Module 1: Introduction to Finite Element Analysis

Lecture 2: Basic Concepts of Finite Element Analysis

1.2.1 Idealization of a Continuum

A continuum may be discretized in different ways depending upon the geometrical configuration of the domain. Fig. 1.2.1 shows the various ways of idealizing a continuum based on the geometry.

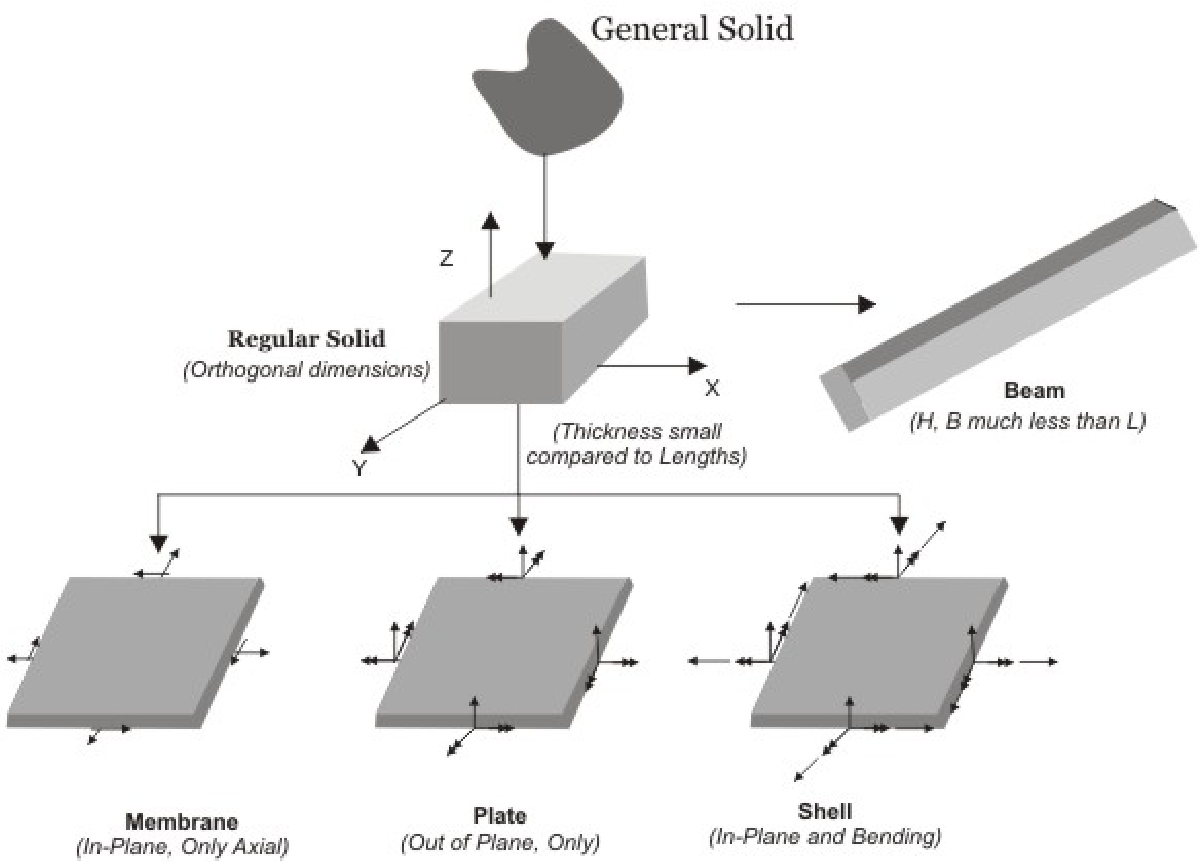

Fig. 1.2.1 Various ways of Idealization of a Continuum

1.2.2 Discretization of Technique

The need of finite element analysis arises when the structural system in terms of its either geometry, material properties, boundary conditions or loadings is complex in nature. For such case, the whole structure needs to be subdivided into smaller elements. The whole structure is then analyzed by the assemblage of all elements representing the complete structure including its all properties.

The subdivision process is an important task in finite element analysis and requires some skill and knowledge. In this procedure, first, the number, shape, size and configuration of elements have to be decided in such a manner that the real structure is simulated as closely as possible. The discretization is to be in such that the results converge to the true solution. However, too fine mesh will lead to extra computational effort. Fig. 1.2.2 shows a finite element mesh of a continuum using triangular and quadrilateral elements. The assemblage of triangular elements in this case shows better representation of the continuum. The discretization process also shows that the more accurate representation is possible if the body is further subdivided into some finer mesh.

