Module 8
SEISMIC SLOPE STABILITY
(Lectures 37 to 40)

Lecture 38

Topics

8.5 STATIC SLOPE STABILITY ANALYSIS
     8.5.1 Limit Equilibrium Analysis
     8.5.2 Stress-Deformation Analyses

8.6 SEISMIC SLOPE STABILITY ANALYSIS
     8.6.1 Analysis of Inertial Instability
     8.6.2 Pseudostatic Analysis
     8.6.3 Selection of Pseudostatic Coefficient
     8.6.4 Limitations of the Pseudostatic Approach
     8.6.5 Discussion

8.5 STATIC SLOPE STABILITY ANALYSIS

Slopes become unstable when the shear stresses required to maintain equilibrium reach or exceed the available shearing resistance on some potential failure surface. For slopes in which the shear stresses required to maintain equilibrium under static gravitational loading are high, the additional dynamic stresses needed to produce instability may be low. Hence the seismic stability of a slope is strongly influenced by its static stability. Because of this and the fact that the most commonly used methods of seismic stability analysis rely on static stability analyses, a brief summary of static slope stability analysis is presented.

The procedures for analysis of slope stability under static conditions are well established. An excellent, concise review of the state of the art for static analysis was presented by Duncan (1992). Detailed descriptions of specific methods of analysis can be found in standard references such as National Research Council (1976), Chowdhury (1978) and Huang (1983). Currently the most commonly used methods of static slope stability analysis are limit equilibrium analyses and stress deformation analyses.

8.5.1 Limit Equilibrium Analysis

Limit equilibrium analyses consider force and/or moment equilibrium of a mass of soil above a potential failure surface. The soil above the potential
failure surface is assumed to be rigid (i.e., shearing an occur only on the potential failure surface). The available shear strength is assumed to be mobilized at the same rate at all points on the potential failure surface. As a result, the factor of safety is constant over the entire failure surface. Because the soil on the potential failure surface is assumed to be rigid-perfectly plastic (figure 8.3), limit equilibrium analysis provide no information on slope deformations.

Slope stability is usually expressed in terms of an index, most commonly the factor of safety, which is usually defined as

\[
FS = \frac{\text{available shear strength}}{\text{shear stress required to maintain equilibrium}}
\]  

(8.1)

Figure 8.3: Stress-strain curves for a rigid-perfectly plastic material. No shear strain occurs until the strength of the material is reached, after which the material strains at constant shear stress

Thus the factor of safety is a ratio of capacity (the shear strength of the soil) to demand (the shear stress induced on the potential failure surface). The factor of safety can also be viewed as the factor by which the strength of the soil would have to be divided to bring the slope to the brink of instability. In contrast to the assumption of limit equilibrium analysis, the strength of the soil in actual slopes is not reached at the same time at all points on the failure surface (i.e., the local factor of safety is not constant).

A variety of limit equilibrium procedures have been developed to analyze the static stability of slopes. Slopes that fail by translation on a planar failure surface (figure 8.4a) such as a bedding plane, rock joint, or seam of weak material can be analyzed quite easily by the Culman method (Taylor, 1948). Slopes in which failure is likely to occur on two or three planes (figure 4b) can be analyzed by wedge methods (e.g., Perlogg and Baron, 1976; Lambe and Whitman, 1969). In homogeneous slopes, the critical failure surface usually has a circular (figure 4c) or log-spiral shape. Since the minimum factors of safety for circular and log-spiral failure surfaces are very close, homogeneous slopes are usually analyzed by methods such as the ordinary method of slices (Fellenius, 1927) or Bishop’s modified method (Bishop, 1955), which assume circular failure surfaces. When subsurface conditions are not homogenous (e.g., when layers with significantly different strength, highly anisotropic strength, or discontinuities exist), failure surfaces are likely to be noncircular (figure 4d). In such cases, methods like those
of Morgenstern and Price (1965), Spencer (1967), and Janbu (1968) may be used. Nearly all limit equilibrium methods are susceptible to numerical problems under certain conditions. These conditions vary for different methods but are most commonly encountered where soils with high cohesive strength are present at the top of a slope and/or when failure surfaces emerge steeply at the base of slopes in soils with high frictional strength (Duncan, 1922).

