Module 3

Lecture 11

SHALLOW FOUNDATIONS: ULTIMATE BEARING CAPACITY

Topics

1.1 BEARING CAPACITY OF LAYERED SOILS-STRONGER SOIL UNDERLAIN BY WEAKER SOIL
   ➢ Special Cases

1.2 BEARING CAPACITY OF FOUNDATIONS ON TOP OF A SLOPE

1.3 SEISMIC BEARING CAPACITY AND SETTLEMENT IN GRANULAR SOIL
BEARING CAPACITY OF LAYERED SOILS-STRONGER SOIL UNDERLAIN BY WEAKER SOIL

The bearing capacity equations presented in the preceding sections involve cases in which the soil supporting the foundation is homogeneous and extends to a considerable depth. Cohesion, angle of friction, and unit weight of soil were assumed to remain constant for the bearing capacity analysis. However, in practice, layered soil profiles are often encountered. In such instances, the failure surface at ultimate load may extend through two or more soil layers. Determination of ultimate bearing capacity in layered soils can be made in only a limited number of cases. This section features the procedure for estimating bearing capacity for layered soils proposed by Meyerhof and Hanna (1978) and Meyerhof (1974).

Figure 3.20 shows a shallow continuous foundation supported by a stronger soil layer underlain by a weaker soil, which extends to a great depth. For the two soil layers, the physical parameters are as follows:

<table>
<thead>
<tr>
<th>Layer</th>
<th>Unit weight</th>
<th>Soil friction angle</th>
<th>Cohesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>$\gamma_1$</td>
<td>$\phi_1$</td>
<td>$c_1$</td>
</tr>
<tr>
<td>Bottom</td>
<td>$\gamma_2$</td>
<td>$\phi_2$</td>
<td>$c_2$</td>
</tr>
</tbody>
</table>

At ultimate load per unit area ($q_u$), the failure surface in soil will be as shown in figure 3.20. If the depth $H$ is relatively small compared to the foundation width $B$, a punching shear failure will occur in the top soil layer followed by a general shear failure in the bottom soil layer. This is shown in figure 3.20a. However, if the depth $H$ is relatively layer, the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity. This is shown in figure 3.20b.

Figure 3.20 Bearing capacity of a continuous foundation on layered soil
The ultimate bearing capacity, \( q_u \), for this problem as shown in figure 3.20a can be given as

\[
q_u = q_b + \frac{2(C_a + P_p \sin \delta)}{B} = \gamma_1 H
\]

[3.60]

Where

\( B \) = width of the foundation

\( C_a \) = adhesive force

\( P_p \) = passive force per unit length of the faces \( aa' \) and \( bb' \)

\( q_b \) = bearing capacity of the bottom soil layer

\( \delta \) = inclination of the passive force \( P_p \) with the horizontal

Note that, in equation (60),

\[
C_a = c_a H
\]

[3.61]

Where

\( c_a \) = adhesion

Equation (60) can be simplified to the form

\[
q_u = q_b + \frac{2c_a H}{B} + \gamma_1 H^2 \left( 1 + \frac{2D_f}{H} \right) \frac{K_{pH} \tan \delta}{B} - \gamma_1 H
\]

[3.62]

Where

\( K_{pH} \) = horizontal component of passive earth pressure coefficient

However, let

\[
K_{pH} \tan \delta = K_s \tan \phi_1
\]

[3.63]

Where

\( K_s \) = punching shear coefficient

So

\[
q_u = q_b + \frac{2c_a H}{B} + \gamma_1 H^2 \left( 1 + \frac{2D_f}{H} \right) \frac{K_s \tan \phi_1}{B} - \gamma_1 H
\]

[3.64]

The punching shear coefficient, \( K_s \), is a function of \( q_2/q_1 \) and \( \phi_1 \), or
\[ K_s = f\left(\frac{q_2}{q_1}, \phi_1\right) \]

Note that \(q_1\) and \(q_2\) are the ultimate bearing capacities of a continuous foundation of width \(B\) under vertical load on the surfaces of homogeneous thick beds of upper and lower soil, or

\[ q_1 = c_1 N_{c(1)} + \frac{1}{2} \gamma_1 B N_{\gamma(1)} \quad [3.65] \]

And

\[ q_2 = c_2 N_{c(2)} + \frac{1}{2} \gamma_2 B N_{\gamma(2)} \quad [3.66] \]

Where

\(N_{c(1)}, N_{\gamma(1)}\) = bearing capacity factors for friction angle \(\phi_1\) (table 4)

\(N_{c(2)}, N_{\gamma(2)}\) = bearing capacity factors for friction angle \(\phi_2\) (table 4)

It is important to note that, for the top layer to be a stronger soil, \(q_2/q_1\) should be less than one.

