y_k} \right) \quad (2) \]

<table>
<thead>
<tr>
<th>Iteration (k)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_k ) (m)</td>
<td>0.100</td>
<td>0.1815</td>
<td>0.1727</td>
</tr>
<tr>
<td>Q (m³/s)</td>
<td>0.1125</td>
<td>0.2684</td>
<td>0.2488</td>
</tr>
</tbody>
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Froude number \( F = \frac{V}{\sqrt{gy}} = \frac{Q/A}{\sqrt{gy}} \)

\[ F = \frac{0.2488/(0.6*0.1727)}{\sqrt{(9.81*0.1727)}} = 1.844 \]

\[ \therefore \text{super critical flow} \]