Objectives

In this section you will learn the following

- Earth Pressure Theories
- Rankine's Earth Pressure Theory
- Active earth pressure
- Passive earth pressure
- Coulomb's Wedge Theory

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 7 : Earth Pressure Theories [Section 7.1 Rankine and Coulomb Theory]

7.1 Earth Pressure Theories

1 Rankine's Earth Pressure Theory

The Rankine's theory assumes that there is no wall friction $(\delta = 0)$, the ground and failure surfaces are straight planes, and that the resultant force acts parallel to the backfill slope.

In case of retaining structures, the earth retained may be filled up earth or natural soil. These backfill materials may exert certain lateral pressure on the wall. If the wall is rigid and does not move with the pressure exerted on the wall, the soil behind the wall will be in a state of *elastic equilibrium* . Consider the prismatic element E in the backfill at depth, z, as shown in Fig. 2.8.

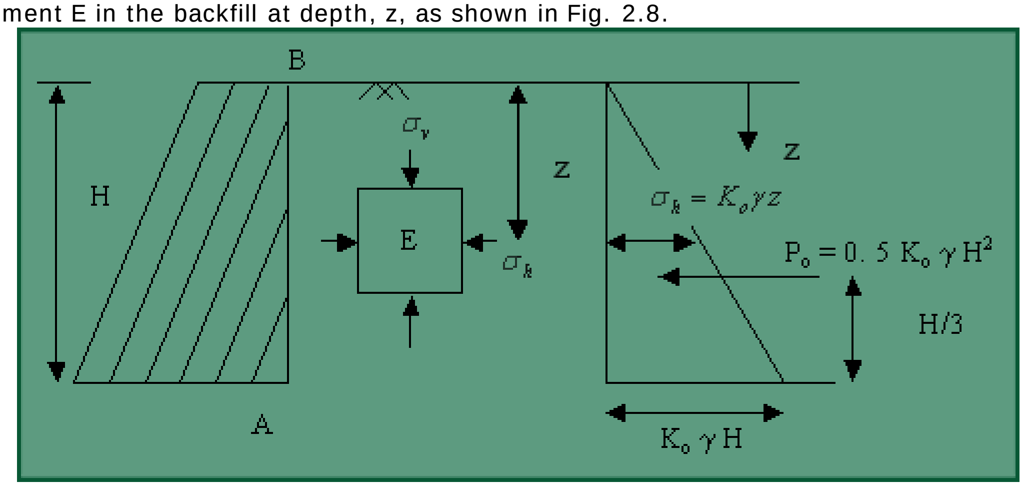

Fig. 2.8 Lateral earth pressure for at rest condition.

The element E is subjected to the following pressures :

Vertical pressure = $\sigma_v = \gamma z$

Lateral pressure = σ_h, where g is the effective unit weight of the soil.

If we consider the backfill is homogenous then both σ_v *and* σ_h increases rapidly with depth z. In that case the ratio of vertical and lateral pressures remain constant with respect to depth, that is $\sigma_h / \sigma_v = \sigma_h / \gamma z$=constant $=K_o$, where K_o is the coefficient of earth pressure for at rest condition.

At rest earth pressure

The at-rest earth pressure coefficient (K_o) is applicable for determining the active pressure in clays for strutted systems. Because of the cohesive property of clay there will be no lateral pressure exerted in the at-rest condition up to some height at the time the excavation is made. However, with time, creep and swelling of the clay will occur and a lateral pressure will develop. This coefficient takes the characteristics of clay into account and will always give a positive lateral pressure.

The lateral earth pressure acting on the wall of height H may be expressed as $\sigma_h = K_o \gamma H$.

The total pressure for the soil at rest condition, $P_o = 0.5 K_o \gamma H^2$.

The value of K_o depends on the relative density of sand and the process by which the deposit was formed. If this process does not involve artificial tamping the value of K_o ranges from 0.4 for loose sand to 0.6 for dense sand. Tamping of the layers may increase it upto 0.8.

From elastic theory, $K_o = \mu/(1-\mu)$. ,where μ is the poisson's ratio.

According to Jaky (1944), a good approximation of K_o is given by , $K_o = 1 - \sin\phi$.

Table 2.4 : Different values of K_o

Soil Type	Typical Value for Poisson's Ratio	K_o
Clay, saturated	0.40 - 0.50	0.67 - 1.00
Clay, unsaturated	0.10 - 0.30	0.11 - 0.42
Sandy Clay	0.20 - 0.30	0.25 - 0.42
Silt	0.30 - 0.35	0.42 - 0.54
Sand - Dense	0.20 - 0.40	0.25 - 0.67
- Coarse (valid upto 0.4 - 0.7)	0.15	0.18
- Fine-grained (valid upto 0.4 - 0.7)	0.25	0.33
Rock	0.10 - 0.40	0.11 - 0.67

Rankine's Earth Pressure Against A Vertical Section With The Surface Horizontal With Cohesionlesss Backfill

Active earth pressure:

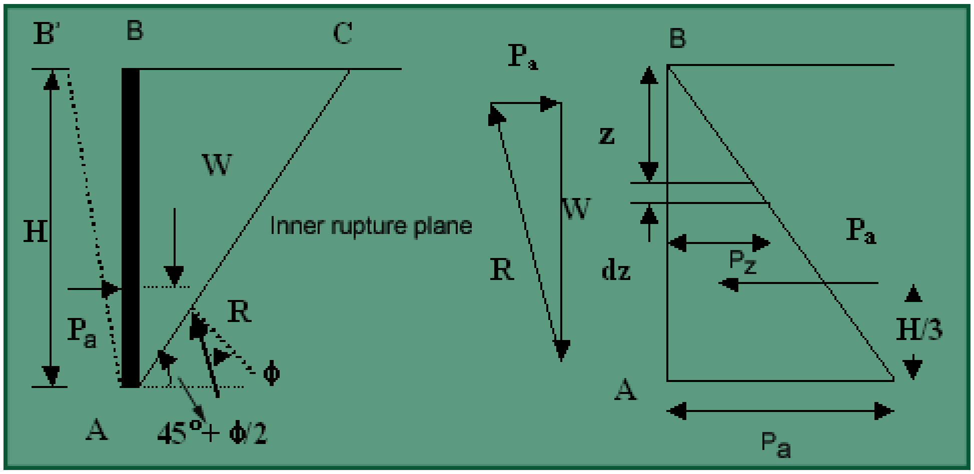

Fig. 2.9 Rankine's active earth pressure in cohesionless soil

The lateral pressure acting against a smooth wall AB is due to mass of soil ABC above the rupture line AC which makes an angle of $\left(45^0+\phi/2\right)$ with the horizontal. The lateral pressure distribution on the wall AB of height H increases in same proportion to depth.

The pressure acts normal to the wall AB.

The lateral active earth pressure at A is $P_a = K_A \gamma H$, which acts at a height H/3 above the base of the wall. The total pressure on AB is therefore calculated as follows:

$$P_a = \int_0^H p_z dz = \int_0^H K_A \gamma z dz = 0.5 K_A \gamma H^2, \text{ where } K_A = \tan^2\left(45^0+\phi/2\right)$$

Passive earth pressure:

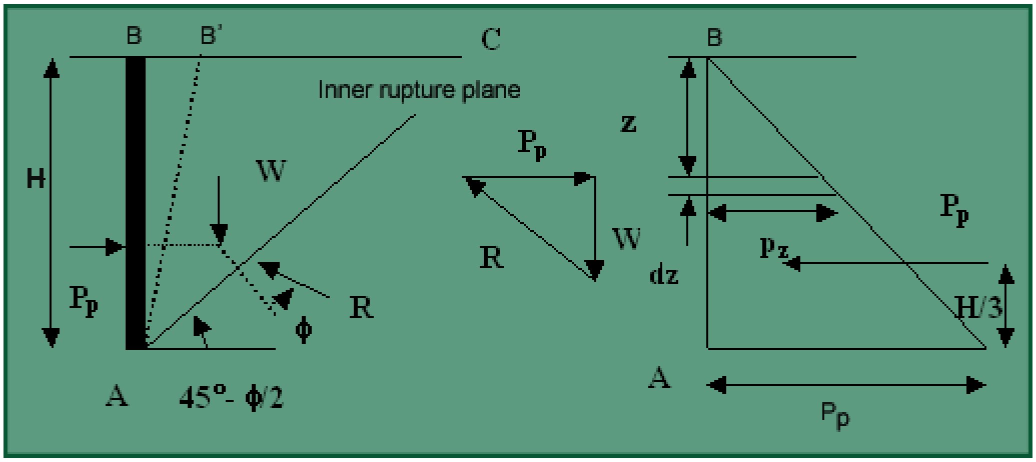

Fig. 2.10 Rankine's passive earth pressure in cohesionless soil

If the wall AB is pushed into the mass to such an extent as to impart uniform compression throughout the mass, the soil wedge ABC in fig. will be in Rankine's Passive State of plastic equilibrium. The inner rupture plane AC makes an angle $\left(45^0 + \phi/2\right)$ with the vertical AB. The pressure distribution on the wall is linear as shown.

