Module 5

Flanged Beams – Theory and Numerical Problems

Version 2 CE IIT, Kharagpur
Lesson 11

Flanged Beams – Numerical Problems

Version 2 CE IIT, Kharagpur
Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify the two types of numerical problems – analysis and design types,
- apply the formulations to analyse the capacity of a flanged beam,
- determine the limiting moment of resistance quickly with the help of tables of SP-16.

5.11.1 Introduction

Lesson 10 illustrates the governing equations of flanged beams. It is now necessary to apply them for the solution of numerical problems. Two types of numerical problems are possible: (i) Analysis and (ii) Design types. This lesson explains the application of the theory of flanged beams for the analysis type of problems. Moreover, use of tables of SP-16 has been illustrated to determine the limiting moment of resistance of sections quickly for the three grades of steel. Besides mentioning the different steps of the solution, numerical examples are also taken up to explain their step-by-step solutions.

5.11.2 Analysis Type of Problems

The dimensions of the beam $b_f$, $b_w$, $D_f$, $d$, $D$, grades of concrete and steel and the amount of steel $A_{st}$ are given. It is required to determine the moment of resistance of the beam.

**Step 1: To determine the depth of the neutral axis $x_u$**

The depth of the neutral axis is determined from the equation of equilibrium $C = T$. However, the expression of $C$ depends on the location of neutral axis, $D_f/d$ and $D_f/x_u$ parameters. Therefore, it is required to assume first that the $x_u$ is in the flange. If this is not the case, the next step is to assume $x_u$ in the web and the computed value of $x_u$ will indicate if the beam is under-reinforced, balanced or over-reinforced.
Other steps:

After knowing if the section is under-reinforced, balanced or over-reinforced, the respective parameter $D/d_o$ or $D/x_u$ is computed for the under-reinforced, balanced or over-reinforced beam. The respective expressions of $C$, $T$ and $M_u$, as established in Lesson 10, are then employed to determine their values. Figure 5.11.1 illustrates the steps to be followed.
5.11.3 Numerical Problems (Analysis Type)

Ex.1: Determine the moment of resistance of the T-beam of Fig. 5.11.2. Given data: \( b_f = 1000 \text{ mm} \), \( D_f = 100 \text{ mm} \), \( b_w = 300 \text{ mm} \), cover = 50 mm, \( d = 450 \text{ mm} \) and \( A_{st} = 1963 \text{ mm}^2 \) (4-25 T). Use M 20 and Fe 415.

Step 1: To determine the depth of the neutral axis \( x_u \)

Assuming \( x_u \) in the flange and equating total compressive and tensile forces from the expressions of \( C \) and \( T \) (Eq. 3.16 of Lesson 5) as the T-beam can be treated as rectangular beam of width \( b_f \) and effective depth \( d \), we get:

\[
x_u = \frac{0.87 f_y A_{st}}{0.36 f_c} = \frac{0.87 (415) (1963)}{0.36 (1000) (20)} = 98.44 \text{ mm} < 100 \text{ mm}
\]

So, the assumption of \( x_u \) in the flange is correct.

\( x_{u, max} \) for the balanced rectangular beam = 0.48 \( d = 0.48 (450) = 216 \text{ mm} \).

It is under-reinforced since \( x_u < x_{u, max} \).

Step 2: To determine \( C, T \) and \( M_u \)

From Eqs. 3.9 (using \( b = b_f \)) and 3.14 of Lesson 4 for \( C \) and \( T \) and Eq. 3.23 of Lesson 5 for \( M_u \), we have:
\[ C = 0.36 \ b_f \ x_u \ f_{ck} \]  
(3.9)

\[ = 0.36 \ (1000) \ (98.44) \ (20) = 708.77 \ \text{kN} \]

\[ T = 0.87 \ f_y \ A_{st} \]  
(3.14)

\[ = 0.87 \ (415) \ (1963) = 708.74 \ \text{kN} \]

\[ M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} b_f d}\right) \]  
(3.23)

\[ = 0.87 \ (415) \ (1963) \ (450) \left\{1 - \frac{(1963) (415)}{(20) (1000) (450)}\right\} = 290.06 \ \text{kNm} \]

This problem belongs to the case (i) and is explained in sec. 5.10.4.1 of Lesson 10.