Figure 8.4: Common failure surface geometries: (a) planar; (b) multi-planar; (c) circular; (d) noncircular

In concept, any slope with a factor of safety above 1.0 should be stable. In practice, however, the level of stability is seldom considered acceptable unless the factor of safety is significantly greater than 1.0. Criteria for acceptable factors of safety recognize (1) uncertainty in the accuracy with which the slope stability analysis represents the actual mechanism of failure, (2) uncertainty in the accuracy with which the input parameters (shear strength, groundwater conditions, slope geometry, etc.) are known, (3) the likelihood and duration of exposure to various types of external loading, and (4) the potential consequence of slope failure. Typical minimum factors of safety used in slope design are about 1.5 for normal long-term loading conditions and about 1.3 for temporary slopes or end-of-construction conditions in permanent slopes (when dissipation of pore pressure increases stability with time).

When the minimum factor of safety of a slope reaches a value of 1.0, the available shear strength of the soil is fully mobilized on some potential failure surface and the slope is at the point of incipient failure. Any additional loading will cause the slope to fail (i.e., to deform until it reaches a configuration in which the shear stresses required for equilibrium are less than or equal to the available shear strength of the soil). The limit equilibrium assumptions of rigid-perfectly plastic behavior suggest that the required deformation will occur in a ductile manner. Many soils however, exhibit brittle, strain softening stress-strain behavior. In such cases the peak shear strength may not be mobilized simultaneously at all points on the failure surface. When the peak strength of a strain softening soil is reached, such as point A in figure 8.5a, the available shearing resistance will drop from the peak to the residual strength. As it does so, shear stresses related to the difference between the peak and residual strength of the soil at point A is transferred to the surrounding soil. These redistributed shear stresses may cause the peak strength in the surrounding soil to be reached (figure 5b) and exceeded, thereby reducing their available shearing resistance to residual values. As the stress redistribution process continues, the zone of failure may grow until the entire slope becomes
unstable. Many instances of such progressive failure have been observed in strain softening soils, even when the limit equilibrium factor of safety (based on peak strength) is well above 1.0. Within the constraints of limit equilibrium analysis, the stability of slopes with strain softening materials can be analyzed reliably only by using residual shear strengths.

Figure 8.5: Development of progressive failure in slope comprised of strain softening materials: (a) exceedance of peak strength at any point (A) reduces strength at that point to residual value; (b) redistribution of shear stresses from failure zone to surrounding area produces failure in surrounding zone (point B). Continued redistribution of stresses can eventually lead to failure of the entire slope (point C and beyond)

Limit equilibrium analyses must be formulated with great care. Since the available shearing resistance of the soil depends on pore water drainage conditions, those conditions must be considered carefully in the selection of shear strength and pore pressure conditions for the analysis. Duncan (1992) provided guidelines for the selection of input parameters for limit equilibrium slope stability analyses.

8.5.2 Stress-Deformation Analyses

Stress-deformation analyses allow consideration of the stress-strain behavior of soil and rock and are most commonly performed using the finite-element method. When applied to slopes, stress-deformation analyses can predict the magnitudes and patterns of stresses, movements, and pore pressures in slopes during and after construction/deposition. Nonlinear stress-strain behavior, complex boundary conditions, irregular geometries, and a variety of construction operations can all be considered in modern finite-element analyses.

For static slope stability analysis-stress-deformation analyses offer the advantages of being able to identify the most likely mode of failure by predicting slope deformations up to (and in some cases beyond) the point of failure, of locating the most critically stressed zones within a slope, and of predicting the effects of slope failures. These advantages come at the cost of increased engineering time for problem formulation, characterization of material properties and interpretation of results, and increased computational effort.
The accuracy of stress-deformation analysis is strongly influenced by the accuracy with which the stress-strain model represents actual material behavior. Many different stress-strain models have been used for stress-deformation analysis of slopes; each has advantages and limitations. The accuracy of simple models is usually limited to certain ranges of strain and/or certain stress paths. Models that can be applied to more general stress and strain conditions are often quite complex and may require a large number of input parameters whose values can be difficult to determine. For many problems, the hyperbolic model (Kondner, 1963; Konder and Zelasko, 1963; Duncan and Chang, 1970; Duncan et al. 1980) offers an appropriate compromise between simplicity and accuracy.