The lateral passive earth pressure at A is $P_p = K_p \gamma H$. which acts at a height H/3 above the base of the wall. The total pressure on AB is therefore

$$P_p = \int_0^H p_z dz = \int_0^H K_p \gamma z dz = 0.5 K_p \gamma H^2, \text{ where } K_p = \tan^2\left(45^0 + \phi/2\right)$$

Rankine's active earth pressure with a sloping cohesionless backfill surface

Fig shows a smooth vertical gravity wall with a sloping backfill with cohesionless soil. As in the case of horizontal backfill, active case of plastic equilibrium can be developed in the backfill by rotating the wall about A away from the backfill. Let AC be the plane of rupture and the soil in the wedge ABC is in the state of plastic equilibrium.

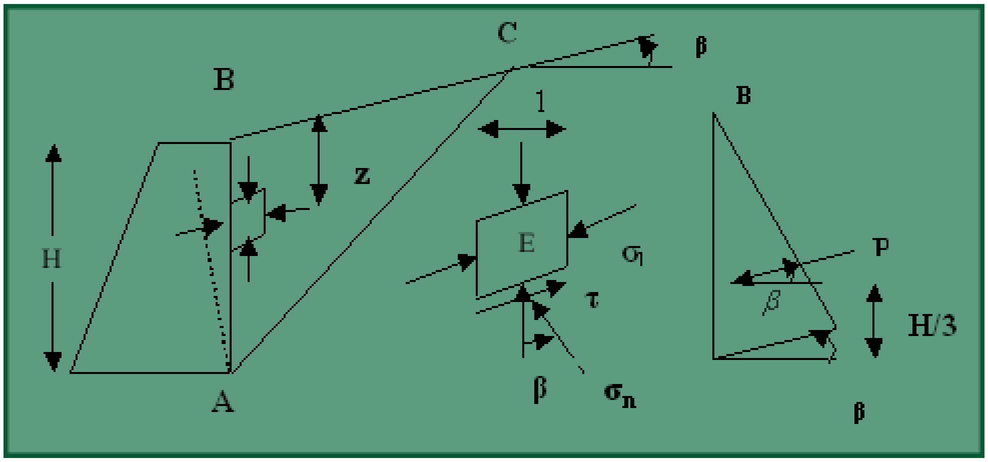

Fig. 2.11 Rankine's active pressure for a sloping cohesionless backfill

The pressure distribution on the wall is shown in fig. The active earth pressure at depth H is $P_a = K_A \gamma H$ which acts parallel to the surface. The total pressure per unit length of the wall is $P_a = 0.5 K_A \gamma H^2$ which acts at a height of H/3 from the base of the wall and parallel to the sloping surface of the backfill. In case of active pressure,

$$K_A = \cos\beta\left(\cos\beta - \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right) / \left(\cos\beta + \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right).$$

In case of passive pressure,

$$K_p = \cos\beta\left(\cos\beta + \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right) / \left(\cos\beta - \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right)$$

Rankine's active earth pressures of cohesive soils with horizontal backfill on smooth vertical walls

In case of cohesionless soils, the active earth pressure at any depth is given by

$P_a = K_A \gamma z$ In case of cohesive soils the cohesion component is included and the expression becomes $P_a = K_A \gamma z - 2c\sqrt{K_A}$.

When $P_a = 0, z = z_o = (2c\sqrt{K_A}) / \gamma$.

This depth is known as the depth of tensile crack. Assuming that the compressive force balances the tensile force (-), the total depth where tensile and compressive force neutralizes each other is $2z_o$. This is the depth upto which a soil can stand without any support and is sometimes referred as the depth of vertical crack or critical depth $(H_c)(H_c = 4c\sqrt{K_A}) / \gamma$.

However Terzaghi from field analysis obtained that $(H_c = 4c\sqrt{K_A}) / \gamma - z_o$, where,

$z_o \approx H_c / 2$ and is not more than that.

The Rankine formula for passive pressure can only be used correctly when the embankment slope angle equals zero or is negative. If a large wall friction value can develop, the Rankine Theory is not correct and will give less conservative results. Rankine's theory is not intended to be used for determining earth pressures directly against a wall (friction angle does not appear in equations above). The theory is intended to be used for determining earth pressures on a vertical plane within a mass of soil.

2 Coulomb's Wedge Theory

The Coulomb theory provides a method of analysis that gives the resultant horizontal force on a retaining system for any slope of wall, wall friction, and slope of backfill provided. This theory is based on the assumption that soil shear resistance develops along the wall and failure plane. The following coefficient is for resultant pressure acting at angle. Since wall friction requires a curved surface of sliding to satisfy equilibrium, the Coulomb formula will give only approximate results as it assumes planar failure surfaces. The accuracy for Coulomb wil1 diminishe with increased depth. For passive pressures the Coulomb formula can also give inaccurate results when there is a large back slope or wall friction angle. These conditions should be investigated and an increased factor of safety considered.

Coulomb (1776) developed a method for the determination of the earth pressure in which he considered the equilibrium of the sliding wedge which is formed when the movement of the retaining wall takes place. The sliding wedge is torn off from the rest of the backfill due to the movement of the wall. In the Active Earth Pressure case, the sliding wedge moves downwards & outwards on a slip surface relative to the intact backfill & in the case of Passive Earth pressure, the sliding wedge moves upward and inwards. The pressure on the wall is, in fact, a force of reaction which it has to exert to keep the sliding wedge in equilibrium. The lateral pressure on the wall is equal and opposite to the reactive force exerted by the wall in order to keep the sliding wedge in equilibrium. The analysis is a type of limiting equilibrium method.

The following assumptions are made

- The backfill is dry, cohesion less, homogeneous, isotropic and ideally plastic material, elastically undeformable but breakable.
- The slip surface is a plane surface which passes through the heel of the wall.
- The wall surface is rough. The resultant earth pressure on the wall is inclined at an angle δ to the normal to the wall, where δ is the angle of the friction between the wall and backfill.
- The sliding wedge itself acts as a rigid body & the value of the earth pressure is obtained by considering the limiting equilibrium of the sliding wedge as a whole.
- The position and direction of the resultant earth pressure are known. The resultant pressure acts on the back of the wall at one third height of the wall from the base and is inclined at an angle δ to the normal to the back. This angle is called the angle of wall friction.
- The back of the wall is rough & relative movement of the wall and the soil on the back takes place which develops frictional forces that influence the direction of the resultant pressure.

In coulomb's theory, a plane failure is assumed and the lateral force required to maintain the equilibrium of the wedge is found using the principles of statics. The procedure is repeated for several trial surfaces. The trial surface which gives the largest force for the active case, and the smallest force for the passive case, is the actual failure surface. This method readily accommodates the friction between the wall & the backfill, irregular backfill, sloping wall, & the surcharge loads etc. Although the initial theory was for dry, cohesion less soil it has now been extended to wet soils and cohesive soils as well. Thus Coulomb's theory is more general than the Rankine's Theory.

Coulomb's Active Pressure in cohesionless soils

Fig 2.12 shows a retaining wall with an inclined back face and sloping dry granular backfill. In active case, the sliding wedge ABD moves downward, and the reaction R acts upward and inclined at an angle f' with the normal.

The sliding wedge ABD is in equilibrium under the three forces:

- Weight of the wedge (W).
- Reaction R on the slip surface BD.
- Reaction P_a from the wall (wall reaction)/Earth pressure.

For the condition of the yield of the wall away from the backfill, the most dangerous or the critical slip surface is that for which the wall reaction is maximum, i.e., the wall must resist the maximum lateral pressure before it moves away from the backfill; the lateral pressure under this condition is the active pressure. The critical slip surface for the case of the passive earth pressure is that for which the wall has to exert a minimum force to tear off a soil wedge by moving towards the backfill.

The main deficiency of this theory is the assumptions that the slip surface is planar, therefore, the force acting on the slide wedge do not generally meet when in static equilibrium condition. The actual slip surface is curved, especially in the lower part. The assumptions of plane slip surface does not affect materially the results in the active case but give very high values in the passive case as compared with the assumptions of curved slip surfaces. The principle of the sliding wedge has been extended for calculating earth pressure of cohesive soils.

