Module 4

(Lecture 16)

SHALLOW FOUNDATIONS: ALLOWABLE BEARING CAPACITY AND SETTLEMENT

Topics

1.1 STRIP FOUNDATION ON GRANULAR SOIL REINFORCED BY METALLIC STRIPS

- Mode of Failure
- Location of Failure Surface
- Force Induced in Reinforcement Ties

1.2 FACTOR OF SAFETY OF TIES AGAINST BREAKING AND PULLOUT

1.3 DESIGN PROCEDURE FOR STRIP FOUNDATION ON REINFORCED EARTH

- Determination of \( q_o \)
- Determination of \( Q_R \)
- Calculation of Tie Force
- Calculation of tie Resistance Due to Friction, \( F_B \)
- Calculation of Tie Thickness to Resist Tie Breaking
- Calculation of Minimum Length of Ties

STRIP FOUNDATION ON GRANULAR SOIL REINFORCED BY METALLIC STRIPS

Mode of Failure
The nature of bearing capacity failure of a shallow strip foundation resting on a compact and homogeneous soil mass was presented in figure 4.1a (from chapter 3). In contrast, if layers of reinforcing strips, or *ties*, are placed in the soil under a shallow strip foundation, the nature of failure in the soil mass will be like that shown in figure 4.36a, *b*, and *c*.

The type of failure in the soil mass shown in figure 4.36a generally occurs when the first layer of reinforcement is placed at a depth, \(d\), greater than about \(\frac{2}{3}B\) (\(B = \) width of the foundation). If the reinforcements in the first layer are strong and they are sufficiently concentrated, they may act as a rigid base at a limited depth. The bearing capacity of foundations in such cases can be evaluated by the theory presented by Mandel and Salencon (1972). Experimental laboratory results for the bearing capacity of shallow foundations resting on a sand layer with a rigid rough base at a limited depth have also been provided by Meyerhof (1974), Pfeifle, and Das (1979), and Das (1981).

The type of failure shown in figure 4.36b could occur if \(d/B\) is less than about \(\frac{2}{3}\) and the numbers of layers of reinforcement, \(N\), is less than about 2-3. In this type of failure, reinforcement tie pullout occurs.

The most beneficial effect of reinforced earth is obtained when \(d/B\) less than about is \(\frac{2}{3}\) and the number of reinforcement layers is greater than 4 but no more than 6-7. In this case, the soil mass fails when the upper ties break (see figure 4.36c).
Figure 4.36 Three modes of bearing capacity failure in reinforced earth (redrawn after Binquet and Lee, 1975b)

**Location of Failure Surface**

**Figure 4.37** shows an idealized condition for development of the failure surface in soil for the condition shown in figure 4.36c. It consists of a central zone-zone I-immediately below the foundation that settles along with the foundation with the application of load. On each side of zone I, the soil is pushed outward and upward-this zone II. The points \( A', A'', A''', \ldots \) and \( B', B'', B''', \ldots \) which define the limiting lines between zones I and II, can be obtained by considering the shear stress distribution, \( \tau_{xz} \), in the soil caused by the foundation load. The term \( \tau_{xz} \) refer to the shear stress developed at a depth \( z \) below the foundation at a distance \( x \) measured from the center line of the foundation. If integration of Boussinesq’s equation is performed, \( \tau_{xz} \) is given by the relation

\[
\tau_{xz} = \frac{4bqRz^2}{\pi[(x^2+z^2-b^2)^2+4b^2z^2]} \quad [4.68]
\]

Where \( q \)
\[ b = \text{half width of the foundation} = \frac{B}{2} \]

\[ B = \text{width of foundation} \]

\[ q_R = \text{load per unit area on the foundation} \]

The variation of \( \tau_{xz} \) at any depth, \( z \), is shown by the broken lines in figure 4.37a. Points \( A' \) and \( B' \) refer to the points at which the value of \( \tau_{xz} \) is maximum at \( z = z_1 \).

Similarly, \( A'' \) and \( B'' \) refer to the points at which \( \tau_{xz} \) is maximum at \( z = z_2 \). The distances \( x = X_o \) at which the maximum value of \( \tau_{xz} \) occurs take a nondimensional form and are shown in figure 4.37b.