The variation of \(K_s\) with \(q_2/q_1\) and \(\phi_1\) is shown in figure 3.21. The variation of \(c_a/c_1\) with \(q_2/q_1\) is shown in figure 3.22. If the height \(H\) is relatively large, then the failure surface in soil will be completely located in the stronger upper-soil layer (figure 3.20b). For this case,

![Figure 3.21](image-url)

Figure 3.21 Meyerhof and Hanna’s punching shear coefficient, \(K_s\)
Figure 3.22 Variation of $c_a/c_1$ vs $q_2/q_1$ based on the theory of Meyerhof and Hanna (1978)

$$q_u = q_t = c_1N_{c(1)} + qN_{q(1)} + \frac{1}{2}BN_{\gamma(1)}$$ \[3.67\]

Where $N_{q(1)}$ = bearing capacity factor for $\phi = \phi_1$ (table 4) and $q = \gamma_1D_f$

Now, combining equations (64 and 67)

$$q_u = q_b + \frac{2c_aH}{B} + \gamma_1H^2 \left( 1 + \frac{2D_f}{H} \right) K_s \frac{\tan \phi_1}{B} - \gamma_1H \leq q_t$$ \[3.68a\]

For rectangular foundations, the preceding equation can be extended to the form

$$q_u = q_b + \left( 1 + \frac{B}{L} \right) \left( \frac{2c_aH}{B} \right) + \gamma_1H^2 \left( 1 + \frac{B}{L} \right) \left( 1 + \frac{2D_f}{H} \right) K_s \frac{\tan \phi_1}{B} - \gamma_1H \leq q_t$$ \[3.68b\]

Where

$$q_b = c_2N_{c(2)}F_{cs(2)} + \gamma_1(D_f + H)N_{q(2)}F_{qs(2)} + \frac{1}{2}\gamma_2BN_{\gamma(2)}F_{\gamma s(2)}$$ \[3.69\]

$$q_t = c_1N_{c(1)}F_{cs(1)} + \gamma_1D_fN_{q(1)}F_{qs(1)} + \frac{1}{2}\gamma_1BN_{\gamma(1)}F_{\gamma s(1)}$$ \[3.70\]

Where

$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors with respect to top soil layer (table 5)

$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors with respect to bottom soil layer (table 5)
Special Cases

1. **Top layer is strong sand and bottom layer is saturated soft clay** ($\phi_2 = 0$). From equations (68, 69, and 70),

\[
q_b = \left(1 + 0.2 \frac{B}{L}\right) 5.14c_2 + \gamma_1(D_f + H) \quad [3.71]
\]

\[
q_t = \gamma_1 D_f N_q(1) F_{qs}(1) + \frac{1}{2} \gamma_1 B N_y(1) F\gamma S(1) \quad [3.72]
\]

Hence

\[
qu = \left(1 + 0.2 \frac{B}{L}\right) 5.14c_2 + \gamma_1 H^2 \left(1 + \frac{B}{L}\right) \left(1 + \frac{2D_f}{H}\right) \frac{K_s \tan \phi_1}{B} + \gamma_1 D_f \leq \gamma_1 D_f N_q(1) F_{qs}(1) + \frac{1}{2} \gamma_1 B N_y(1) F\gamma S(1) \quad [73]
\]

For determination of $K_s$ from figure 3.21,

\[
\frac{q_2}{q_1} = \frac{c_2 N_{c(2)}}{\frac{5.14c_2}{0.5\gamma_1 B N_y(1)}} \quad [3.74]
\]

2. **Top layer is stronger sand and bottom layer is weaker sand** ($c_1 = 0, c_2 = 0$). The ultimate bearing capacity can be given as

\[
qu = \left[\gamma_1(D_f + H)N_q(2)F_{qs}(2) + \frac{1}{2} \gamma_1 B N_y(2) F\gamma S(2)\right] + \gamma_1 H^2 \left(1 + \frac{B}{L}\right) \left(1 + \frac{2D_f}{H} \right) \frac{K_s \tan \phi_1}{B} \leq q_t \quad [3.75]
\]

Where

\[
q_t = \gamma_1 D_f N_q(1) F_{qs}(1) + \frac{1}{2} \gamma_1 B N_y(1) F\gamma S(1) \quad [3.76]
\]

\[
\frac{q_2}{q_1} = \frac{2\gamma_2 B N_y(2)}{\frac{\gamma_2 N_{y(2)}}{\gamma_1 N_y(1)}} \quad [3.77]
\]