Fig 2.12 shows the force triangle. As the magnitude of one force (weight W) and the directions of all the three forces are known, the force triangle can be completed. The magnitude of P_a is determined from the force triangle. The pressure acting on the wall is equal and opposite of P_a

The procedure is repeated after assuming another failure surface. The surface that gives the maximum value of P_a is the critical failure plane; the corresponding force is the active force.

Coulomb's method does not give the point of application of the earth pressure (P_a).The point of application is found to be approximately at the point of intersection E of the back of the retaining wall with a line CE drawn from the centroid C of the failure wedge and parallel to the surface. As this procedure is cumbersome, for convenience, the pressure distribution is some times assumed to be hydrostatic on the back of the wall, and the resultant pressure P_a is assumed to act at one third the height of the wall from the base.

The following points should be carefully noted while using coulomb's theory:

- For most practical cases, the backfill moves down relative to the wall in the Active case, and, therefore, the active force P_a is inclined δ at an angle below the normal. However, if the wall is supported on a soft, compressible soil, it may settle to such extent that the movement of the wall be downward relative to the backfill and the relative movement of the wedge will be upward. In such a case, the force P_a would be inclined at an angle δ above the normal to the wall.
- The angle δ is the friction angle between the soil and the wall. It may be determined by means of a direct test. For concrete walls δ is generally taken as $(2/3)\phi'$. The value of δ can not exceed ϕ', because in that case the failure will occur in soil. If the friction angle δ is zero, for a vertical wall and the horizontal ground surface, the coulomb method gives identical results with Rankine Method.
- Coulombs method assumes the failure surface to be a plane. The actual failure surface is slightly curved. Fortunately, for the active case, the error is small, and the failure surface may be assumed to be planar without any significant error.

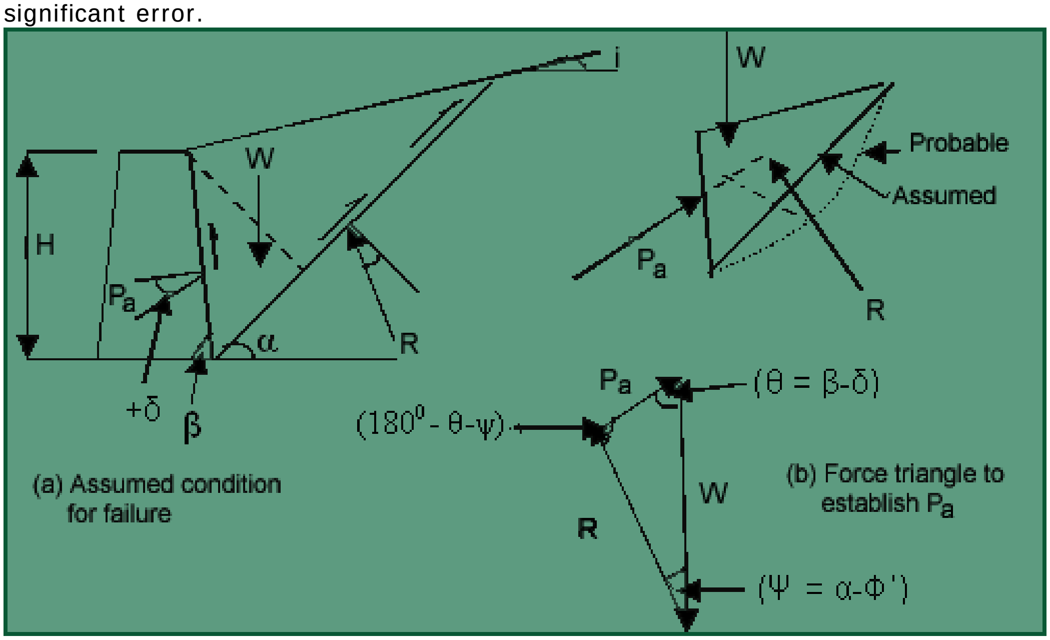

Fig. 2.12 Expression for Active Pressure for cohesion less soil

Using the law of sine
The active force is the component of the weight vector as from fig , we obtain

$$\frac{P_a}{\sin(\alpha - \phi')} = \frac{W}{\sin(180^0 - \beta - \alpha + \phi' - \delta)}$$

$$P_a = \frac{W.\sin(\alpha - \phi')}{\sin(180^0 - \beta - \alpha + \phi' + \delta)} \qquad \text{------------(a)}$$

Since from figure

$$BE = \frac{AB.\sin(\beta + i)}{\sin(\alpha - i)}$$

$$AG = AB.\sin(\alpha + \beta)$$

$$AB = \frac{H}{\sin\beta}$$

Therefore
The weight of the soil wedge is

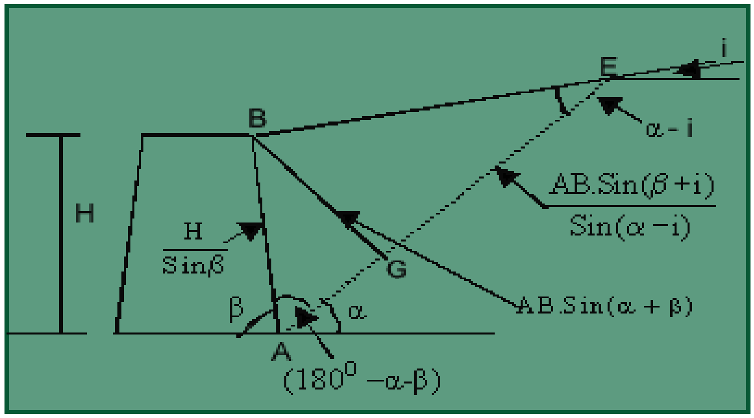

Fig. 2.13 Failure wedge

$$W = \frac{1}{2} \times BE \times AG \times \gamma \times 1$$

$$W = \frac{\gamma H^2}{2.\sin^2\beta}\left[\sin(\beta + \alpha)\right] \times \frac{\sin(\beta + i)}{\sin(\alpha - i)} \qquad \text{-----------(b)}$$

From eq. (a) it can be seen that the value of P_a = f (α); that is, all other terms for a given problem are constant, and the value of P_a of primary interest is the largest possible value. Combining the eq. (a) and (b), we obtain

$$P_a = \frac{\gamma H^2}{2.\sin^2\beta}\left[\sin(\beta+\alpha)\frac{\sin(\beta+i)}{\sin(\alpha-i)}\right].\frac{\sin(\alpha-\phi')}{\sin(180-\beta-\alpha+\phi'+\delta)} \quad \text{---------(c)}$$

Equating the first derivative to zero, $\frac{dP_a}{d\alpha} = 0$

The maximum value of active wall Force P_a is found to be

$$P_a = \frac{\gamma H^2}{2}\times\frac{\sin^2(\beta+\phi')}{\sin^2\beta.\mathrm{Sin}(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi'+\delta).\sin(\phi'-i)}{\sin(\beta-\delta).\sin(\beta+i)}}\right]^2} \quad \text{--------(d)}$$

If i = δ = 0 and ß = 90^0 (a smooth vertical wall with horizontal backfill)

Then Eq. (d) simplifies to

$$P_a = \frac{\gamma H^2}{2}\times\frac{1-\sin\phi'}{1+\sin\phi'} = \frac{\gamma H^2}{2}\tan(45-\frac{\phi'}{2}) \quad \text{--------(e)}$$

This is also the Rankine equation for active earth pressure equation. Equation takes the general form

$$P_a = \frac{\gamma H^2}{2}.k_a$$

Where

$$K_a = \frac{\sin^2(\beta+\phi')}{\sin^2\beta.\sin(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi'+\delta).\sin(\phi'-\alpha)}{\sin(\beta-\delta).\sin(\beta+i)}}\right]^2}$$

K_a = coefficient which considered β, i, δ and ϕ', but independent of γ and H.

Passive earth pressure is derived similarly except that the inclination at the wall and the force triangle will be as shown in Fig. 2.14.