**Force Induced in Reinforcement Ties**

Assumptions needed to obtain the tie force at any given depth are as follows:

1. Under the application of bearing pressure by the foundation, the reinforcing ties at points \( A', A'', A''', \ldots \) and \( B; B'', B''', \ldots \) take the shape shown in figure 4.37c. That is, the ties take two right-angle turns on each side of zone I around two frictionless rollers.

2. For \( N \) reinforcing layers, the ratio of the load per unit area on the foundation supported by reinforced earth, \( q_R \), to the load per unit area on the foundation supported by unreinforced earth, \( q_o \), is constant irrespective of the settlement level, \( S \) (see figure 4.38). Benquet and Lee (1975a) proved this relation in laboratory experiments.

**Figure 4.39a** shows a continuous foundation supported by unreinforced soil and subjected to a load of \( q_o \) per unit area. Similarly, figure 4.39b shows a continuous foundation supported by a reinforced soil layer (one layer of reinforcement, or \( N = 1 \)) and subjected to a load of \( q_R \) per unit area. (Due to symmetry only one-half of the foundation is shown in figure 4.39). In both cases—that is, in figure 4.39a and 4.39b—let the settlement equal \( S_c \). For one-half of each foundation under consideration, the following are the forces per unit length on a soil element of thickness \( \Delta H \) located at a depth \( z \).
Figure 4.38 Relationship between load per unit area and settlement for foundations resting on reinforced and unreinforced soil.
Figure 4.39 Derivation of equation (87)
Unreinforced Case $F_1$ and $F_2$ are the vertical forces and $S_1$ is the shear force. Hence, for equilibrium,

$$F_1 - F_2 - S_1 = 0 \quad [4.69]$$

Reinforced Case Here, $F_3$ and $F_4$ are the vertical forces, $S_2$ is the shear force, and $T_{(N=1)}$ is the tensile force developed in the reinforcement. The force $T_{(N=1)}$ is vertical because of the assumption made for the deformation of reinforcement as shown in figure 4.37c. So

$$F_3 - F_4 - S_2 - T_{(N=1)} = 0 \quad [4.70]$$

If the foundation settlement, $S_c$, is the same in both cases,

$$F_2 = F_4 \quad [4.71]$$

Subtracting equation (69) from equation (70) and using the relationship given in equation (71), we obtain

$$T_{(N=1)} = F_3 - F_1 - S_2 + S_1 \quad [4.72]$$

Note that the force $F_1$ is caused by the vertical stress, $\sigma$, on the soil element under consideration as a result of the load $q_o$ on the foundation. Similarly, $F_3$ is caused by the vertical stress imposed on the soil element as a result of the load $q_R$. Hence
\[ F_1 = \int_0^{x_o} \sigma(q_o) \, dx \]  
\[ F_3 = \int_0^{x_o} \sigma(q_R) \, dx \]  
\[ S_1 = \tau_{xz}(q_o)\Delta H \]  
\[ S_2 = \tau_{xz}(q_R)\Delta H \]

Where

\[ \sigma(q_o) \] and \( \sigma(q_R) \) are the vertical stresses at a depth \( z \) caused by the loads \( q_o \) and \( q_R \) on the foundation

\[ \tau_{xz}(q_o) \] and \( \tau_{xz}(q_R) \) are the shear stresses at a depth \( z \) and at a distance \( X_o \) from the center line caused by the loads \( q_o \) and \( q_R \)

Integrating Boussinesq’s solution yields

\[ \sigma(q_o) = \frac{q_o}{\pi} \left[ \tan^{-1} \frac{z}{x-b} - \tan^{-1} \frac{z}{x+b} - \frac{2b(x^2-z^2-b^2)}{(x^2+z^2-b^2)^2+4b^2z^2} \right] \]  
\[ \sigma(q_R) = \frac{q_R}{\pi} \left[ \tan^{-1} \frac{z}{x-b} - \tan^{-1} \frac{z}{x+b} - \frac{2b(x^2-z^2-b^2)}{(x^2+z^2-b^2)^2+4b^2z^2} \right] \]  
\[ \tau_{xz}(q_o) = \frac{4bq_oX_o z^2}{\pi[(X_o^2+z^2-b^2)^2+4b^2z^2]} \]  
\[ \tau_{xz}(q_R) = \frac{4bq_R X_o z^2}{\pi[(X_o^2+z^2-b^2)^2+4b^2z^2]} \]