3. **Top layer is stronger saturated clay** ($\phi_1 = 0$) and **bottom layer is weaker saturated clay** ($\phi_2 = 0$). The ultimate bearing capacity can be given as

\[
qu = \left(1 + 0.2 \frac{B}{L}\right) 5.14c_2 + \left(1 + \frac{B}{L}\right) \left(\frac{2c_a H}{B}\right) + \gamma_1 D_f \leq q_t \quad [3.78]
\]

\[
q_t = \left(1 + 0.2 \frac{B}{L}\right) 5.14c_1 + \gamma_1 D_f \quad [3.79]
\]

For this case
\[
\frac{q_2}{q_1} = \frac{5.14c_2}{5.14c_1} = \frac{c_2}{c_1}
\]  

**Example 8**

A foundation 1.5 m × 1 m is located at a depth, \(D_f\) of 1 m in stronger clay. A softer clay layer is located at a depth, \(H\), of 1 m measured from the bottom of the foundation. For the top clay layer,

Undrained shear strength = 120 kN/m²

Unit weight = 16.8 kN/m³

And for the bottom clay layer,

Undrained shear strength = 48 kN/m²

Unit weight = 16.2 kN/m³

Determine the gross allowable load for the foundation with an FS of 4.

**Solution**

For this problem, equations (78, 79 and 80) will apply, or

\[
q_u = \left(1 + 0.2 \frac{B}{L}\right)5.14c_2 + \left(1 + \frac{B}{L}\right)\left(\frac{2c_aH}{B}\right) + \gamma_1 D_f \leq \left(1 + 0.2 \frac{B}{L}\right)5.14c_1 + \gamma_1 D_f
\]

Given:

\(B = 1\) m \hspace{1cm} \(H = 1\) m \hspace{1cm} \(D_f = 1\) m

\(L = 1.5\) m \hspace{1cm} \(\gamma_1 = 16.8\) kN/m³

From figure 3.22, \(c_2/c_1 = 48/120 = 0.4\), the value of \(c_a/c_1 \approx 0.9\), so

\(c_a = (0.9)(120) = 108\) kN/m²

\[q_u = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(48) + \left(1 + \frac{1}{1.5}\right)\left[\frac{(2)(108)(1)}{1}\right] + (16.8)(1) = 279.6 + 360 + 16.8 = 656.4\) kN/m²

Check: From equation (79),

\[q_u = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(120) + (16.8)(1) = 699 + 16.8 = 715.8\) kN/m²

Thus \(q_u = 656.4\) kN/m² (that is, the smaller of the two values calculated above) and
\[ q_{\text{all}} = \frac{q_u}{FS} = \frac{656.4}{4} = 164.1 \text{ kN/m}^2 \]

The total allowable load is
\[ (q_{\text{all}})(1 \times 1.5) = 246.15 \text{ kN} \]

**Example 9**

Refer to figure 3.20. Assume that the top layer is sand and the bottom layer is soft saturated clay. Given:

*For the sand: \( \gamma_1 = 117 \text{ lb/ft}^3; \ \phi_1 = 40^\circ \)*

*For the soft clay (bottom layer): \( c_2 = 400 \text{ lb/ft}^2; \ \phi_2 = 0 \)*

*For the foundation: \( B = 3 \text{ ft}; \ D_f = 3 \text{ ft}; L = 4.5 \text{ ft}; H = 4 \text{ ft} \)*

Determine the gross ultimate bearing capacity of the foundation.

**Solution**

For this case equations (73 and 74) apply. For \( \phi_1 = 40^\circ \), from table 4, \( N_\gamma = 109.41 \) and

\[ q_2 = \frac{c_2 N_\gamma(2)}{0.5 \gamma_1 BN_\gamma(1)} = \frac{(400)(5.14)}{(0.5)(117)(3)(109.41)} = 0.107 \]

From figure 3.21, for \( c_2 N_\gamma(2)/0.5 \gamma_1 BN_\gamma(1) = 0.107 \) and \( \phi_1 = 40^\circ \), the value of \( K_s \approx 2.5 \). Equation (73) gives

\[ q_u = \left(1 + 0.2 \frac{B}{L}\right) 5.14 c_2 + \left(1 + \frac{B}{L}\right) \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) K_s \frac{\tan \phi_1}{B} + \gamma_1 D_f \]

\[ = \left[1 + (0.2) \left(\frac{3}{4.5}\right)\right] (5.14)(400) + \left(1 + \frac{3}{4.5}\right) (117)(4)^2 \times \left[1 + \frac{(2)(3)}{4}\right] (2.5) \frac{\tan 40}{3} + \]

\[ (117)(3) = 2330 + 5454 + 351 = 8135 \text{ lb/ft}^2 \]