**Ex.2:** Determine \( A_{st,\text{lim}} \) and \( M_{u,\text{lim}} \) of the flanged beam of Fig. 5.11.3. Given data are: \( b_f = 1000 \ \text{mm}, D_f = 100 \ \text{mm}, b_w = 300 \ \text{mm}, \) cover = 50 mm and \( d = 450 \ \text{mm}. \) Use M 20 and Fe 415.

**Step 1:** To determine \( D_f/d \) ratio

For the limiting case \( x_u = x_{u,\text{max}} = 0.48 \ (450) = 216 \ \text{mm} > D_f. \) The ratio \( D_f/d \) is computed.
\[ D/d = 100/450 = 0.222 > 0.2 \]

Hence, it is a problem of case (ii b) and discussed in sec. 5.10.4.2 b of Lesson 10.

**Step 2: Computations of \( y_f \), \( C \) and \( T \)**

First, we have to compute \( y_f \) from Eq.5.8 of Lesson 10 and then employ Eqs. 5.9, 10 and 11 of Lesson 10 to determine \( C \), \( T \) and \( Mu \), respectively.

\[ y_f = 0.15 \ x_{u,max} + 0.65 \ D_f = 0.15 \ (216) + 0.65 \ (100) = 97.4 \ mm. \ (\text{from Eq. 5.8}) \]

\[ C = 0.36 \ f_{ck} \ b_w \ x_{u,max} + 0.45 \ f_{ck} \ (b_f - b_w) \ y_f \]
\[ = 0.36 \ (20) \ (300) \ (216) + 0.45 \ (20) \ (1000 - 300) \ (97.4) = 1,080.18 \ kN. \]

\[ T = 0.87 \ f_y \ A_{st} = 0.87 \ (415) \ A_{st} \]
\[ (5.10) \]

Equating \( C \) and \( T \), we have

\[ A_{st} = \frac{(1080.18) \ (1000) \ N}{0.87 \ (415) \ N/mm^2} = 2,991.77 \ mm^2 \]

Provide 4-28 T (2463 mm\(^2\)) + 3-16 T (603 mm\(^2\)) = 3,066 mm\(^2\)

**Step 3: Computation of \( M_u \)**

\[ M_{u,lim} = 0.36 \left( \frac{x_{u,max}}{d} \right) \{1 - 0.42 \left( \frac{x_{u,max}}{d} \right)\}f_{ck} \ b_w \ d^2 \]
\[ + 0.45f_{ck}(b_f - b_w)y_f (d - y_f / 2) \]
\[ = 0.36 \ (0.48) \left\{1 - 0.42 \ (0.48)\right\} \ (20) \ (300) \ (450)^2 \]
\[ + 0.45 \ (20) \ (1000 - 300) \ (97.4) \ (450 - 97.4/2) = 413.87 \ kNm \]
Ex. 3: Determine the moment of resistance of the beam of Fig. 5.11.4 when $A_{st} = 2,591 \text{ mm}^2$ (4- 25 T and 2- 20 T). Other parameters are the same as those of Ex.1: $b_f = 1,000 \text{ mm}$, $D_f = 100 \text{ mm}$, $b_w = 300 \text{ mm}$, cover = 50 mm and $d = 450 \text{ mm}$. Use M 20 and Fe 415.

**Step 1: To determine $x_u$**

Assuming $x_u$ to be in the flange and the beam is under-reinforced, we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87 (415) (2591)}{0.36 (100) (20)} = 129.93 \text{ mm} > 100 \text{ mm}$$

Since $x_u > D_f$, the neutral axis is in web. Here, $D_f/d = 100/450 = 0.222 > 0.2$. So, we have to substitute the term $y_f$ from Eq. 5.15 of Lesson 10, assuming $D_f/x_u > 0.43$ in the equation of $C = T$ from Eqs. 5.16 and 17 of sec. 5.10.4.3 b of Lesson 10. Accordingly, we get:

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

or

$$0.36 (20) (300) (x_u) + 0.45 (20) (1000 - 300) \{0.15 x_u + 0.65 (100)\}$$

$$= 0.87 (415) (2591)$$

or

$$x_u = 169.398 \text{ mm} < 216 \text{ mm} \ (x_{u,max} = 0.48 x_u = 216 \text{ mm})$$
So, the section is under-reinforced.