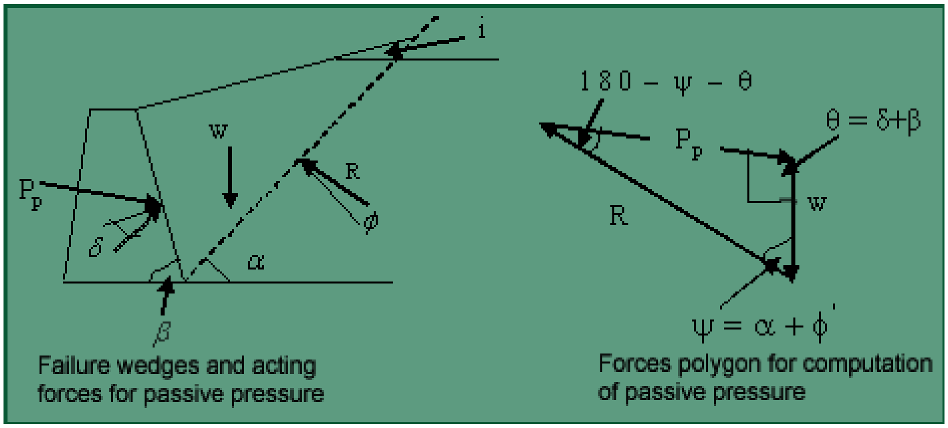

Fig. 2.14 Failure wedges

From above Fig. 2.14 the weight of the assumed failure mass is

$$W = \frac{\gamma H^2}{2} \times \frac{\sin(\beta + i)}{\sin(\alpha - i)}$$

and from the force triangle, using the law of sines

$$P_p = W.\frac{\sin(\alpha + \phi')}{\sin(180 - \alpha - \phi' - \delta - \beta)}$$

Setting the derivative($\frac{dP_a}{d\alpha} = 0$) gives the minimum value of the P_p as

$$P_p = \frac{\gamma H^2}{2} \times \frac{\sin^2(\beta - \phi')}{\sin^2\beta.\ \text{in}(\beta + \delta)\left[1 - \sqrt{\frac{\sin(\phi' + \delta).\sin(\phi' + i)}{\sin(\beta + \delta).\sin(\beta + i)}}\right]^2}$$

For a smooth vertical wall with the horizontal backfill (ß =90 ^{0}, δ = i = 0),

$$P_p = \frac{\gamma H^2}{2} \times \frac{1 + \sin\phi'}{1 - \sin\phi'} = \frac{\gamma H^2}{2} \times \tan(45 + \frac{\phi'}{2})$$

Equation can also be written as

$$P_p = \frac{\gamma H^2}{2} \times K_p$$

Where

$$K_p = \frac{\sin^2(\beta - \phi')}{\sin^2\beta . \sin(\beta + \delta)\left[1 - \sqrt{\frac{\sin(\phi' + \delta).\sin(\phi' + i)}{\sin(\beta + \delta).\sin(\beta + i)}}\right]^2}$$

Graphical solutions for lateral Earth Pressure

- Culman's solution
- The trial wedge method
- The logarithmic spiral

The Culmann solution

The Culmann's solution considered wall friction δ, irregularity of the backfill, any surcharges (either concentrated or distributed loads), and the angle of internal friction of the soil. Here we are describing the solution which is applicable to the cohesion less soils, although with some modification it can be used for the soils with cohesion. This method can be adapted for stratified deposits of varying densities, but the angle of internal friction must be the same throughout the soil mass. A rigid, plane rupture is assumed. Essentially the solution is a graphical determination of the maximum value of the soil pressure, and a given problem may have several graphical maximum points, of which the largest value is chosen as the design value. A solution can be made for both Active and Passive pressure.

Steps in Culmann's solution for Active Pressure are as follows:

- Draw the retaining wall to any convenient scale, together with the ground line, location of surface irregularities, point loads, surcharges, and the base of the wall when the retaining wall is a cantilever type.

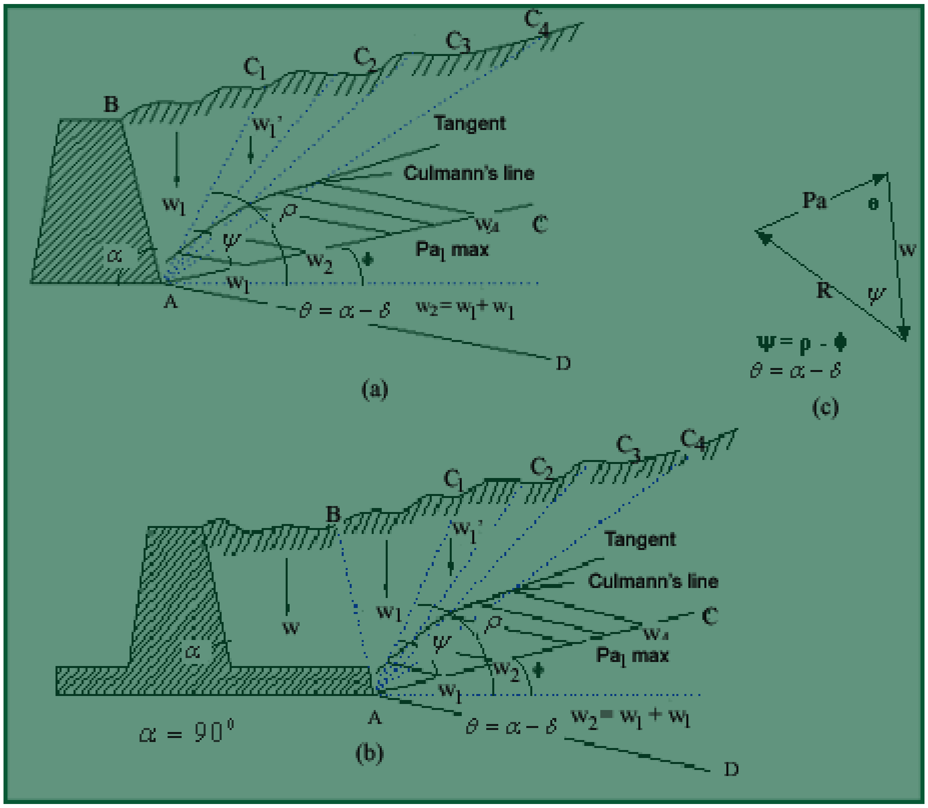

Fig 2.15a Graphical Solution By Culmann's Method

- From the point A lay off the angle ϕ with the horizontal plane, locating the line AC. Note that in the case of a cantilever wall, the point A is at the base of the heel, as shown in the fig. 2.15.
- Layoff the line AD at an angle of θ with line AC. The angle is $\theta = \alpha - \delta$.
  Where α = angle the back of the wall makes with horizontal, δ = angle of wall friction.
- Draw assumed failure wedges as $ABC_1, ABC_2, \ldots, ABC_n$. These should be made utilizing the backfill surface as a guide, so that geometrical shapes such as triangles and rectangles formed.

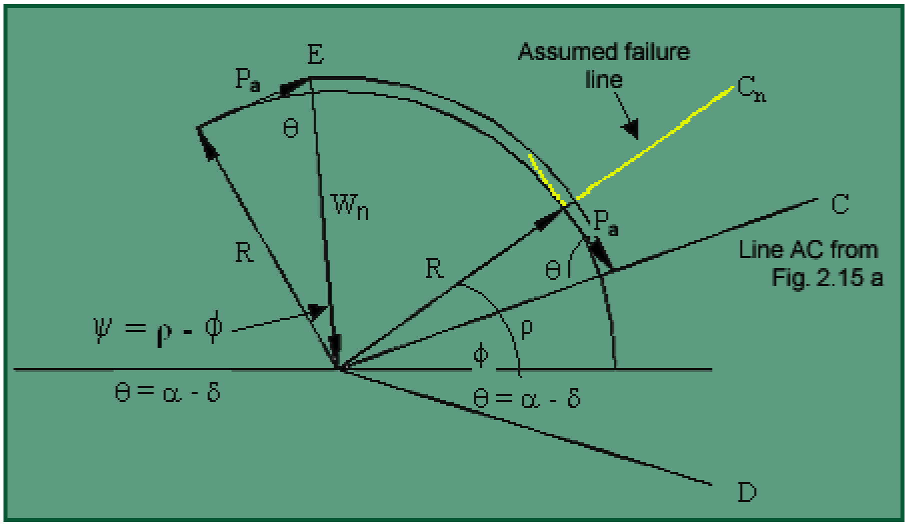

Fig. 2.15b. Analysis of Forces in Culmann's Method

- Find the weight W_n of each of the wedges by treating as triangles, trapezoidal or rectangles, depending on the soil stratification, water in the soil, and other condition and geometry.
- Along the line AC, plot to a convenient weight scale, the wedge weights, locating the points $w_1, w_2, \ldots, w_n$.
- Through the points just established in previous step, draw lines parallel to AD to intersect the corresponding side of the triangle, as W_1 to side AC_1, w_2 to side AC_1, AC_2, ..., w_n to side AC_n Through the locus of points established on the assumed failure wedges, draw a smooth curve (the Culmann line). Draw the tangent to this curve and parallel to the line AC .
- Through the tangent point established in above step, project a line back to the AC line, which is also parallel to the line AD. The value of this to the weight scale is P_a, and a line through the tangent point from A is the failure surface. When several tangents are drawn, choose the largest value of P_a .