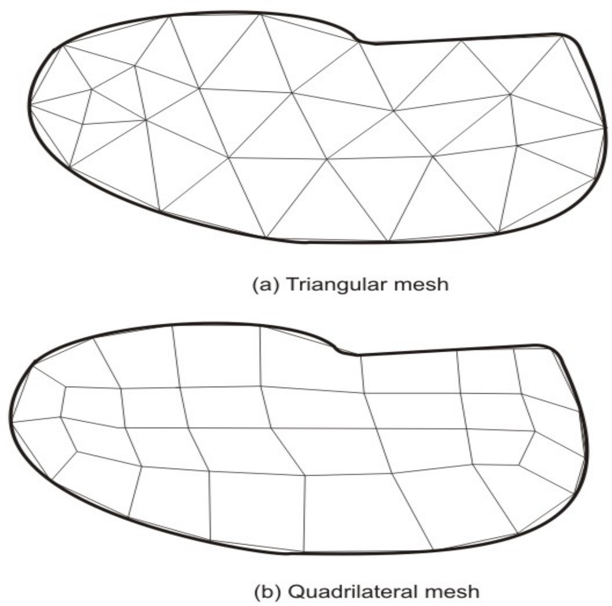

Fig. 1.2.2 Discretization of a continuum

1.2.3 Concepts of Finite Element Analysis

FEA consists of a computer model of a continuum that is stressed and analyzed for specific results. A continuum has infinite particles with continuous variation of material properties. Therefore, it needs to simplify to a finite size and is made up of an assemblage of substructures, components and members. Discretization process is necessary to convert whole structure to an assemblage of members/elements for determining its responses. Fig. 1.2.3 shows the process of idealization of actual structure to a finite element form to obtain the response results. The assumptions are required to be made by the experienced engineer with finite element background for getting appropriate response results. On the basis of assumptions, the appropriate constitutive model can be constructed.

For the linear-elastic-static analysis of structures, the final form of equation will be made in the form of *F=Kd*where *F*, *K* and *d* are the nodal loads, global stiffness and nodal displacements respectively.