Where

\[ b = B/2 \]

The procedure for derivation of equations (77 to 80) is not presented here; for this information see a soil mechanics textbook (for example, Das, 1997). Proper substitution of equations (77 to 80) into equations (73 to 76) and simplification yields

\[ F_1 = A_1 q_o B \]  
\[ F_3 = A_1 q_R B \]  
\[ S_1 = A_2 q_o \Delta H \]  
\[ S_2 = A_2 q_R \Delta H \]

Where

\[ A_1 \text{ and } A_2 = f(z/B) \]
The variations of \( A_1 \) and \( A_2 \) with nondimensional depth \( z \) are given in figure 4.40. Substituting equations (81)-(84) into equation (72) gives

\[
T_{(N=1)} = A_1 q R B - A_1 q_o B - A_2 q R \Delta H \quad A_2 q_o \Delta H = A_1 B (q R - q_o) - A_2 \Delta H (q R - q_o) = q_o \left( \frac{q R}{q_o} - 1 \right) (A_1 B - A_2 \Delta H)
\]

Note that the derivation of equation (85) was based on the assumption that there is only one layer of reinforcement under the foundation shown in figure 4.39b. However, if there are \( N \) layers of reinforcement under the foundation with center-to-center spacing of \( \Delta H \), as shown in figure 4.39c, the assumption can be made that

\[
T_{(N)} = \frac{T_{(N=1)}}{N}
\]

Combining equations (85 and 86) gives

\[
T_{(N)} = \frac{1}{N} \left[ q_o \left( \frac{q R}{q_o} - 1 \right) (A_1 B - A_2 \Delta H) \right]
\]

The unit of \( T_{(N)} \) in equation (87) is lb/ft (or kN) per unit length of foundation.

**FACTOR OF SAFETY OF TIES AGAINST BREAKING AND PULLOUT**

Once the tie forces that develop in each layer as the result of the foundation load are determined from equation (87), an engineer must determine whether the ties at any depth \( z \) will fail either by *breaking* or by *pullout*. The factor of safety against tie breaking at any depth \( z \) below the foundation can be calculated as
\[ FS_B = \frac{wtn f_y}{T(N)} \] \[ \text{[4.88]} \]

Where

\( FS_B \) = factor of safety against the breaking

\( w \) = width of a single tie

\( t \) = thickness of each tie

\( n \) = number of ties per unit length of the foundation

\( f_y \) = yield or breaking strength of the tie material

The term \( wn \) may be defined as the linear density ratio, \( LDR \), so

\[ FS_B = \left[ \frac{tf_y}{T(N)} \right] (LDR) \] \[ \text{[4.89]} \]

The resistance against the tie being pulled out derives from the frictional resistance between the soil and the ties at any depth. From the fundamental principles of statics, we know that the frictional force per unit length of the foundation resisting tie pullout at any depth \( z \) (figure 4.41).

\[ F_B = 2 \tan \phi_u \text{[normal force]} \] \[ \text{[4.90]} \]

\[ = \frac{2 \tan \phi_u}{\text{two sides of tie (i.e., top and bottom)}} \left[ \int_{x_o}^{L_o} (LDR) \sigma(q_h) dx + (LDR)(\gamma)(L_o - X_o)(z + D_f) \right] \]
Where

\( \gamma \) = unit weight of soil

\( D_f \) = depth of foundation

\( \phi_u \) = tie – soil friction angle

The relation for \( \sigma(q_R) \) was defined in equation (78). The value of \( x = L_o \) is generally assumed to be the distance at which \( \sigma(q_R) \) equals to 0.1\( q_R \). The value of \( L_o \) as a function of depth \( z \) is given in figure 4.42. Equation (90) may be simplified as

\[
F_B = 2 \tan \phi_y (LDR) \left[ A_3 B q_o \left( \frac{q_R}{q_o} \right) + \gamma (L_o - X_o)(z + D_f) \right]
\]

[4.91]
Figure 4.42 Variation of $L_o/B$ with $z/B$ (after Binquet and Lee, 1975b)

Where

$A_3$ is a nondimensional quantity that may be expressed as a function of depth ($z/B$) (see figure 4.40)