The basis of the Culmann procedure is the solution of the force triangle shown in the figure. The triangle is rotated so that the location of the trial failure wedge automatically yields the angle ψ without recourse to measuring ψ each time. The line AD is laid off for use in projecting the instant value of P_a at the proper slope since θ is constant for a particular problem. The slope of R is automatically established from the slope of the weight line AC; thus, with all slopes and one side w_n known, the force triangle is readily solved.

To find the point of application of P_a, the following procedure (Terzaghi (1943)) is recommended.

Case1. No concentrated loads (fig a), but may have other surcharges.

- Find the centre of gravity of the failure wedge graphically or by trimming a cardboard model and hanging by a thread at the two or three points.
- Through the centre of gravity and parallel to the failure surface draw a line of action of P_a until it intercepts AB (wall or plane through the heel of the cantilever wall). P_a acts at an angle of δ or β to a perpendicular to AB.

Case 2. Concentrated load or line load within the failure wedge.(Fig b)

- Parallel to AC draw line Vc', and parallel to AC_f draw V c_f'.
- Take one- third of distance $c'c_f'$ from c' for the point of application of P_a.

Case 3. Concentrated load or line load outside the failure wedge (fig c).

- Draw a line from the concentrated load to A(V A).
- Draw V_c' parallel to Ac.
- Take one third of c'A from c' as point of application of P_a. If the surcharge falls out of Zone ABC, the problem should be treated as if no surcharge were present.

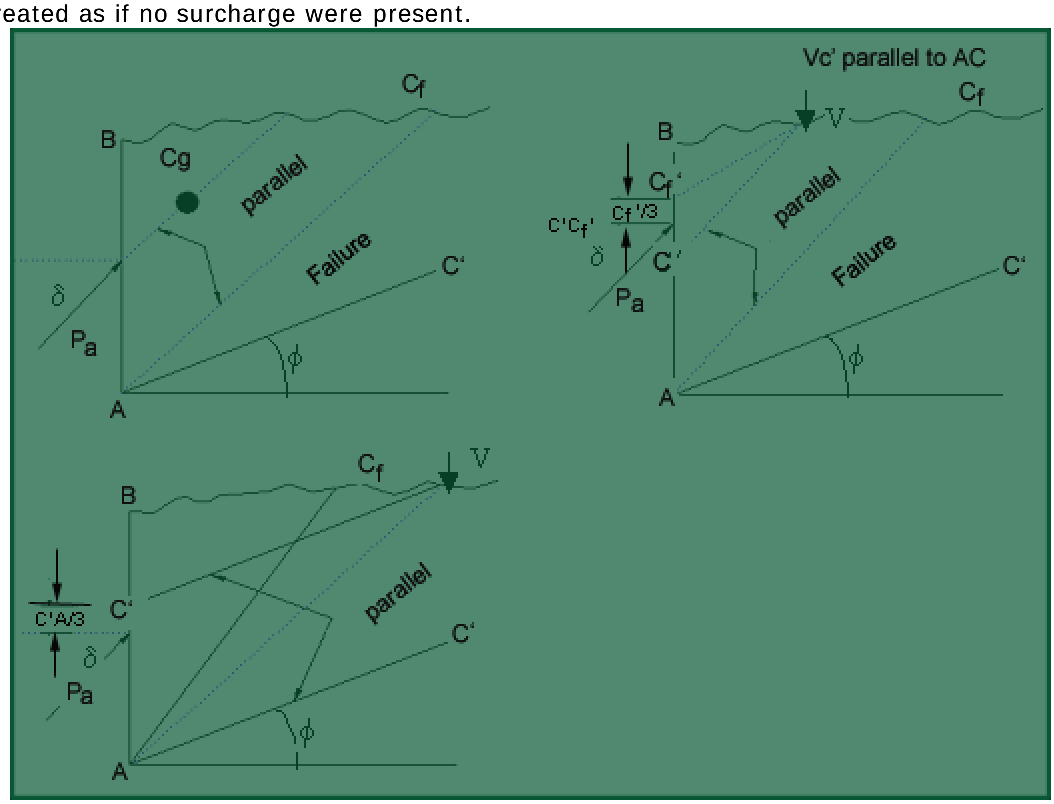

Fig. 2.16 Failure wedge

Recap

In this section you have learnt the following

- Earth Pressure Theories
- Rankine's Earth Pressure Theory
- Active earth pressure
- Passive earth pressure
- Coulomb's Wedge Theory

Objectives

In this section you will learn the following

- Friction Circle Method
- Terzaghi's Analysis
- Log-Spiral Theory

7.2.1 FRICTION CIRCLE METHOD

- This method is based on total stress analysis, but it enables the angle of shearing resistance to be taken into account.
- It should be noted that some soils, such as saturated silts and unsaturated clays, do exhibit a F value under un-drained conditions.

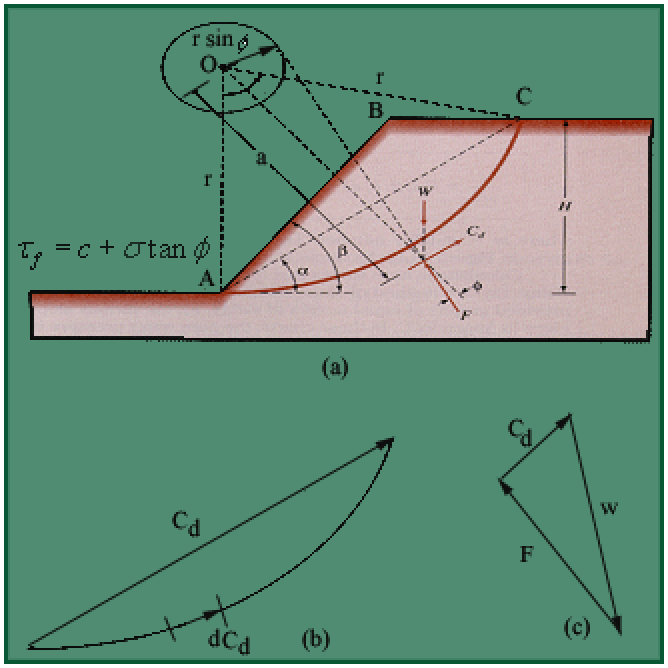

Fig. 2.17 Depiction of friction circle

- The friction circle method assumes a circular slip surface; it is in the form of a circular arc of radius R with its centre at ‘O’.
- Let the slip surface arc AC (=L) be considered to be made up of a number of elementary arcs of length ΔL. The elementary cohesive force acting along this element of length ΔL opposing the probable movement of the soil mass is $dC_d L_d$. Where dC_d is the unit mobilized cohesion which is assumed to be constant along the slip surface. The total cohesive force $C_d = dC_d$ L is considered to be made up of elementary cohesive force $dC_d \Delta L$ representing a force polygon. The closing link of which must represent the magnitude and direction of the resultant cohesive force, which is equal to $dC_d L_d$.

- The position of the resultant $dC_d L_d$ is determined by equating the sum of the moments of all the elementary cohesive forces along the slip circle about the centre of rotation 'O' to the moment of the resistant about 'O'.
- $\tau_f = c' + \sigma' \tan\phi'$
  - $\overline{AC}$ is a trial circular arc that passes through the toe of the slope, and O is the center of the circle.
  - Weight of soil wedge ABC = W = (Area of ABC)(γ)
  - For equilibrium (Figure 2.17)

  C_d - resultant of the cohesive force

$$C_d = c'_d(\overline{AC})$$

$$C_d(a) = c'_d(AC)r$$

$$a = \frac{c'_d(AC)r}{C_d} = \frac{AC}{\overline{\overline{AC}}}r$$

where 'a' is the moment arm of the cohesive force $dC_d AC$.