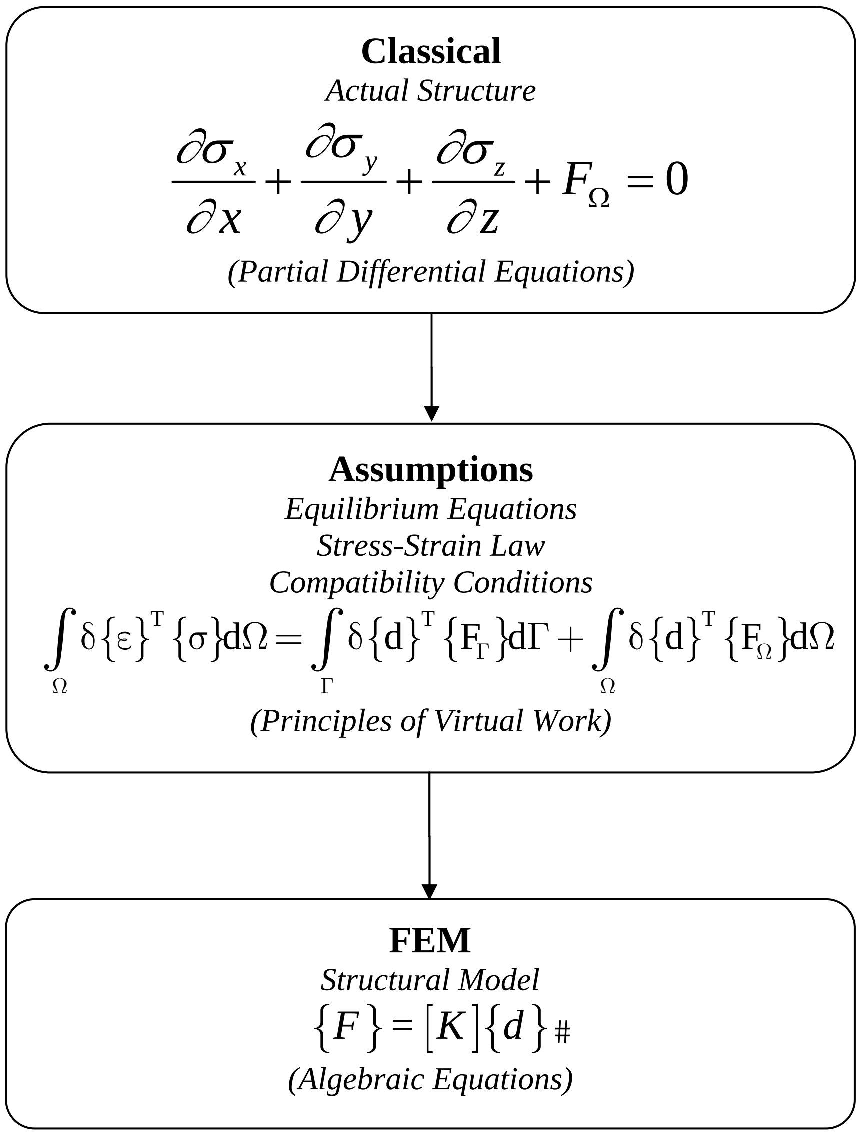

Fig. 1.2.3Fromclassical to FE solution

Varieties of engineering problem like solid and fluid mechanics, heat transfer can easily be solved by the concept of finite element technique. The basic form of the equation will become as follows where action, property and response parameter will vary for case to case as outlined in Table 1.2.1.

$$\{F\}=[K]\{d\} \quad OR \quad \{d\}=[K]^{-1}\{F\}$$

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Table 1.2.1 Response parameters for different cases

	Property	**Action**	**Response**
Solid	Stiffness	Load	Displacement
Fluid	Viscosity	Body force	Pressure/Velocity
Thermal	Conductivity	Heat	Temperature

1.2.4 Advantages of FEA

1. The physical properties, which are intractable and complex for any closed bound solution, can be analyzed by this method.
2. It can take care of any geometry (may be regular or irregular).
3. It can take care of any boundary conditions.
4. Material anisotropy and non-homogeneity can be catered without much difficulty.
5. It can take care of any type of loading conditions.
6. This method is superior to other approximate methods like Galerkine and Rayleigh-Ritz methods.
7. In this method approximations are confined to small sub domains.
8. In this method, the admissible functions are valid over the simple domain and have nothing to do with boundary, however simple or complex it may be.
9. Enable to computer programming.

1.2.5 Disadvantages of FEA

1. Computational time involved in the solution of the problem is high.
2. For fluid dynamics problems some other methods of analysis may prove efficient than the FEM.

1.2.6 Limitations of FEA

1. Proper engineering judgment is to be exercised to interpret results.
2. It requires large computer memory and computational time to obtainintend results.
3. There are certain categories of problems where other methods are more effective, e.g., fluid problems having boundaries at infinity are better treated by the boundary element method.
4. For some problems, there may be a considerable amount of input data. Errors may creep up in their preparation and the results thus obtained may also appear to be acceptable which indicates deceptive state of affairs. It is always desirable to make a visual check of the input data.
5. In the FEM,many problems lead to round-off errors. Computer works with a limited number of digits and solving the problem with restricted number of digits may not yield the desired degree of accuracy or it may give total erroneous results in some cases. For many problems the increase in the number of digits for the purpose of calculation improves the accuracy.

1.2.7 Errors and Accuracy in FEA

Every physical problem is formulated by simplifying certain assumptions. Solution to the problem, classical or numerical, is to be viewed within the constraints imposed by these simplifications. The material may be assumed to be homogeneous and isotropic; its behavior may be considered as linearly elastic; the prediction of the exact load in any type of structure is next to impossible. As such the true behavior of the structure is to be viewed with in these constraints and obvious errors creep in engineering calculations.

1. The results will be erroneous if any mistake occurs in the input data. As such, preparation of the input data should be made with great care.
2. When a continuum is discretised, an infinite degrees of freedom system is converted into a model having finite number of degrees of freedom. In a continuum, functions which are continuous are now replaced by ones which are piece-wise continuous within individual elements. Thus the actual continuum is represented by a set of approximations.
3. The accuracy depends to a great extent on the mesh grading of the continuum. In regions of high strain gradient, higher mesh grading is needed whereas in the regions of lower strain, the mesh chosen may be coarser. As the element size decreases, the discretisation error reduces.
4. Improper selection of shape of the element will lead to a considerable error in the solution. Triangle elements in the shape of an equilateral or rectangular element in the shape of a square will always perform better than those having unequal lengths of the sides. For very long shapes, the attainment of convergence is extremely slow.
5. In the finite element analysis, the boundary conditions are imposed at the nodes of the element whereas in an actual continuum, they are defined at the boundaries. Between the

nodes, the actual boundary conditions will depend on the shape functions of the element forming the boundary.

6. Simplification of the boundary is another source of error. The domain may be reduced to the shape of polygon. If the mesh is refined, then the error involved in the discretized boundary may be reduced.
7. During arithmetic operations, the numbers would be constantly round-off to some fixed working length. These round–off errors may go on accumulating and then resulting accuracy of the solution may be greatly impaired.