Again, from equation (73)

\[ q_t = \gamma_1 D_f N_q(1) F_{qs}(1) + \frac{1}{2} \gamma_1 BN_\gamma(1) F_{ys}(1) \]

From table 4, for \( \phi_1 = 40^\circ, N_\gamma = 109.4, N_q = 64.20 \)

From table 5,

\[ F_{qs}(1) = \left(1 + \frac{B}{L}\right) \tan \phi_1 = \left(1 + \frac{3}{4.5}\right) \tan 40 = 1.4 \]

\[ F_{ys}(1) = 1 - 0.4 \frac{B}{L} = 1 - (0.4) \left(\frac{3}{4.5}\right) = 0.733 \]
\[ q_t = (117)(3)(64.20)(1.4) + \left(\frac{1}{2}\right)(117)(3)(109.4)(0.733) = 45,622 \text{ lb/ft}^2 \]

Hence

\[ q_u = 8135 \text{ lb/ft}^2 \]

**BEARING CAPACITY OF FOUNDATIONS ON TOP OF A SLOPE**

In some instances, shallow foundations need to be constructed on top of a slope (figure 3.23). In figure 3.23, the height of the slope is \( H \), and the slope makes an angle \( \beta \) with the horizontal. The edge of the foundation is located at a distance \( b \) from the top of the slope. At ultimate load, \( q_u \), the failure surface will be as shown in the figure 3.23..

![Shallow foundation on top of a slope](image-url)

Figure 3.23 Shallow foundation on top of a slope
Figure 3.24 Meyerhof bearing capacity factor, $N_{eq}$, for granular soil ($c = 0$)
Example 10

Refer to figure 3.23. For a shallow continuous foundation in a clay, the following are given: \( B = 1.2 \text{ m} \); \( D_f = 1.2 \text{ m} \); \( b = 0.8 \text{ m} \); \( H = 6.2 \text{ m} \); \( \beta = 30^\circ \); unit weight of soil = 17.5 kN/m\(^3\); \( \phi = 0 \); \( c = 50 \text{ kN/m}^2 \). Determine the gross allowable bearing capacity with a factor of safety \( FS = 4 \).

Solution

Since \( B < H \), we will assume the stability number \( N_s = 0 \). From equation (83),

\[ q_u = cN_{cq} \]

Given
For $\beta = 30^\circ$, $D_f/B = 1$ and $b/B = 0.75$, figure 3.25 gives $N_{cq} = 6.3$. Hence

\[ q_u = (50)(6.3) = 315 \text{ kN/m}^2 \]

\[ q_{all} = \frac{q_u}{FS} = \frac{315}{4} = 78.8 \text{ kN/m}^2 \]

**SEISMIC BEARING CAPACITY AND SETTLEMENT IN GRANULAR SOIL**

In some instances shallow foundations may fail during seismic events. Published studies relating to the bearing capacity of shallow foundations in such instances are rare. Recently, however, Richads et al. (1993) develop a seismic bearing capacity theory that is presented in this section. It needs to be pointed out that this theory has not yet been supported by field data.

**Figure 3.26** shows the nature of failure in soil assumed for this analysis for static conditions. Similarly, **figure 3.27** shows the failure surface under earthquake conditions. Note that, in figure 3.26 and 3.27.

\[ \alpha_A = 45 + \phi/2 \text{ and } \alpha_p = 45 - \phi/2 \]

**Figure 3.26** Failure surface in soil for static bearing capacity analysis; note: $\alpha_A = 45 + \phi/2$ and $\alpha_p = 45 - \phi/2$

**Figure 3.27** Failure surface in soil seismic bearing capacity analysis

$\alpha_A, \alpha_{AE}$ = inclination angles for active pressure conditions

$\alpha_p, \alpha_{pE}$ = inclination angles for passive pressure conditions
According to this theory, the ultimate bearing capacities for *continuous foundations* in granular soil are:

**Static conditions:**

\[ q_u = qN_q + \frac{1}{2} \gamma BN_f \]  \[3.85a\]

**Earthquake conditions:**

\[ Q_{uE} = qN_{qE} + \frac{1}{2} \gamma BN_{fE} \]  \[3.85b\]

Where

\[ N_q, N_f, N_{qE}, N_{fE} \] = bearing capacity factors

\[ q = \gamma D_f \]

Note that

\[ N_q \text{ and } N_f = f(\phi) \]

And

\[ N_{qE} \text{ and } N_{fE} = f(\phi, \tan \theta) \]

Where

\[ \tan \theta = \frac{k_h}{1 - k_v} \]

\[ k_h \] = horizontal coefficient of acceleration due to an earthquake

\[ k_v \] = vertical coefficient of acceleration due to an earthquake

The variations of \( N_q \) and \( N_f \) with \( \phi \) are shown in **Figure 3. 28.** **Figure 3. 29** shows the variations of \( N_{fE}/N_f \) and \( N_{qE}/N_q \) with \( \tan \theta \) and the soil friction angle \( \phi \).