- If it is assumed that the frictional resistance is fully mobilized along the slip surface, the soil reaction dC_d on any elementary arc has its direction opposing the probable movement of the sliding mass and is inclined at ϕ to the normal at the point of action of dC_d on the slip surface. Line of action of dC_d will thus be a tangent to a circle of radius 'r sin ϕ',drawn with 'O' as centre. This circle is referred to as the *friction circle or the* ϕ*-circle.*
- The three forces considered in the equilibrium of the sliding mass can now be drawn to form a force triangle to determine the required value of dC_d from this force triangle, with the known values of 'W' and 'F' magnitudally with some direction , equal to $dC_d = C / L_d$.
- The factor of safety with respect to the cohesion is given by, $F_c = C_u / dC_d$. the minimum factor of safety is obtained by locating the critical slip circle. If the real factor of safety with respect to shear strength F s is required, a trial factor of safety with respect to friction F_ϕ is assumed. The friction circle is now constructed with a reduced radius 'r sin ϕ', where $\tan\phi_m = \tan\phi_u / F_\phi$. By carrying out the frictional circle analysis, the factor of safety with respect to cohesion, F_c is determined. If $F_c = F_\phi$, then this is the true factor of safety F_s . If not, a different F_ϕ is assumed and the procedure repeated till we obtain $F_\phi = F_c = F_s$.
- For an effective stress analysis, the total weight W is combined with the resultant boundary water force on the failure mass and the effective stress parameters c ' and ϕ' are used. This analysis is done to investigate the long form stability of a slope.

7.2.2 TERZAGHI'S ANALYSIS

The earth pressure increases rapidly with increasing values of internal friction ϕ . In the case of wall friction angle $\delta < \phi/3$, the error is around 5%. However, if δ is greater than about $\phi/3$, the surface of sliding is strongly curved. As a consequence, the error due to Coulomb's assumption of a plane surface increases rapidly. For $\delta = \phi$ it may be as great as 30 percent. Hence, for values of δ greater than $\phi/3$, the curvature of the surface of sliding must be taken into consideration. For passive pressures, the Coulomb formula can also give inaccurate results when there is a large back slope or wall friction angle. These conditions should be investigated and an increased factor of safety considered.

Log-Spiral Theory

A Log-spiral theory was developed because of the unrealistic values of earth pressures that are obtained by theories which assume a straight line failure plane. The difference between the Log-Spiral curved failure surface and the straight line failure plane can be large and on the unsafe side for Coulomb passive pressures (especially when wall friction exceeds $\phi/3$). The following figure shows a comparison of the Coulomb and Log-Spiral failure surfaces:

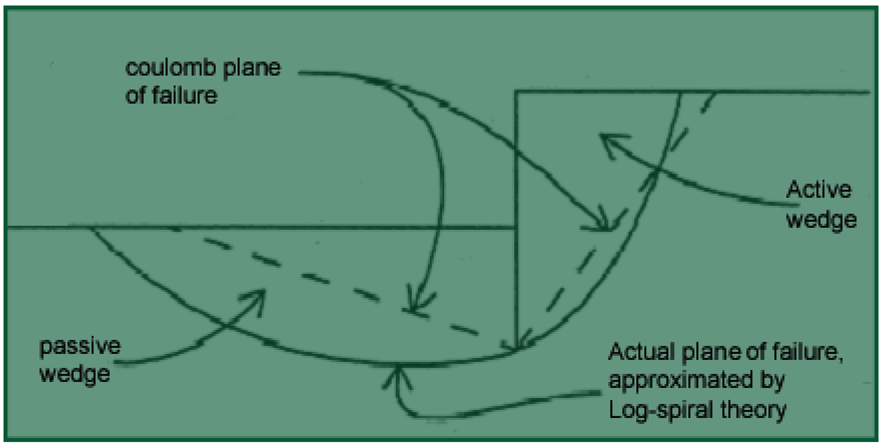

Fig. 2.18 Comparision of Coulomb's Theory and Log-Spiral Theory

Considering these factors, the theory put forward by Terzaghi is preferred for analyzing passive earth pressure when $\delta > \phi/3$. Terzaghi in 1943 published a theory for the computation of passive earth pressure.

Following are the assumptions made in this theory:

- Backfill is considered to be horizontal with a uniform surcharge density *q*. The shearing resistance of soil is given by the equation

  $s = c + \sigma \tan\phi$

- Rupture surface is assumed to be made up of a curved part *bd* and a straight line part *de* (fig 2.9). The soil within the triangle *ade* is in the passive Rankine state. Therefore, the shear stresses on the vertical section *df* are zero and the pressure on this section will be horizontal. The curve *bd* is taken as the log-spiral with the equation:

  $r = r_0 e^{\theta \tan\phi}$ ------------ (1)

  Centre of the log spiral is assumed to lie on the line *ad* at the tip *a*

- Three cases of loading are considered and the superposition of limit stresses for these three cases are considered to calculate the minimum resulting passive pressure.

  (a) $c=q=0$, $\gamma \neq 0$ (b) $c \neq 0$, $\gamma = q = 0$ (c) $q \neq 0$, $\gamma = c = 0$ holds good.

  where, c=cohesion, $\gamma =$ density of soil, q=intensity of surcharge.

  The following figure (2.19) illustrates the assumptions on which the theory of passive earth pressure against rough contact faces is based. Here, the lines *ab* and *ad* represents the initial (r_0) and final radius (r) of the log spiral respectively. The failure zone *abd* is thus divided into two parts viz ;(1) log spiral zone (2) Planar zone.

According to equation (1), every radius of the spiral makes an angle ϕ with the normal to the spiral at the point where it intersects the curve. Since ϕ is the angle of internal friction, the resultant *F* of the normal stress and the frictional resistance on any element of the surface of sliding also makes an angle ϕ with the normal to the element, and its direction coincides with that of the radius that the element subtends. Since every radius of the spiral passes through point a, the resultant F of the normal and frictional forces on *bd* also passes through the centre a.

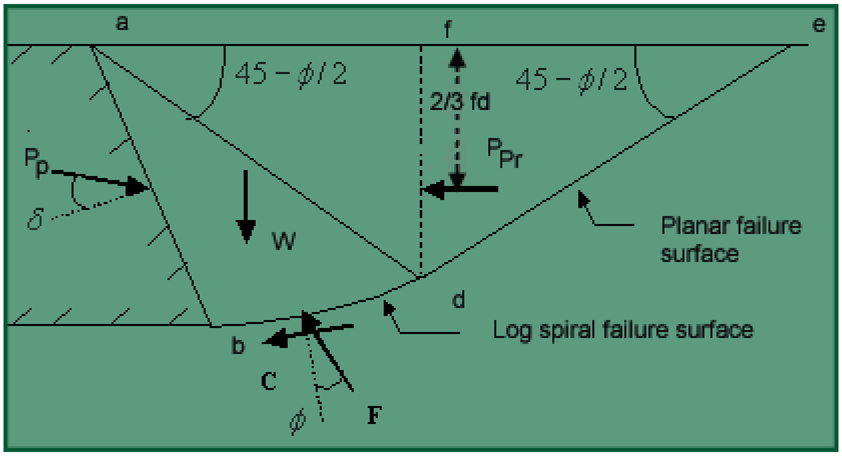

Fig. 2.19 Rupture Surface assumed in Terzaghi's Wedge Theory

Case (a): $c = q = 0, \gamma \neq 0$

In order to compute the passive pressure, we arbitrarily select a surface of sliding, *bde*

(**Figure** 2.21) consisting of the logarithmic spiral *bd* with its centre at *a*, and the straight line *de'*, which makes an angle of 45- $\phi/2$ with the horizontal. The surcharge q and the cohesion c are not taken into consideration in this case. The lateral pressure required to produce a slip on this surface is designated as P_p. The Rankine's passive earth pressure P_{pr} which acts at the lower third-point of *fd*, is calculated as

$$P_{pr} = \frac{1}{2}\gamma H^2 N_\phi \text{ -} \qquad \text{---------(2)}$$

where H is the height of the wall, $N_\phi = \tan^2(45 + \phi/2)$

Taking moments of the forces about a,

$$P_p l_1 \cos\delta = W.l_2 + P_{PR.}l_3 \qquad \text{---------(3)}$$

Since the force F acts along the radius of the log spiral failure surface, its moment with respect to point a is zero.

The value of P_p is plotted to scale as $P_{p\gamma}$. This value corresponds to the failure surface *be'*. This process is repeated and the passive pressure for each assumed failure surface is found out. The curve resulting from the passive pressures corresponding to each failure surface is used to get the minimum passive pressure ($P_{p\gamma}$).