**Step 2: To determine \( M_u \)**

\[
D_f/x_u = 100/169.398 = 0.590 > 0.43
\]

This is the problem of case (iii b) of sec. 5.10.4.3 b. The corresponding equations are Eq. 5.15 of Lesson 10 for \( y_f \) and Eqs. 5.16 to 18 of Lesson 10 for \( C, T \) and \( M_u \), respectively. From Eq. 5.15 of Lesson 10, we have:

\[
y_f = 0.15 x_u + 0.65
\]

\[
D_f = 0.15 (169.398) + 0.65 (100) = 90.409 \text{ mm}
\]

From Eq. 5.18 of Lesson 10, we have

\[
M_u = 0.36(x_u/d)(1 - 0.42(x_u/d)) f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f (d - y_f/2)
\]

or

\[
M_u = 0.36 (169.398/450) (1 - 0.42 (169.398/450)) (20) (300) (450) (450)
\]

\[
+ 0.45 (20) (1000 - 300) (90.409) (450 - 90.409/2)
\]

\[
= 138.62 + 230.56 = 369.18 \text{ kNm.}
\]

**Ex.4:** Determine the moment of resistance of the flanged beam of Fig. 5.11.5 with \( A_{st} = 4,825 \text{ mm}^2 \) (6-32 T). Other parameters and data are the same as those of Ex.1: \( b_f = 1000 \text{ mm}, D_f = 100 \text{ mm}, b_w = 300 \text{ mm}, \) cover = 50 mm and \( d = 450 \text{ mm.} \) Use M 20 and Fe 415.
Step 1: To determine \(x_u\)

Assuming \(x_u\) in the flange of under-reinforced rectangular beam we have from Eq. 3.16 of Lesson 5:

\[
x_u = \frac{0.87 \cdot f_y \cdot A_{st}}{0.36 \cdot b_f \cdot f_{ck}} = \frac{0.87 \cdot (415) \cdot (4825)}{0.36 \cdot (1000) \cdot (20)} = 241.95 \text{ mm} > D_f
\]

Here, \(D_f/d = 100/450 = 0.222 > 0.2\). So, we have to determine \(y_f\) from Eq. 5.15 and equating \(C\) and \(T\) from Eqs. 5.16 and 17 of Lesson 10.

\[
y_f = 0.15 x_u + 0.65 D_f \quad (5.15)
\]

\[
0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st} \quad (5.16 \text{ and } 5.17)
\]

or

\[
0.36 \cdot (20) \cdot (300) \cdot (x_u) + 0.45 \cdot (20) \cdot (1000 - 300) \cdot [0.15 x_u + 0.65 (100)]

= 0.87 \cdot (415) \cdot (4825)
\]

or

\[
2160 x_u + 945 x_u = -409500 + 1742066
\]

or

\[
x_u = 1332566/3105 = 429.17 \text{ mm}
\]

\[
x_{u,\text{max}} = 0.48 (450) = 216 \text{ mm}
\]

Since \(x_u > x_{u,\text{max}}\), the beam is over-reinforced. Accordingly.

\[
x_u = x_{u,\text{max}} = 216 \text{ mm}.
\]

Step 2: To determine \(M_u\)

This problem belongs to case (iv b), explained in sec.5.10.4.4 b of Lesson 10. So, we can determine \(M_u\) from Eq. 5.11 of Lesson 10.

\[
M_u = 0.36 (x_{u,\text{max}}/d) \{1 - 0.42(x_{u,\text{max}}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f (d - y_f)/2
\]

where \(y_f = 0.15 x_{u,\text{max}} + 0.65 D_f = 97.4 \text{ mm}\) (5.8)

From Eq. 5.11, employing the value of \(y_f = 97.4 \text{ mm}\), we get:

\[
M_u = 0.36 \cdot (0.48) \cdot (1 - 0.42 \cdot (0.48)) \cdot (20) \cdot (300) \cdot (450) \cdot (450)
\]
It is seen that this over-reinforced beam has the same $M_u$ as that of the balanced beam of Example 2.

