Lesson 37

1. **What is an integral equation?**

   An integral equation is an equation in which the unknown function appears under an integral sign. If in addition the equation includes only linear functions of the unknown function it is a linear integral equation, otherwise it is a nonlinear integral equation.

2. **What is a Fredholm equation?**

   A linear integral equation of the following form is known as a Fredholm equation:
   \[ \alpha(x)y(x) = F(x) + \lambda \int_{a}^{b} K(x, \xi) y(\xi) \, d\xi \] (*)

   Here \( y(x) \) is the unknown function while \( K(x, \xi), \alpha(x) \) and \( F(x) \) are known functions while \( \lambda, a \) and \( b \) are known constants. The function \( K(x, \xi) \) which depends on the independent variable \( x \) as well as the auxiliary variable \( \xi \) is known as the kernel of the integral equation.

3. **What is a Volterra Equation?**

   If the upper limit of the integral in (*) is the independent variable \( x \) rather than the constant 'b', the integral equation is known as Volterra's equation:
   \[ \alpha(x)y(x) = F(x) + \lambda \int_{a}^{x} K(x, \xi) y(\xi) \, d\xi \] (**)

4. **What is an integral equation of the first kind?**

   It is clear from both the Fredholm equation and the Volterra equations that when the function \( \alpha \) is zero, the unknown function \( y \) appears only under the integral sign. In that case, the integral equations are known as integral equations of the first kind.

5. **What is an integral equation of the second kind?**

   If \( \alpha(x) \) is not zero, for both the Fredholm equation and the Volterra equation, by a suitable manipulation and change of variables, the integral equation can be written in a form such that \( \alpha = 1 \). Integral equations with \( \alpha = 1 \) are known as integral equations of the second kind.
Why do integral equations contain a complete formulation of the problem?

In general, an integral equations contain the complete formulation of a problem, including initial and boundary conditions. Integral equation formulations are therefore different from differential equations where initial and boundary conditions have to be specified separately.