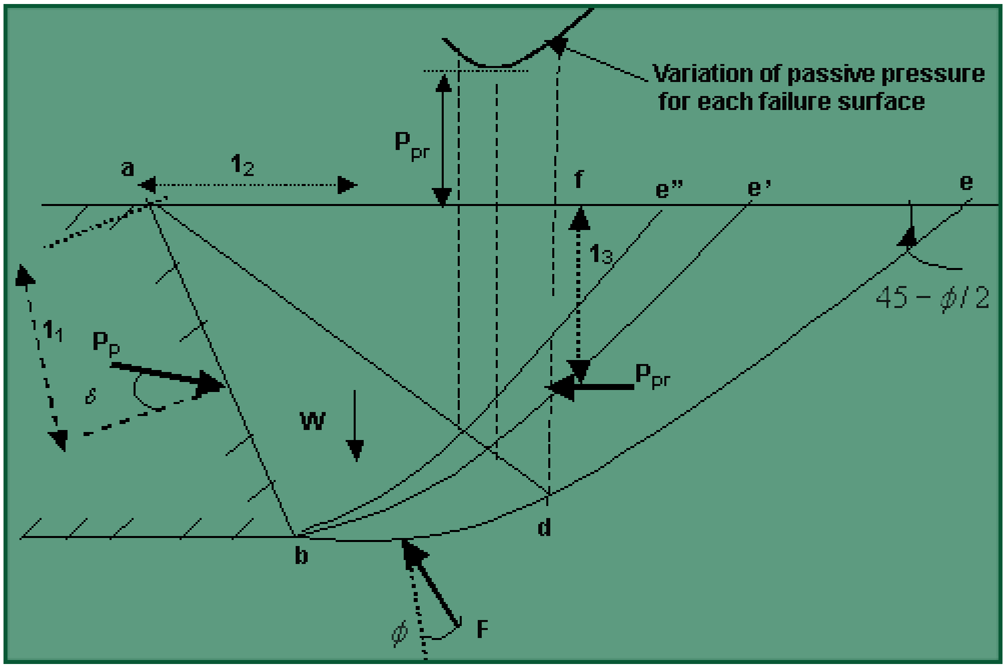

Fig. 2.20 Logarithmic Spiral Method for Determining Passive Earth Pressure

Case (b): $c \neq 0$, $\gamma = q = 0$

Here,the cohesive forces of the soil are used for the analysis while, the unit weight and surcharge of the soil are neglected. The adhesive force between the soil and the wall C_a is also taken into account.

The Rankine's passive pressure due to cohesion P_{PRC} is calculated as:

$$P_{PRC} = 2cH\sqrt{N_\varphi} \qquad \text{---------(4)}$$

Moment due to cohesion C, $M_c = \dfrac{c}{2\tan\phi}(r_f^{\,2} - r_0^{\,2})$ ---------(5)

The value of $\dfrac{c_a}{c}$ ranges from 0 to $\dfrac{\tan\delta}{\tan\phi}$. The shear strength corresponding to soil and wall are given as:

$\tau = c + \sigma\tan\phi$ (For soil) ----------(6)

$\tau = c_a + \sigma\tan\delta$ (For wall) ----------(7)

Adhesive force between the wall and the soil is given by

$$C_a = c_a.ab \qquad \text{---------(8)}$$

$$P_p.l_1\cos\delta = M_c + P_{PRC}l_3 \qquad \text{---------(9)}$$

After calculating the passive pressure for each failure surface, the minimum passive pressure corresponding to cohesion (P_{PC})is obtained in the same manner as in case (a).

Case (c) : $q \neq 0$, $\gamma = c = 0$

The equilibrium equation is given as:

$$P_p l_1 Cos\delta = Q.af + P_{PR}.l_3 \qquad \text{---------(10)}$$

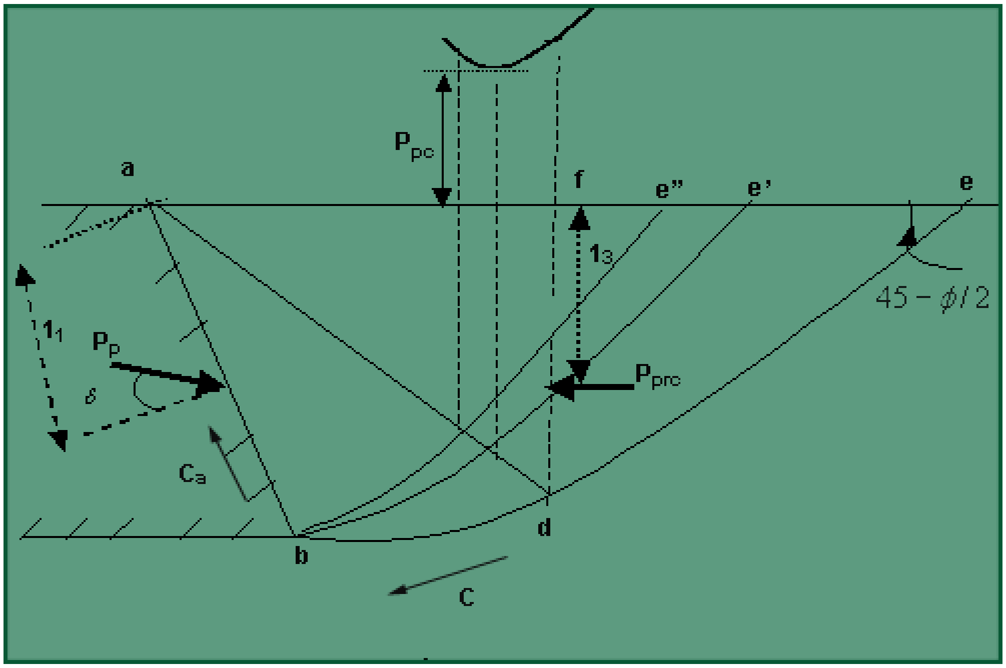

Fig. 2.21 Logarithmic Spiral Method for Determining Passive Earth Pressure

Case (a): $c \neq 0, \gamma = q = 0$

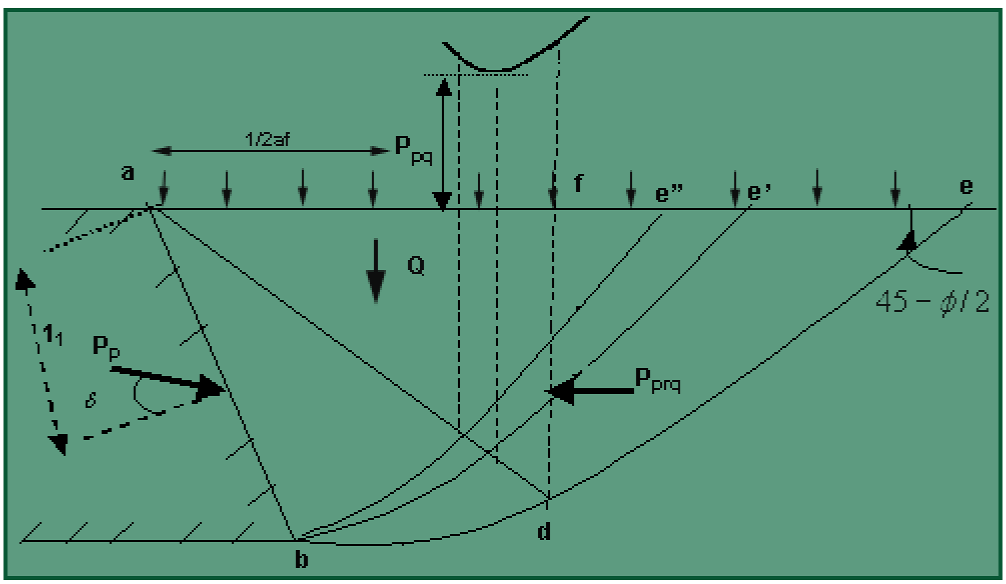

Fig. 2.22 Logarithmic Spiral Method for Determining Passive Earth Pressure

Case(c): $q \neq 0, c = \gamma = 0$

After getting the minimum values of passive pressure for the three cases, the absolute minimum is found out by summation of the three pressures. But the actual minimum pressure (P_{min}) obtained by superimposing each value of the three minimum passive pressures. Hence a single failure surface is not obtained for (P_{min}) (figure 2.23)

Absolute minimum pressure $P_{abs} = P_{PC} + P_{p\gamma} + P_{PQ}$

Error in using the method of superposition= $\dfrac{P_{min} - P_{abs}}{P_{min}} \times 100\% \approx \text{maximum } 30\%$