The factor of safety against tie pullout, $FS_{(P)}$, is

$$FS_{(P)} = \frac{F_B}{T(N)}$$  \[4.92\]

**DESIGN PROCEDURE FOR STRIP FOUNDATION ON REINFORCED EARTH**

Following is a step-by-step procedure for the design of a strip foundation supported by granular soil reinforced by metallic strips:

1. Obtain the total load to be supported per unit length of the foundation.

   Also obtain the quantities

   a. Soil-friction angle, $\phi$
   b. Soil-tie friction angle, $\phi_{\mu}$
   c. Factor of safety against bearing capacity failure
   d. Factor of safety against tie breaking, $FS_{(B)}$
   e. Factor of safety against tie pullout, $FS_{(P)}$
f. Breaking strength of reinforcement ties, \( f_y \)
g. Unit weight of soil, \( \gamma \)
h. Modulus of elasticity of soil, \( E_s \)
i. Poisson's ratio of soil, \( \mu_s \)
j. Allowable settlement of foundation, \( S_c \)
k. Depth of foundation, \( D_f \)

2. Assume a width of foundation, \( B \), and also \( d \) and \( N \). The value of \( d \) should be less than \( \frac{3}{4}B \). Also, the distance from the bottom of the foundation to the lowest layer of the reinforcement should be about \( 2B \) or less. Calculate \( \Delta H \).

3. Assume a value of \( L/LR \)

4. For width \( Bi \) (step 2) determine the ultimate bearing capacity, \( q_{ult} \), for unreinforced soil [equation (3 from chapter 3); note: \( c = 0 \)]. Determine \( q_{all \ (1)} \):

\[
q_{all \ (1)} = \frac{q_u}{FS \text{ against bearing capacity failure}} \tag{4.93}
\]

5. Calculate the allowable load, \( q_{all \ (2)} \) based on the tolerable settlement, \( S_c \), assuming that the soil is not reinforced [equation (32a)]:

\[
S_c = \frac{b q_{all \ (2)}}{E_s} (1 - \mu_s^2) \alpha_r
\]

For \( L/B = \infty \), the value of \( \alpha_r \) may be taken as 2, or

\[
q_{all \ (2)} = \frac{E_s S_c}{B(1-\mu_s^2)\alpha_r} \tag{4.94}
\]

(The allowable load for a given settlement, \( S_c \), could have also been determined from equations that relate to standard penetration resistances).

6. Determine the lower of the two value of \( q_{all} \) obtained from steps 4 and 5. The lower value of \( q_{all} \) equals \( q_o \).

7. Calculate the magnitude of \( q_R \) for the foundation supported by reinforced earth:

\[
q_R = \frac{\text{load on foundation}}{B} \text{ per unit length} \tag{4.95}
\]

8. Calculate the tie force, \( T_{(N)} \), in each layer of reinforcement by using equation (87) (note: unit of \( T_{(N)} \) as kN/m of foundation).

9. Calculate the frictional resistance of ties for each layer per unit length of foundation, \( F_B \), by using equation (91). For each layer, determine whether \( F_B/T_{(N)} \geq FS_{(P)} \). If \( F_B/T_{(N)} < FS_{(P)} \), the length of the reinforcing strips for a layer may be increased. That will increase the value of \( F_B \) and thus \( FS_{(P)} \), and so equation (91) must be rewritten as

\[
F_B = 2 \tan \phi \mu (LDR) \left[ A_3 B q_o \left( \frac{q_B}{q_o} \right) + \gamma (L_0 - X_0)(z + D_f) \right] \tag{4.96}
\]
Where

\[ L = \text{the required length to obtain the desired value of } F_B \]

10. Use equation (89) to obtain the tie thickness for each layer. Some allowance should be made for the corrosion effect of the reinforcement during the life of the structure.

11. If the design is unsatisfactory, repeat steps 2-10.

The following example demonstrates the application of these steps.