**Figure 3. 28** Variation of \( N_q \) and \( N_f \) based on failure surface assumed in figure 3. 26
Figure 3.28, 3.29 Variation of $N_{yE}/N_y$ and $N_{qE}/N_q$ (after Richards et al., 1993)

For static conditions, bearing capacity failure can lead to substantial sudden downward movement of the foundation. However, bearing capacity-related settlement in an earthquake takes place when the ratio $k_h/(1-k_v)$ reaches a critical value $(k_h/1-k_v)^*$. If $k_v = 0$, then $(k_h/1-k_v)^*$ becomes equal to $k_h^*$; figure 3.30 shows the variation of $k_h^*$ (for $k_v = 0$ and $c = 0$; granular soil) with the factor of safety (FS) applied to the ultimate static bearing capacity [equation (84)], $\phi$, and $D_f/B$.

Figure 3.30 Critical acceleration $k_h^*$ for $c = 0$ (after Richards et al., 1993)
The settlement of a strip foundation due to an earthquake \( S_{Eq} \) can be estimated (Richards et al., 1993) as

\[
S_{Eq} = (m) = 0.174 \frac{V^2}{Ag} \left| k_h^* \right|^{-4} \tan \alpha_{AE} \tag{3.86}
\]

Where

- \( V \) = peak velocity for the design earthquake (m/sec)
- \( A \) = acceleration coefficient for the design earthquake
- \( g \) = acceleration due to gravity (9.18 m/sec\(^2\))

The values of \( k_h^* \) and \( \alpha_{AE} \) can be obtained from figure 3.30 and 3.31, respectively.

![Figure 3.31 Variation of tan \( \alpha_{AE} \) with \( k_h^* \) and soil friction angle, \( \phi \) (after Richards et al., 1993)]

**Example 11**

A strip foundation is to be constructed on a sandy soil with \( B = 2 \) m, \( D_f = 1.5 \) b, \( \gamma = 18 \) kN/m\(^3\), \( \phi = 30^\circ \). Determine the gross ultimate bearing capacity \( q_{uE} \). Assume \( k_v = 0 \) and \( k_h = 0.176 \).

**Solution**

From figure 3.28, for \( \phi = 30^\circ \), \( N_q = 16.51 \) and \( N_\gamma = 23.76 \).

\[
\tan \theta = \frac{k_h}{1-k_v} = 0.176
\]

For \( \tan \theta = 0.176 \). Figure 3.29 gives

\[
\frac{N_{\gamma E}}{N_\gamma} = 0.4 \quad \text{and} \quad \frac{N_{q E}}{N_q} = 0.6
\]
Thus

\[ N_{\gamma E} = (0.4)(23.76) = 9.5 \]

\[ N_{qE} = (0.6)(16.51) = 9.91 \]

\[ q_{ue} = q_{NqE} + \frac{1}{2} \gamma BN_{\gamma} = (1.5 \times 18)(9.91) + \left( \frac{1}{2} \right)(18)(9.5) = 438.6 \text{ kN/m}^2 \]

**Example 12**

Refer to example 11. If the design earthquake parameters are \( V = 0.4 \text{ m/sec} \) and \( A = 0.32 \), determine the seismic settlement of the foundation. Use \( FS = 3 \) for obtaining static allowable bearing capacity.

**Solution**

For the foundation

\[ \frac{D_f}{B} = \frac{1.5}{2} = 0.75 \]

From figure 3.30, for \( \phi = 30^\circ \), \( FS = 3 \), and \( D_f/B = 0.75 \), the value of \( k_h^* = 0.26 \). Also from figure 3.31, for \( k_h^* = 0.26 \) and \( \phi = 30^\circ \), the value of \( \tan \alpha_{AE} = 0.88 \). From equation (86)

\[ S_{Eq} = 0.174 \left| \frac{k_h^*}{A} \right|^{-4} \tan \alpha_{AE} \left( \frac{v^2}{A_g} \right) \]

\[ = 0.174 \left( \frac{(0.4)^2}{(0.32)(9.81)} \right) \left( \frac{0.26}{0.32} \right)^{-4} \left( 0.88 \right) = 0.0179 \text{ m} = 17.9 \text{ mm} \]