8.6 SEISMIC SLOPE STABILITY ANALYSIS

The previously described procedure for static slope stability analysis have been used for many years and calibrated against many actual slope failures. The database against which seismic slope stability analyses can be calibrated is much smaller. Analysis of the seismic stability of slopes is further complicated by the need to consider the effects of (1) dynamic stresses induced by earthquake shaking, and (2) the effects of those stresses on the strength and stress-strain behavior of the slope materials.

Seismic slope instabilities may be grouped into two categories on the basis of which of these effects are predominant in a given slope. In inertial instabilities, the shear strength of the soil remains relatively constant, but slope deformations are produced by temporary exceedances of the strength by dynamic earthquake stresses. Weakening instabilities are those in which the earthquake serves to weaken the soil sufficiently that are cannot remain stable under earthquake induced stresses. Flow liquefaction and cyclic mobility are the most common causes of weakening instability. A number of analytical techniques, based on both limit equilibrium and stress deformation analyses are available for both categories of seismic instability.

8.6.1 Analysis of Inertial Instability

Earthquake motions can induce significant horizontal and vertical dynamic stresses in slopes. These stresses produce dynamic normal and shear stresses along potential failure surfaces within a slope. When superimposed upon the previously existing static shear stresses, the dynamic shear stresses may exceed the available shear strength of the soil and produce inertial instability of the slope. A number of techniques for the analysis of inertial instability have been proposed. These techniques differ primarily in the accuracy with which the earthquake motion and the dynamic response of the slope are represented. The following section describe several common approaches to the analysis is inertial instability. The first, pseudostatic analysis produces a factor of safety against seismic slope failure in much the same way that static limit equilibrium analyses produce factors of safety against static slope failure. All the other approaches attempts to evaluate permanent slope displacement produced by earthquake shaking.

8.6.2 Pseudostatic Analysis
Beginning in the 1920s, the seismic stability of earth structures has been analyzed by a pseudostatic approach in which the effects of an earthquake are represented by constant horizontal and/or vertical accelerations. The first explicit application of the pseudostatic approach to the analysis of seismic slope stability has been attributed to Terzaghi (1950).

In their most common form, pseudostatic analyses represent the effects of earthquake shaking by pseudostatic accelerations that produce inertial forces, \( F_h \) and \( F_v \), which act through the centroid of the failure mass (figure 8.6). The magnitudes of the pseudostatic forces are where \( a_h \) and \( a_v \) are horizontal and vertical pseudostatic accelerations, \( k_h \) and \( k_v \) are dimensionless horizontal and vertical pseudostatic coefficients, and \( W \) is the weight of the failure mass.

![Figure 8.6: Forces acting on triangular wedge of soil above planar failure surface in pseudostatic slope stability analysis](image)

\[
F_h = \frac{a_h W}{g} = k_h W \quad (8.2a)
\]
\[
F_v = \frac{a_v W}{g} = k_v W \quad (8.2b)
\]

The magnitudes of the pseudostatic accelerations should be related to the severity of the anticipated ground motion; selection of pseudostatic accelerations for design is, as discussed in the next section, not a simple matter. Resolving the forces on the potential failure mass in a direction parallel to the failure surface.

\[
FS = \frac{\text{resisting force}}{\text{driving force}} = \frac{c l_{ab} + [(W - F_v) \cos \beta - F_h \sin \beta] \tan \phi}{(W - F_v) \sin \beta + F_h \cos \beta} \quad (8.3)
\]

Where \( c \) and \( \phi \) are the Mohr-Coulomb strength parameters that describe the shear strength on the failure plane and \( l_{ab} \) is the length of the failure plane. The horizontal pseudostatic force clearly decreases the factor of safety-it reduces the resisting force (for \( \phi > 0 \)) and increases the driving force. The vertical pseudostatic force typically has less influence on the factor of safety since it reduces (or increases, depending on its direction) both the driving force and the resisting force-as a result, the effects of vertical accelerations are frequently neglected in pseudostatic analyses. The pseudostatic approach can be used to evaluate pseudostatic factors of safety for planar, circular, and noncircular failure surfaces.
Many commercially available computed programs for limit equilibrium slope stability analysis have the option of performing pseudostatic analyses.