**Example 10**

Design a strip foundation that will carry a load of 1.8 MN/m. Use the following parameters:

- **Soil**: \( y = 17.3 \text{ kN/m}^3; \phi = 35^\circ; E_s = 3 \times 10^4 \text{kN/m}^2; \mu_s = 0.35 \)
- **Reinforcement ties**: \( f_y = 2.5 \times 10^5 \text{kN/m}^2; \phi_{\mu} = 28^\circ; F_{S(R)} = 3; F_{S(P)} = 2.5 \)
- **Foundation**: \( D_f = 1 \text{m}; \text{Factor of safety against baring capacity failure} = 3. \)

Tolerable settlement = \( S_c = 25 \text{ mm}; \text{desired life of structure} = 50 \text{ years} \)

**Solution**

Let

\[ B = 1 \text{ m} \]

\( d = \text{depth from the bottom of the foundation to the first reinforcing layer} = 0.5 \)

\( \Delta H = 0.5 \text{ m} \)

\( N = 5 \)

\( LDR = 65\% \)

If the reinforcing strips used are 75 mm wide, then

\[ wn = LDR \]

Or

\[ n = \frac{LDR}{w} = \frac{0.65}{0.075 \text{ m}} = 8.67/\text{m} \]
Hence each layer will contain 8.67 strips per meter length of the foundation.

**Determination of** $q_o$

For an unreinforced foundation

$$q_u = \gamma D_f N_q + \frac{1}{2} \gamma B N_r$$

From table 4 (from chapter 3) for $\phi = 35^\circ$, $N_q = 33.30$ and $N_r = 48.03$. Thus

$$q_u = (17.3)(1)(33.3) + \frac{1}{2} (17.3)(1)(48.03)$$

$$= 576.09 + 415.46 = 991.55 \approx 992 \text{ kN/m}^2$$

$$q_{all(1)} = \frac{q_u}{F_S} = \frac{992}{3} = 330.7 \text{ kN/m}^2$$

From equation (94)

$$q_{all(2)} = \frac{(E_s)(S_c)}{B(1-\mu s^2)} = \frac{(30,000 \text{ kN/m}^2)(0.025 \text{ m})}{(1 \text{ m})(1-0.35^2)(2)} = 427.35 \text{ kN/m}^2$$

As $q_{all(1)} < q_{all(2)}$, $q_o = q_{all(1)} = 330.7 \text{ kN/m}^2$

**Determination of** $Q_R$

From equation (95),

$$Q_R = \frac{1.8 \text{ MN/m}}{B} = \frac{1.8 \times 10^3}{1} = 1.8 \times 10^3 \text{ kN/m}^2$$

**Calculation of Tie Force**

From equation (87),

$$T_{(N)} = \left(\frac{q_o}{N}\right)\left(\frac{Q_R}{q_o} - 1\right)(A_1B - A_2A_1)$$

The tie forces for each layer are given in the following table.

<table>
<thead>
<tr>
<th>Layer no</th>
<th>$\left(\frac{q_o}{N}\right)\left(\frac{Q_R}{q_o} - 1\right)$</th>
<th>$z$(m)</th>
<th>$\frac{z}{B}$</th>
<th>$A_1B$</th>
<th>$A_2A_1$</th>
<th>$A_1B - A_2A_1$</th>
<th>$T_{(N)}$(kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>293.7</td>
<td>0.5</td>
<td>0.5</td>
<td>0.35</td>
<td>0.125</td>
<td>0.225</td>
<td>66.08</td>
</tr>
<tr>
<td>2</td>
<td>293.7</td>
<td>1.0</td>
<td>1.0</td>
<td>0.34</td>
<td>0.09</td>
<td>0.25</td>
<td>73.43</td>
</tr>
</tbody>
</table>
Calculation of tie Resistance Due to Friction, $F_B$

Use equation (91):

$$F_B = 2 \tan \phi_u (LDR) \left[ A_3 B q_o \left( \frac{q_u}{q_o} \right) + \gamma (L_o - X_o) (z + D_f) \right]$$

The following table shows the magnitude of $F_B$ for each layer:

<table>
<thead>
<tr>
<th>Layer number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 tan $\phi_u$ (LDR)</td>
<td>0.691</td>
<td>0.691</td>
<td>0.691</td>
<td>0.691</td>
<td>0.691</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.125</td>
<td>0.14</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$A_3 B q_o (q_u/q_o)$</td>
<td>225.0</td>
<td>252.0</td>
<td>270.0</td>
<td>270.0</td>
<td>270.0</td>
</tr>
<tr>
<td>$z$ (m)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>$z/B$</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>$L_o$ (m)</td>
<td>1.55</td>
<td>2.6</td>
<td>3.4</td>
<td>3.85</td>
<td>4.2</td>
</tr>
<tr>
<td>$X_o$ (m)</td>
<td>0.55</td>
<td>0.8</td>
<td>1.1</td>
<td>1.4</td>
<td>1.65</td>
</tr>
<tr>
<td>$L_o - X_o$ (m)</td>
<td>1.0</td>
<td>1.8</td>
<td>2.3</td>
<td>2.45</td>
<td>2.55</td>
</tr>
<tr>
<td>$z + D_f$ (m)</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>$\gamma (L_o - X_o) (z + D_f)$ (m)</td>
<td>25.95</td>
<td>62.28</td>
<td>99.48</td>
<td>127.16</td>
<td>154.4</td>
</tr>
<tr>
<td>$F_B$ (kN/m)</td>
<td>173.4</td>
<td>217.2</td>
<td>255.1</td>
<td>274.4</td>
<td>292.3</td>
</tr>
<tr>
<td>$FS_{(P)} = F_B / T(N)$</td>
<td>2.62</td>
<td>2.96</td>
<td>3.16</td>
<td>3.34</td>
<td>3.57</td>
</tr>
</tbody>
</table>

Note: $A_3$ is from figure 4.40; $X_o$ is from figure 4.37; $L_o$ is from figure 4.42; $T(N)$ is from
The minimum factor of safety is greater than the required values of $F_s(P)$ which is 2.5.

**Calculation of Tie Thickness to Resist Tie Breaking**

From equation (89),

$$FS(B) = \frac{f_y}{t(N)} (LDR)$$

Here, $f_y = 2.5 \times 10^5$ kN/m$^2$, $LDR = 0.65$ and $F_s(B) = 3$, so

$$t = \left(\frac{3}{(2.5 \times 10^5)(0.65)}\right) T(N) = (1.846 \times 10^5)T(N)$$

So, for layer 1

$$t = (1.846 \times 10^{-5})(66.08) = 0.00122m = 1.22 \text{ mm}$$

For layer 2

$$t = (1.846 \times 10^{-5})(73.43) = 0.00136m = 1.36 \text{ mm}$$

Similarly, for layer 3

$$t = 0.00149 = 1.49 \text{ mm}$$

For layer 4

$$t = 1.52 \text{ mm}$$

For layer 5

$$t = 1.52 \text{ mm}$$

Thus in each layer ties with a thickness of 1.6 mm will be sufficient. However, if galvanized steel is used, the rate of corrosion is about 0.025 mm/yr, so $t$ should be 1.6 + (0.025)(50) = 2.85 mm.
Calculation of Minimum Length of Ties

The minimum length of ties in each layer should equal $2L_o$. Following is the length of ties in each layer:

<table>
<thead>
<tr>
<th>Layer no.</th>
<th>Minimum length of the tie, $2L_o$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.1</td>
</tr>
<tr>
<td>2</td>
<td>5.2</td>
</tr>
<tr>
<td>3</td>
<td>6.8</td>
</tr>
<tr>
<td>4</td>
<td>7.7</td>
</tr>
<tr>
<td>5</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Figure 4.43 is a diagram of the foundation with the ties. The design could be changed by varying $B$, $d$, $N$, and $\Delta H$ to determine the most economical combination.

Example 11

Refer to example 10. For the loading given, determine the width of the foundation that is needed for unreinforced earth. Note that the factor of safety against bearing capacity failure is 3 and that the tolerable settlement is 25 mm.
Solution

Bearing Capacity Consideration

For a continuous foundation,

$$q_u = \gamma Df N_q + \frac{1}{2} \gamma BN_r$$

For $\phi = 35^\circ, \quad N_q = 33.3$ and $N_r = 48.03$, so

$$q_{all} = \frac{q_u}{FS} = \frac{1}{FS} \left[ \gamma Df N_q + \frac{1}{2} \gamma BN_r \right]$$

Or

$$q_{all} = \frac{1}{3} \left[ (17.3)(1)(33.3) + \frac{1}{2} (17.3)(B)(48.03) \right] = 192.03 + 138.5B \quad [a]$$

However,

$$q_{all} = \frac{1.8 \times 10^3 \text{kN}}{(B)(1)} \quad [b]$$

Equating the right-hand sides of equations (a) and (b) yields

$$\frac{1800}{(B)(1)} = 192.03 + 138.5B$$

Solving the preceding equation gives $B \approx 3$ m, so with $B = 3$ m, $q_{all} = 600$ kN/m$^2$.

