6.10 NUMERICAL MODELLING FOR STRESSES AROUND UG OPENINGS

Numerical methods of stress analysis may be adopted as most underground excavations are irregular in shape and are frequently grouped close to other excavations. These groups of excavations can form a set of complex three-dimensional shapes. In addition, because of the presence of geological features such as faults and dykes, the rock properties are seldom uniform within the rock volume of interest. Consequently, closed form solutions are of limited value in calculating the stresses, displacements and failure of the rock mass surrounding underground excavations. A number of computer-based numerical methods have been developed over the past few decades and these methods provide the means for obtaining approximate solutions to these problems. Some of the numerical techniques extensively used for numerical analysis of underground openings are, boundary element method (BEM), finite element method (FEM), finite difference method (FDM), distinct element method (DEM) and some hybrid methods. In this section, some numerical analysis results for underground openings using boundary element method (BEM) have been discussed.

The boundary element method is derived through the discretisation of an integral equation that is mathematically equivalent to the original partial differential equation (PDE). The essential re-formulation of the PDE that underlies the BEM consists of an integral equation that is defined on the boundary of the domain and an integral that relates the boundary solution to the solution at points in the domain. The former is termed a boundary integral equation (BIE) and the BEM is often referred to as the boundary integral equation method or boundary integral method. The advantages in the boundary element method arises from the fact that only the boundary (or boundaries) of the domain of the PDE requires subdivision. (In the finite element method or finite difference method the whole domain of the PDE requires discretisation). Thus the dimension of the problem is effectively reduced by one, for example an equation governing a three-dimensional region is transformed into one over its surface.
Here, numerical results are shown with comparison with close from solution for circular openings. BEM software, Examined-2D were used to derive the solution of these underground openings.

Figure 6.25: A typical radial stress distribution for a circular opening using BEM
Figure 6.26: Radial and tangential stress distribution for an underground opening on X-axis together with close form solution
Figure 6.27: Radial and tangential stress distribution for an underground opening on Y-axis together with close form solution
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Figure 6.28: Radial stress distribution for a circular underground opening

Figure 6.29: Radial stress distribution for a elliptical underground opening
Figure 6.30: Radial stress distribution for a square underground opening with rounded corner

Figure 6.31: Radial stress distribution for a modified horse shoe shape underground opening
Figure 6.32: Radial stress distribution variation for different shape of openings, high peak for sharp corners of square opening may be seen

6.11 EFFECT OF WIDTH TO HEIGHT (W/H) RATIO

Larger the W/H ratio, smaller the stress concentration. Fig. 6 shows the effect of W/H ratio on various shapes. From the above figure, (a), (b), (c) it is evident that when the W/H ratio increases, the tangential stresses decreases for the various shapes. Also, (d) shows that when there is a sharp peak, the tangential stresses can really shoot up. The oval opening minimizes the tangential boundary stresses and reduces the localization of compressive stress concentrations, around the opening. The rectangular opening has high compressive stress concentrations at the corners, almost ensuring that excavation damage will occur at these locations.
Figure 6.33: Comparison of tangential boundary stresses around openings with axes aligned with the principal stress directions. W/H ratios shown on each curve (Read et al. 1990)

6.12 TUNNELING IN WEAK ROCK

The tunneling in weak rocks and the corresponding stresses and deformation around circular openings are relatively complex. Numerical modelling using 3D FEM of the failure and deformation of the rock mass surrounding the face of an advancing circular tunnel is shown in Figure 6.35. The plot shows displacement vectors as well as the shape of the deformed tunnel profile. There is observed some inward deformation at the face of the advancing tunnel. The radial displacement reaches almost about one half a tunnel diameter ahead of the advancing face as shown in Figure 6.36 and the displacement reaches one third of its final value at the tunnel face. Behind the face, a radial displacement of final value may be observed at a distance of one to one and half diameter (Figure 6.36). The final deformation
and tunnel shape is usually defined within a week and afterwards, there may be some/little additional long term deformation due to creep effects (Figure 6.37).

**Figure 6.34**: Displacement vectors and the shape of the deformed tunnel in a weak rock.

**Figure 6.35**: Numerically captured radial displacement in advancing tunnel ahead of the advancing face.
6.13 EFFECTS OF PLANES OF WEAKNESS ON STRESS DISTRIBUTION (Brady and Brown, 1985)

The discontinuity is assumed to have zero tensile strength, and is non-dilatant in shear, with a shear strength defined by

$$\tau = \sigma_n \tan \varphi$$

Case 1. (Figure 6.37)

From the Kirsch equations, for $\theta = 0$, the shear stress component $\sigma_{r\theta} = 0$, for all $r$. Thus $\sigma_{rr}$, $\sigma_{\theta\theta}$ are the principal stresses $\sigma_{xx}$, $\sigma_{yy}$ and $\sigma_{xy}$ is zero. The shear stress on the plane of weakness is zero, and there is no tendency for slip on it. The plane of weakness therefore has no effect on the elastic stress distribution.
Case 2. (Figure 6.38a)

When $\theta = \pi/2$, no shear stress is mobilised on the plane of weakness. If $K < 1/3$ a possibility of separation on the plane of weakness arises as tensile stress develop in the crown of the opening. If $K \geq 1/3$, the elastic stress distribution is unaltered by either slip or separation.

Figure 6.38: A plane of weakness intersecting a circular opening and oriented parallel to the major principal stress, developing destressed zone
**Case 3. (Figure 6.39)**

A flat-lying feature whose trace on the excavation boundary is located at an angle above the horizontal diameter is shown in Figure 6.39a, the normal and shear stress components on the plane of weakness are given by

\[
\sigma_n = \sigma_{\theta\theta} \cos^2 \theta \\
\tau = \sigma_{\theta\theta} \sin \theta \cos \theta
\]

For a limiting condition of

\[
\tau = \sigma_n \tan \phi
\]

the condition for slip is satisfied on the plane of weakness as \( \theta = \phi \).

![Figure 6.39: A flat lying plane of weakness intersecting a circular excavation non diametrically](image)

**Case 4. (Figure 6.40)**

For an arbitrarily inclined plane of weakness intersecting an opening inclined at an angle of 45°, the normal and shear stress components are obtained by substitution in the Kirsch solution

\[
\sigma_n = \sigma_{\theta\theta} = \frac{P}{2} \times 1.5(1 + \frac{a^2}{r^2}) \\
\tau = \sigma_{r\theta} = \frac{P}{2} \times 0.5(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4})
\]
If the rock mass were in a state of limiting equilibrium, during excavation an extensive zone of slip could develop along the plane of weakness.

Figure 6.40: An inclined plane of weakness intersecting a circular excavation