1. Why is extracting the eigen values and eigen values of an integral equation useful?

The usefulness of extracting the eigen values and eigen functions of such an integral equation stems from the following result: Any function \( f(x) \) generated from a continuous function \( \psi(x) \) and a continuous real symmetric kernel \( K(x, \xi) \) as: \( f(x) = \int_{a}^{b} K(x, \xi) \psi(\xi) d\xi \) can be represented in the interval \((a, b)\) as a linear combination of the eigen functions of the homogeneous Fredholm integral equation that has \( K(x, \xi) \) as its kernel. In such a case it is possible to write

\[
f(x) = \sum_{n=1}^{N} C_{n} y_{n}(x) \quad \text{where} \quad y_{n}(x) = \lambda \int_{a}^{b} K(x, \xi) y_{n}(\xi) d\xi
\]

2. Why are iterative procedures necessary to determine the eigen values and eigen functions of an integral equation?

If the kernel is separable the eigen values and eigen functions can be obtained analytically if the coefficient matrix \((I - \lambda A)\) is of order 4 or less. However for integral equations with separable kernels that give rise to larger systems, or for integral equations with kernels that are not separable kernels, iterative procedures are adopted to find the eigen values and eigen functions.