Settlement Consideration

For a friction angle of $\phi = 35^\circ$, the corrected average standard penetration number is about 10-15 (equation 11 from chapter 2). From equation (53) for the higher value, $N_{cor} = 15$,

$$q_{all} = 11.98N_{cor} \left( \frac{3.28B+1}{3.28} \right)^2 \left( 1 + \frac{0.33D_f}{B} \right)$$

for a settlement of about 25 mm. now, we can make a few trials:

<table>
<thead>
<tr>
<th>Assumed $B$(m)(1)</th>
<th>$q_{all} = 11.98N_{cor} \left( \frac{3.28B+1}{3.28} \right)^2 \left( 1 + \frac{0.33D_f}{B} \right)$ (kN/m$^2$) (2)</th>
<th>$Q = (B)(q_{all}) = \text{Col.1} \times \text{Col.2}$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>209</td>
<td>1254</td>
</tr>
<tr>
<td>9</td>
<td>199</td>
<td>1791</td>
</tr>
</tbody>
</table>

Note: $D_f = 1$ m
Required 1800 kN/m

For $N_{cor} = 15$, the width of the foundation should be 9m or more. Based on the consideration of bearing capacity failure and tolerable settlement, the latter criteria will control, so $B$ is about 9 m.

Note: At first, the results of this calculation may show the use of reinforced earth for foundation construction to be desirable. However, several factors must be considered before a final decision is made. For example, reinforced earth needs overexcavation and backfilling. Hence, under many circumstances, proper material selection and compaction may make the construction of foundations on unreinforced soils more economical.

**PROBLEMS**

1. A flexible circular area is subjected to a uniformly distributed load of 3000 lb/ft$^2$. The diameter of the loaded area is 9.5 ft. Determine the stress increase in a soil mass at a point located 7.5 ft below the center of the loaded area.
2. Refer to figure 5, which shows a flexible rectangular area. Given: $B_1 = 1.2$ m, $B_2 = 3$ m, $L_1 = 3$ m, and $L_2 = 6$ m. If the area is subjected to a uniform load of 110 kN/m$^2$, determine the stress increase at a depth of 8 m located immediately below point $O$.
3. Repeat problem 2 with the following

   $B_1 = 5$ ft, $B_2 = 10$ ft
   $L_1 = 7$ ft, $L_2 = 12$ ft

   Uniform load on the flexible area = 2500 lb/ft$^2$

4. Refer to figure P-1. Using the procedure outlined in section 5, determine the average stress increase in the clay layer below the center of the foundation due to the net foundation load of 900 kN.
5. **Figure P-2** shows an embankment load on a silty clay soil layer. Determine the stress increase at points $A$, $B$, and $C$ which are located at a depth of 15 ft below the ground surface.

6. A flexible load area (**figure P-3**) is $2\text{m} \times 3\text{m}$ in plan and carries a uniformly distributed load of $210 \text{kN/m}^2$. Estimate the elastic settlement below the center of the loaded area. Assume $D_f = 0$ and $H = \infty$. 
7. A continuous foundation on a deposit of sand layer is shown in figure P-4 along with the variation of the modulus of elasticity of the soil \( E_s \). Assuming \( \gamma = 115 \text{ lb/ft}^3 \) and \( C_2 = 10 \text{ yr} \), calculate the elastic settlement of the foundation using the strain influence factor.

![Figure P-4](image)

8. Tow plate load tests with square plates were conducted in the field. At 1-in. settlement, the results were

<table>
<thead>
<tr>
<th>Width of plane (in.)</th>
<th>Load (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>8,070</td>
</tr>
<tr>
<td>24</td>
<td>25,800</td>
</tr>
</tbody>
</table>

What size of square footing is required to carry a net load of 150,000 lb at a settlement of 1 in.?

9. The tie forces under a continuous foundation are given by equation (87). For the foundation described in problem 23, \( q_o = 200 \text{ kN/m}^2 \) and \( q_R/q_o = 4.5 \). Determine the tie forces, \( T_{(N)} \), in kN/m for each layer of reinforcement.