**Example 2**

Assuming \( k_h = 0.1 \) and \( k_v = 0.0 \), compute the static and pseudostatic factors of safety for the 30-ft high 2:1 (H:V) slope shown in figure 8.7.

![Figure 8.7](image.png)

**Solution**

Using a simple moment equilibrium analysis the factor of safety can be defined as the ratio of the moment that resist rotation of a potential failure mass about the center of a circular potential failure surface to the moment that is driving the rotations. The critical failure surface, defined as that which has the lowest factor of safety, is identified by analyzing a number of potential failure surfaces. Shown below are the factor-of-safety calculations for one potential failure surface which may not be the critical failure surface.

Computations of the factor of safety require evaluation of the overturning the resisting moments for both static and pseudostatic conditions. The overturning moment for static conditions results from the weight of the soil above the potential failure surface. The overturning moment for pseudostatic conditions is equal to the sum of the overturning moment for static conditions and he overturning moment produced by the pseudostatic forces. The horizontal pseudostatic forces are assumed to act in directions that produce positive (clockwise, in this case) driving moments. In the calculations shown in tabular form below, the soil above the potential failure mass is divided into two sections.

### Overturning moments:

<table>
<thead>
<tr>
<th>Section</th>
<th>Area (ft²)</th>
<th>( \gamma ) (kip/ft³)</th>
<th>( W ) (kips)</th>
<th>Mome nt Arm (ft)</th>
<th>Static Mome nt (kip-ft)</th>
<th>( k_h ) W (kip-ft)</th>
<th>Mome nt Arm (ft)</th>
<th>Pseudosta tic Moment (kip-ft/ft)</th>
<th>Total Mome nt (kip-ft)</th>
</tr>
</thead>
</table>
Resisting moment:

<table>
<thead>
<tr>
<th>Section</th>
<th>Length (ft)</th>
<th>Force (kips)</th>
<th>Moment Arm (ft)</th>
<th>Moment (kip-ft/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>11.5</td>
<td>600</td>
<td>6.9</td>
<td>78</td>
</tr>
<tr>
<td>B</td>
<td>129.3</td>
<td>1000</td>
<td>129.3</td>
<td>78</td>
</tr>
</tbody>
</table>

Factor of safety:

\[
\text{Static FS} = \frac{\text{resisting moment}}{\text{static overturning moment}} = \frac{10,623.6}{5925.5} = 1.79
\]

\[
\text{Pseudostatic FS} = \frac{\text{resisting moment}}{\text{static + pseudostatic overturning moment}} = \frac{10,623.6}{8281.1} = 1.28
\]

### 8.6.3 Selection of Pseudostatic Coefficient

The results of pseudostatic analyses are critically dependent on the value of seismic coefficient, \( k_h \). Selection of an appropriate pseudostatic coefficient is the most important, and most difficult, aspect of a pseudostatic stability analysis. The seismic coefficient controls the pseudostatic force on the failure mass, so its value should be related to some measure of the amplitude of the inertial force induced in the potentially unstable material. If the slope material was rigid, the inertial force induced on a potential slide would be equal to the product of the actual horizontal acceleration and the mass of the unstable material. This inertial force would reach its maximum value when the horizontal acceleration reached its maximum value. In recognition of the fact that actual slopes are not rigid and that the peak acceleration exists for only a very short time, the pseudostatic coefficients used in practice generally correspond to acceleration values well below \( a_{\text{max}} \). Terzaghi (1950) originally suggested the use of \( k_h = 0.1 \) for “severe” earthquake (Rossi-Forel IX), \( k_h = 0.2 \) for “violent, destructive” earthquakes (Rossi-Forel X), and \( k_h = 0.5 \) for “catastrophic” earthquakes, Seed (1979) listed pseudostatic design criteria for 14 dams in 10 seismically active countries; 12 required minimum factors of safety of 1.0 to 1.5 with pseudostatic coefficients of 0.10 to 0.12. Marcuson (1981) suggested that appropriate pseudostatic coefficients for dams should correspond to one-third to one-half of the maximum acceleration, including amplification or de-amplification effects to which the dam is subjected. Using
shear beam models, Seed and Martin (1966) and Dakoulas and Gazetas (1986) showed that the inertial force on a potentially unstable slope in an earth dam depends on the response of the dam and that the average seismic coefficient for a deep failure surface is substantially smaller than that of a failure surface that does not extend for below the crest. Seed (1979) also indicated that deformations of earth dams constructed of ductile soils (defined as those that do not generate high pore pressure or show more than 15% strength loss upon cyclic loading) with crest acceleration less than 0.75g would be acceptably small for pseudostatic factors of safety of at least 1.15 with \( k_h = 0.10 \) \((M = 6.5)\) to \( k_h = 0.15 \) \((M = 8.25)\). This criteria would allow the use of pseudostatic accelerations as small as 13 to 20% of the peak crest acceleration. Hynes-Griffin and Franklin (1984) applied the Newmark sliding block analysis described in the following section to over 350 accelerograms and concluded that earth dams with pseudostatic factors of safety greater than 1.0 using \( k_h = 0.5a_{max}/g \) would not develop “dangerously large” deformations.