### 5.11.4 Summary of Results of Examples 1-4

The results of four problems (Exs. 1-4) are given in Table 5.1 below. All the examples are having the common data except $A_{st}$.

**Table 5.1 Results of Examples 1-4 (Figs. 5.11.2 – 5.11.5)**

<table>
<thead>
<tr>
<th>Ex. No.</th>
<th>$A_{st}$ (mm$^2$)</th>
<th>Case</th>
<th>Section No.</th>
<th>$M_u$ (kNm)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,963</td>
<td>(i)</td>
<td>5.10.4.1</td>
<td>290.06</td>
<td>$x_u = 98.44$ mm $&lt; x_{u,\text{max}} (= 216$ mm$)$, $x_u &lt; D_f (= 100$ mm$)$, Under-reinforced, (NA in the flange).</td>
</tr>
<tr>
<td>2</td>
<td>3,066</td>
<td>(ii b)</td>
<td>5.10.4.2 (b)</td>
<td>413.87</td>
<td>$x_u = x_{u,\text{max}} = 216$ mm, $D_f/d = 0.222 &gt; 0.2$, Balanced, (NA in web).</td>
</tr>
<tr>
<td>3</td>
<td>2,591</td>
<td>(iii b)</td>
<td>5.10.4.3 (b)</td>
<td>369.18</td>
<td>$x_u = 169.398$ mm $&lt; x_{u,\text{max}} (= 216$ mm$)$, $D_f/x_u = 0.59 &gt; 0.43$, Under-reinforced, (NA in the web).</td>
</tr>
<tr>
<td>4</td>
<td>4,825</td>
<td>(iv b)</td>
<td>5.10.4.4 (b)</td>
<td>413.87</td>
<td>$x_u = 241.95$ mm $&gt; x_{u,\text{max}} (= 216$ mm$)$, $D_f/d = 0.222 &gt; 0.2$, Over-reinforced, (NA in web).</td>
</tr>
</tbody>
</table>

It is clear from the above table (Table 5.1), that Ex.4 is an over-reinforced flanged beam. The moment of resistance of this beam is the same as that of balanced beam of Ex.2. Additional reinforcement of 1,759 mm$^2$ ($= 4,825$ mm$^2 - 3,066$ mm$^2$) does not improve the $M_u$ of the over-reinforced beam. It rather prevents the beam from tension failure. That is why over-reinforced beams are to be avoided. However, if the $M_u$ has to be increased beyond 413.87 kNm, the flanged beam may be doubly reinforced.

### 5.11.5 Use of SP-16 for the Analysis Type of Problems

Using the two governing parameters ($b_f/b_w$) and ($D_f/d$), the $M_{u,\text{lim}}$ of balanced flanged beams can be determined from Tables 57-59 of SP-16 for the

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three grades of steel (250, 415 and 500). The value of the moment coefficient $M_{u,\text{lim}}/b_w d^2 f_{ck}$ of Ex.2, as obtained from SP-16, is presented in Table 5.2 making linear interpolation for both the parameters, wherever needed. $M_{u,\text{lim}}$ is then calculated from the moment coefficient.

Table 5.2 $M_{u,\text{lim}}$ of Example 2 using Table 58 of SP-16

Parameters: (i) $b_l/b_w = 1000/300 = 3.33$
(ii) $D_l/d = 100/450 = 0.222$

<table>
<thead>
<tr>
<th>$D_l/d$</th>
<th>$b_l/b_w$</th>
<th>$(M_{u,\text{lim}}/b_w d^2 f_{ck})$ in N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>3.33</td>
</tr>
<tr>
<td>0.22</td>
<td>0.309</td>
<td>0.395</td>
</tr>
<tr>
<td>0.23</td>
<td>0.314</td>
<td>0.402</td>
</tr>
<tr>
<td>0.222</td>
<td>0.31*</td>
<td>0.3964*</td>
</tr>
</tbody>
</table>

* by linear interpolation

So, from Table 5.2, 

$$\frac{M_{u,\text{lim}}}{b_w d^2 f_{ck}} = 0.339$$

$$M_{u,\text{lim}} = 0.339 b_w d^2 f_{ck} = 0.339 (300) (450) (450) (20) 10^{-6} = 411.88 \text{ kNm}$$

$M_{u,\text{lim}}$ as obtained from SP-16 is close to the earlier computed value of $M_{u,\text{lim}} = 413.87 \text{ kNm}$ (see Table 5.1).