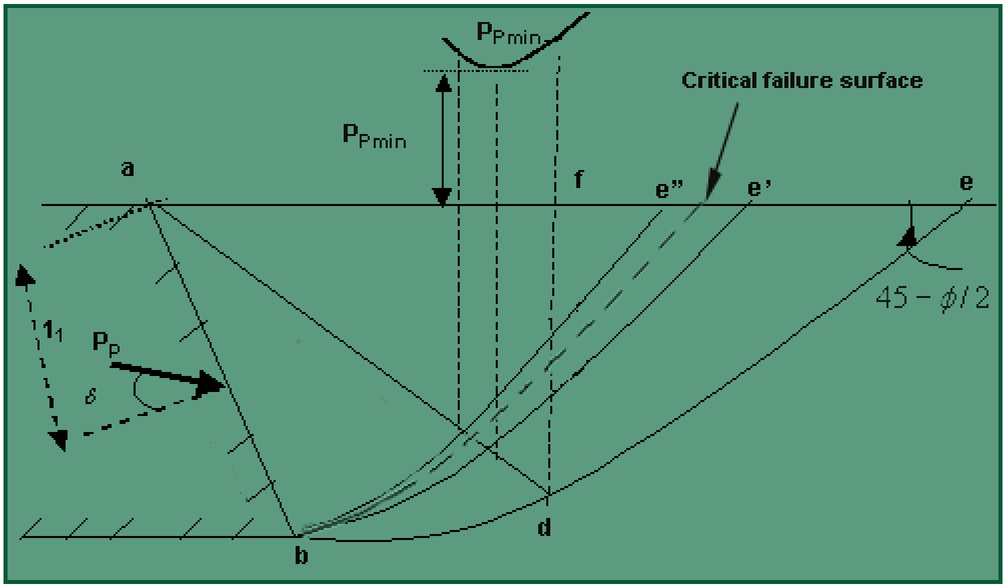

Fig. 2.23 Calculation of Minimum Passive Pressure.

Negative wall friction

In this case the passive pressure makes a negative angle of wall friction with the normal to the surface. The soil moves downwards with respect to wall. For negative wall friction only the log spiral failure surface is considered since the aim is to get minimum passive pressure. Kumar and Subbha Rao (1997) had done analysis of passive pressure under static condition using a composite failure surface for the negative friction angle case.

Seismic passive resistance in soil for negative wall friction

Choudhury and Subba Rao (2002) have analysed the seismic passive resistance for negative wall friction. In this analysis it was assumed that the rigid retaining structure is supporting dry, homogenous backfill with surcharge and that the occurrence of an earthquake does not affect the basic soil parameters. Uniform seismic accelerations were assumed to be acting at a particular time in both horizontal and vertical directions. The *limit equilibrium* method is used in the analysis of the earth pressure coefficients in soils for negative wall friction in the presence of pseudo-static seismic forces.

A retaining wall AB of vertical height H, wall batter angle α ,ground slope β, wall friction angle δ ,soil friction angle ϕ, coefficient of seismic horizontal acceleration K_h and coefficient of seismic vertical acceleration K_v was considered in the analysis (fig 2.24). The seismic passive resistance P_{pd} is split into three components as (i) unit weight component $c=q=0$, $\gamma \neq 0$ (ii) cohesion component $c \neq 0$, $\gamma = q = 0$ (iii) surcharge component $q \neq 0$, $\gamma = c = 0$.

The seismic passive resistance $P_{pd} = P_{pqd} + P_{pcd} + P_{p\gamma d}$ ---------(11)

Unlike the case of positive wall friction, focus point of the log spiral lies at the point F. The focus point is a variable and it is determined by varying η , where η is the inclination of the final radius r_f of the log spiral with the horizontal, so as to result in the minimum seismic passive resistance. The exit angle ξ at the point J on the ground is given by the following equation:

$$\xi = \frac{\pi}{4} - \frac{\phi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{K_h}{1-K_v}\right) + \frac{\beta}{2} + \frac{1}{2}\sin^{-1}\left\{\frac{\sin\left[\tan^{-1}\left(\frac{K_h}{1-K_v}\right)+\beta\right]}{\sin\phi}\right\}$$ ---------(12)

for $K_h = K_v = 0$,eq(12) reduces to the same as Rankine's exit angle.

The forces considered are the seismic passive force P_{pd} acting on AB; the uniform surcharge pressure of q acting on AJ ;the weight W of the soil mass ABJ; the cohesive force C on the failure surface BJ;the normal force N on the failure surface BJ; the adhesive force C_a on the retaining wall-soil interface AB;the pseudo static force due to the seismic weight component for zone ABJ as Wk_h and Wk_v in the horizontal and vertical directions, respectively; and the pseudo-static forces due to Qk_h and Qk_v in the horizontal and vertical directions, respectively,on AJ.

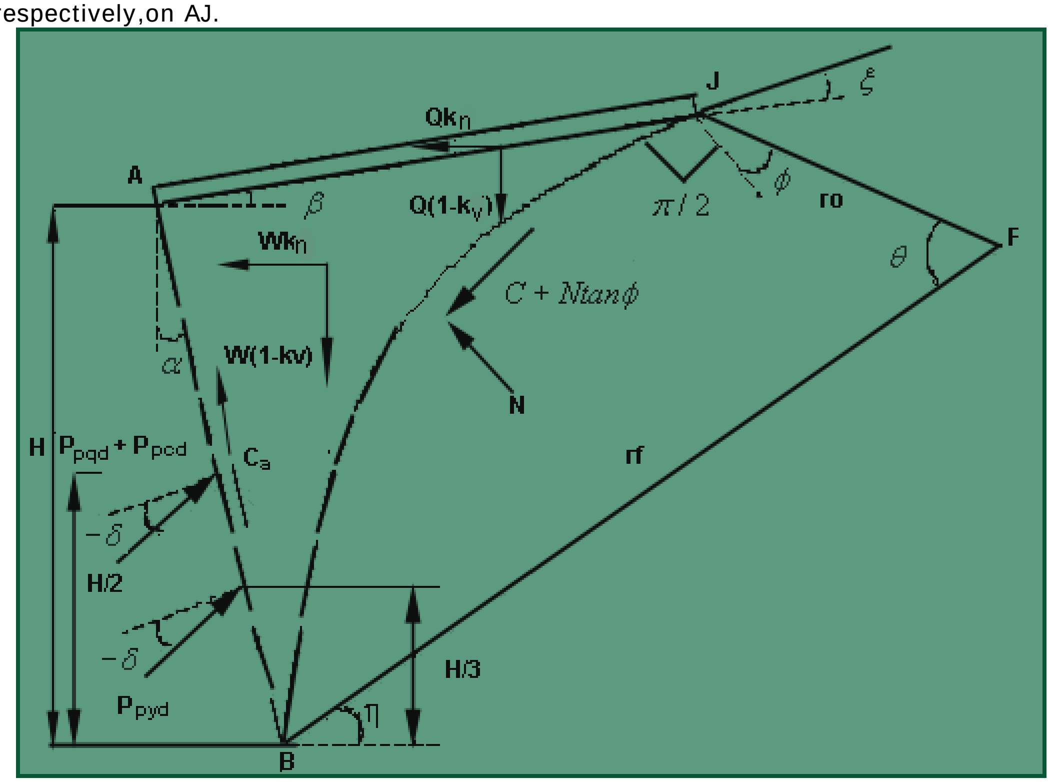

Fig. 2.24 Logarithmic Failure Surface and the Forces Considered

Total seismic passive pressure P_{pd} is given by:

$$P_{pd} = \left(2cHK_{pcd} + qHK_{pqd} + \frac{1}{2}\gamma H^2 K_{p\gamma d}\right)\frac{1}{\cos\delta} \quad \text{----------(13)}$$

where $K_{P\gamma d}, K_{pcd}, K_{pqd}$ are the seismic passive earth pressures coefficients corresponding to the individual values of $P_{p\gamma d}$, P_{pcd} and P_{pqd} respectively.

The values of $K_{P\gamma d}, K_{pcd}, K_{pqd}$ are given by:

$$K_{P\gamma d} = \frac{2P_{p\gamma d}\cos\delta}{\gamma H^2} \quad \text{----------(14)}$$

$$K_{pcd} = \frac{P_{pcd}\cos\delta}{2cH} \quad \text{----------(15)}$$

$$K_{pqd} = \frac{P_{pqd}\cos\delta}{qH} - \quad \text{----------(16)}$$

The analysis showed that the seismic passive pressure coefficients always decrease with the increase in the vertical seismic acceleration whereas, the horizontal seismic acceleration results in either an increase or decrease in the passive earth pressure coefficients, depending on the combinations of α, β and $\frac{\delta}{\phi}$.

Recap

In this section you have learnt the following

- Friction Circle Method
- Terzaghi's Analysis
- Log-Spiral Theory

Congratulations, you have finished Lecture 7. To view the next lecture select it from the left hand side menu of the page