As the preceding discussion indicates, there are no hard and fast rules for selection of a pseudostatic coefficient for design. It seems clear, however, that the pseudostatic coefficient should be based on the actual anticipated level of acceleration in the failure mass (including any amplification or de-amplification effects) and that is should correspond to some fraction of the anticipated peak acceleration. Although engineering judgments’ is required for all cases, the criteria of Hynes-Griffin and Franklin (1984) should be appropriate for most slopes.

8.6.4 Limitations of the Pseudostatic Approach

Representation of the complex, transient, dynamic effects of earthquake shaking by a single constant unidirectional pseudostatic acceleration is obviously quite crude. Even in its infancy, the limitations of the pseudostatic approach were clearly recognized. Terzaghi (1950) stated that “the concept it convey is earthquake effects on slope sis very inaccurate to say the least” and that a slope could be unstable even if the computed pseudostatic factor of safety was greater than 1. Detailed analysis of historical and recent earthquake induced landslides (e.g., Seed et al., 1969, 1975; Marcuson et al., 1979) has illustrated significant shortcomings of the pseudostatic approach. Experience has clearly shown, for example, that pseudostatic analyses can be unreliable for soils that build up large pore pressures or show more than about 15% degradation of strength due to earthquake shaking. As illustrated in table 4, pseudostatic analyses produced factors of safety well above 1 for a number of dams that later failed during earthquakes. These cases illustrate the inability of the pseudostatic method to reliably evaluate the stability of slope susceptible to weakening instability. Nevertheless the pseudostatic approach can provide at least a crude index of relative, if not absolute stability.

8.6.5 Discussion
The pseudostatic approach has a number of attractive features. The analysis is relatively simple and straightforward; indeed, its similarly to the static limit equilibrium analyses routinely conducted by geotechnical engineers makes its computations easy to understand and perform. It produces a scalar index of stability (the factor of safety) that is analogous to that produce by static stability. It must always be recognized, however, that the accuracy of the pseudostatic approach is governed by the accuracy with which the simple pseudostatic inertial forces represent the complex dynamic inertial forces that actually exist in an earthquake. Difficulty in the assignment of appropriate pseudostatic coefficients and in interpretation of pseudostatic factors of safety, compiled with the development of more realistic methods of analysis, have reduced the use of the pseudostatic approach for seismic slope stability analyses. Methods based on evaluation of permanent slope deformation, such as those described in the following sections, are being used increasingly for seismic slope stability analysis.

Table 8.4 Results of Pseudostatic Analyses of Earth Dam That Failed during Earthquakes

<table>
<thead>
<tr>
<th>Dam</th>
<th>$k_h$</th>
<th>FS</th>
<th>Effect of Earthquakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheffield Dam</td>
<td>0.10</td>
<td>1.2</td>
<td>Complete failure</td>
</tr>
<tr>
<td>Lower San Fernando Dam</td>
<td>0.15</td>
<td>1.3</td>
<td>Upstream slope failure</td>
</tr>
<tr>
<td>Upper San Fernando Dam</td>
<td>0-15</td>
<td>-2-2.5</td>
<td>Downstream shell, including crest slipped about 6 ft downstream</td>
</tr>
<tr>
<td>Tailings dam (Japan)</td>
<td>0.20</td>
<td>-1.3</td>
<td>Failure of dam with release of tailings</td>
</tr>
</tbody>
</table>