5.11.6 Practice Questions and Problems with Answers
Q.1: Determine the moment of resistance of the simply supported doubly reinforced flanged beam (isolated) of span 9 m as shown in Fig. 5.11.6. Assume M 30 concrete and Fe 500 steel.

A.1: Solution of Q.1:

Effective width

\[ b_f = \frac{l_o}{(l_o/b) + 4} + b_w = \frac{9000}{(9000/1500) + 4} + 300 = 1200 \text{ mm} \]

Step 1: To determine the depth of the neutral axis

Assuming neutral axis to be in the flange and writing the equation \( C = T \), we have:

\[ 0.87 f_y A_{st} = 0.36 f_{ck} b_f x_u + (f_{sc} A_{sc} - f_{cc} A_{sc}) \]

Here, \( d'/d = 65/600 = 0.108 \) = 0.1 (say). We, therefore, have \( f_{sc} = 353 \text{ N/mm}^2 \).

From the above equation, we have:

\[ x_u = \frac{0.87 (500) (6509) - \{(353) (1030) - 0.446 (30) (1030)\}}{0.36 (30) (1200)} = 191.48 \text{ mm} > 120 \text{ mm} \]

So, the neutral axis is in web.

\[ D_f/d = 120/600 = 0.2 \]

Assuming \( D_f/x_u < 0.43 \), and Equating \( C = T \)

\[ 0.87 f_y A_{st} = 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) D_f + (f_{sc} - f_{cc}) A_{sc} \]

\[ x_u = \frac{0.87 (500) (6509) - 1030\{(353 - 0.446 (30)} - 0.446 (30) (1200 - 300 \text{ (120)} \}

\[ = 319.92 > 276 \text{ mm} \ (x_{u,max} = 276 \text{ mm}) \]

So, \( x_u = x_{u,max} = 276 \text{ mm} \) (over-reinforced beam).

\[ D_f/x_u = 120/276 = 0.4347 > 0.43 \]

Let us assume \( D_f/x_u > 0.43 \). Now, equating \( C = T \) with \( y_f \) as the depth of flange having constant stress of 0.446 \( f_{ck} \). So, we have:

\[ y_f = 0.15 x_u + 0.65 D_f = 0.15 x_u + 78 \]

\[ 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st} \]

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\[0.36 \times 10^3 \times 300 \times x_u + 0.446 \times 10^3 \times 900 \times (0.15 x_u + 78)\]
\[= 0.87 \times 10^3 \times (6509) - 1030 \{353 - 0.446 \times 30\}\]

or \[x_u = 305.63 \text{ mm} > x_{u,max}. \quad (x_{u,max} = 276 \text{ mm})\]

The beam is over-reinforced. Hence, \(x_u = x_{u,max} = 276 \text{ mm}\). This is a problem of case (iv), and we, therefore, consider the case (ii) to find out the moment of resistance in two parts: first for the balanced singly reinforced beam and then for the additional moment due to compression steel.

**Step 2: Determination of \(x_{u,lim}\) for singly reinforced flanged beam**

Here, \(D_{fr}/d = 120/600 = 0.2\), so \(y_f\) is not needed. This is a problem of case (ii a) of sec. 5.10.4.2 of Lesson 10. Employing Eq. 5.7 of Lesson 10, we have:

\[M_{u,lim} = 0.36 \left(\frac{x_{u,max}}{d}\right) \{1 - 0.42 \left(\frac{x_{u,max}}{d}\right)\} f_{ck} b_w d^2\]
\[+ 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2)\]
\[= 0.36(0.46) \{1 - 0.42(0.46)\} (30) (300) (600) (600)\]
\[+ 0.45(30) (900) (120) (540)\]
\[= 1,220.20 \text{ kNm}\]

\[A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y d \{1 - 0.42 \left(\frac{x_{u,max}}{d}\right)\}}\]
\[= \frac{(1220.20) (10^6)}{(0.87) (500) (600) (0.8068)} = 5,794.6152 \text{ mm}^2\]

**Step 3: Determination of \(M_{u2}\)**

Total \(A_{st} = 6,509 \text{ mm}^2\), \(A_{st,lim} = 5,794.62 \text{ mm}^2\)

\(A_{st2} = 714.38 \text{ mm}^2\) and \(A_{sc} = 1,030 \text{ mm}^2\)

It is important to find out how much of the total \(A_{sc}\) and \(A_{st2}\) are required effectively. From the equilibrium of \(C\) and \(T\) forces due to additional steel (compressive and tensile), we have:

\[(A_{st2}) (0.87) (f_y) = (A_{sc}) (f_{sc})\]

If we assume \(A_{sc} = 1,030 \text{ mm}^2\)
\[ A_{st2} = \frac{1030 \times 353}{0.87 \times 500} = 835.84 \text{ mm}^2 > 714.38 \text{ mm}^2 , \] (714.38 mm\(^2\) is the total \(A_{st2}\) provided). So, this is not possible.

Now, using \(A_{st2} = 714.38 \text{ mm}^2\), we get \(A_{sc}\) from the above equation.

\[ A_{sc} = \frac{(714.38) (0.87)(500)}{353} = 880.326 < 1,030 \text{ mm}^2 , \] (1,030 mm\(^2\) is the total \(A_{sc}\) provided).

\[ M_{u2} = A_{sc} f_{sc} (d - d') = (880.326)(353)(600 - 60) = 167.807 \text{ kNm} \]

Total moment of resistance = \(M_{u,lim} + M_{u2} = 1,220.20 + 167.81 = 1,388.01 \text{ kNm} \)

Total \(A_{st}\) required = \(A_{st,lim} + A_{st2} = 5,794.62 + 714.38 = 6,509.00 \text{ mm}^2\),
(provided \(A_{st} = 6,509 \text{ mm}^2\))

\[ A_{sc} \text{ required} = 880.326 \text{ mm}^2 \text{ (provided 1,030 mm}^2\).}

**5.11.7 References**


5.11.8 Test 11 with Solutions

Maximum Marks = 50,   Maximum Time = 30 minutes

Answer all questions.

TQ.1: Determine $M_{u,lim}$ of the flanged beam of Ex. 2 (Fig. 5.11.3) with the help of SP-16 using (a) M 20 and Fe 250, (b) M 20 and Fe 500 and (c) compare the results with the $M_{u,lim}$ of Ex. 2 from Table 5.2 when grades of concrete and steel are M 20 and Fe 415, respectively. Other data are: $b_f = 1000$ mm, $D_f = 100$ mm, $b_w = 300$ mm, cover = 50 mm and $d = 450$ mm.

$M_{u,lim}$, of TQ.1 (PART a for M 20 and Fe 250)

\[
\begin{array}{c|c|c|c}
\hline
\text{Parameter} & \frac{b_f}{b_w} & D_f/d & M_{u,lim} \\
\hline
(i) & 1000/300 = 3.33 & 100/450 = 0.222 & \\
(ii) & 0.22 & 0.23 & 0.222 \\
\hline
\end{array}
\]

For part (a): When Fe 250 is used, the corresponding table is Table 57 of SP-16. The computations are presented in Table 5.3 below:

<table>
<thead>
<tr>
<th>$D_f/d$</th>
<th>$b_f/b_w$</th>
<th>$M_{u,lim}/b_w d^2 f_{ck}$ in N/mm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>3.33</td>
</tr>
<tr>
<td>0.22</td>
<td>0.324</td>
<td>0.411</td>
</tr>
<tr>
<td>0.23</td>
<td>0.330</td>
<td>0.421</td>
</tr>
<tr>
<td>0.222</td>
<td>0.3252*</td>
<td>0.413*</td>
</tr>
</tbody>
</table>

\[M_{u,lim}/b_w d^2 f_{ck} = 0.354174 = 0.354 \text{ (say)}\]

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So, $M_{u,\text{lim}} = (0.354) (300) (450) (450) (20) \text{ N mm} = 430.11 \text{ kNm}$

**For part (b):** When Fe 500 is used, the corresponding table is Table 59 of SP-16. The computations are presented in Table 5.4 below:

**Table 5.4** $(M_{u,\text{lim}}/b_w d^2 f_{ck})$ in N/mm² Of TQ.1 (PART b for M 20 and Fe 500)

<table>
<thead>
<tr>
<th>$D_l/d$</th>
<th>$b_f/b_w$</th>
<th>$b_f/b_w$</th>
<th>$D_f/d$</th>
<th>$D_f/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>3.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.22</td>
<td>0.302</td>
<td>0.386</td>
<td>0.23</td>
<td>0.306</td>
</tr>
<tr>
<td>0.23</td>
<td>0.393</td>
<td></td>
<td>0.222</td>
<td>0.3028*</td>
</tr>
<tr>
<td></td>
<td>0.3874*</td>
<td></td>
<td></td>
<td>0.330718*</td>
</tr>
</tbody>
</table>

* by linear interpolation

$M_{u,\text{lim}}/b_w d^2 f_{ck} = 0.330718 = 0.3307$ (say)

So, $M_{u,\text{lim}} = (0.3307) (300) (450) (450) (20) \text{ mm} = 401.8 \text{ kNm}$

**For part (c):** Comparison of results of this problem with that of Table 5.2 (M 20 and Fe 415) is given below in Table 5.5.

**Table 5.5** Comparison of results of $M_{u,\text{lim}}$

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Grade of Steel</th>
<th>$M_{u,\text{lim}}$ (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fe 250</td>
<td>430.11</td>
</tr>
<tr>
<td>2</td>
<td>Fe 415</td>
<td>411.88</td>
</tr>
<tr>
<td>3</td>
<td>Fe 500</td>
<td>401.80</td>
</tr>
</tbody>
</table>

It is seen that $M_{u,\text{lim}}$ of the beam decreases with higher grade of steel for a particular grade of concrete.

**TQ.2:** With the aid of SP-16, determine separately the limiting moments of resistance and the limiting areas of steel of the simply supported isolated, singly reinforced and balanced flanged beam of Q.1 as shown in Fig. 5.11.6 if the span = 9 m. Use M 30 concrete and three grades of steel, Fe 250, Fe 415 and Fe 500, respectively. Compare the results obtained above with that of Q.1 of sec. 5.11.6, when balanced.

(15 + 5 = 20 marks)

**A.TQ.2:** From the results of Q.1 sec. 5.11.6, we have:

Parameters: (i) $b_f/b_w = 1200/300 = 4.0$

(ii) $D_l/d = 120/600 = 0.2$
For Fe 250, Fe 415 and Fe 500, corresponding tables are Table 57, 58 and 59, respectively of SP-16. The computations are done accordingly. After computing the limiting moments of resistance, the limiting areas of steel are determined as explained below. Finally, the results are presented in Table 5.6 below:

\[ A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y d \left(1 - 0.42 \left(\frac{x_{u,max}}{d}\right)\right)} \]

<table>
<thead>
<tr>
<th>Grade of Fe / Q.1 of sec. 5.11.6</th>
<th>( \frac{M_{u,lim}}{b_w d^2 f_{ck}} )</th>
<th>( M_{u,lim} ) (kNm)</th>
<th>( A_{st,lim} ) (mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe 250</td>
<td>0.39</td>
<td>1,263.60</td>
<td>12,455.32</td>
</tr>
<tr>
<td>Fe 415</td>
<td>0.379</td>
<td>1,227.96</td>
<td>7,099.78</td>
</tr>
<tr>
<td>Fe 500</td>
<td>0.372</td>
<td>1,205.28</td>
<td>5,723.76</td>
</tr>
<tr>
<td>Q.1 of sec. 5.11.6 (Fe 415)</td>
<td></td>
<td>1,220.20</td>
<td>5,794.62</td>
</tr>
</tbody>
</table>

The maximum area of steel allowed is \( .04 b D = (.04) (300) (660) = 7,920 \) mm\(^2\). Hence, Fe 250 is not possible in this case.

5.11.9 Summary of this Lesson

This lesson mentions about the two types of numerical problems (i) analysis and (ii) design types. In addition to explaining the steps involved in solving the analysis type of numerical problems, several examples of analysis type of problems are illustrated explaining all steps of the solutions both by direct computation method and employing SP-16. Solutions of practice and test problems will give readers the confidence in applying the theory explained in Lesson 10 in solving the numerical problems.