Module 8

Lecture 5: Reliability analysis
Reliability

- It is defined as the probability of non-failure, $p_s$, at which the resistance of the system exceeds the load;

$$p_s = P(L \leq R)$$

where $P()$ denotes the probability.

- The failure probability, $p_f$, is the compliment of the reliability which can be expressed as

$$p_f = P(L \geq R) = 1 - p_s$$

The resistance or strength ($R$) is the ability to accomplish the intended mission satisfactorily without failure when subjected to loading of demands or external stresses ($L$). Failure occurs when the resistance of the system is exceeded by the load (floods, storms etc.)
Measurements of Reliability

- **Recurrence interval**
  \[ T = \frac{1}{1-F}, \quad F = P(X \leq x_T) \]
  - No consideration for the interaction with the system resistance

- **Safety Margin**
  - It is defined as the difference between the resistance (R) and the anticipated load (L)
  \[ SM = R - L \]

- **Safety Factor**
  - It is the ratio of resistance to load
  \[ SF = \frac{R}{L} \]

(Tung, 2004)
Recurrence Interval

- Assume independence of occurrence of events and the hydraulic structure design for an event of T-year return period.
- $1/T$ is the probability of exceedance for the hydrologic event in any one year.
  - Failure probability over an n-year service period, $p_f$, is
    - $p_f = 1-(1-1/T)^n$ (using Binomial distribution)
    - or $p_f = 1-exp(-n/T)$ (using Poisson distribution)

- Types of problem:
  - (a) Given $T$, $n$, find $p_f$
  - (b) Specify $p_f$ & $T$, find $n$
  - (c) Specify $p_f$ & $n$, find $T$
Probabilistic Approaches to Reliability

- **Statistical analysis** of data of past failure records for similar systems

- **Reliability analysis**, which considers and combines the contribution of each factor potentially influencing the failure with the steps as
  1. to identify and analyze the uncertainties of each contributing factor;
  2. to combine the uncertainties of the stochastic factors to determine the overall reliability of the structure.
Uncertainties in Hydraulic Engineering Design

- **Hydrologic uncertainty**
  (Inherent, parameter, or model uncertainties)

- **Hydraulic uncertainty**
  (Uncertainty in the design and analysis of hydraulic structures)

- **Structural uncertainty**
  (Failure from structural weaknesses)

- **Economic uncertainty**
  (Uncertainties in various cost items, inflation, project life, and other intangible factors)
Techniques for Uncertainty Analysis

- **Analytical Technique**
  - Fourier and Exponential Transforms
  - Mellin Transform
- **Approximate Technique**
  - First-Order Variance Estimation (FOVE) Method
  - Rosenblueth’s Probabilistic Point Estimation (PE) Method
  - Harr’s Probabilistic Point Estimation (PE) Method
- **Reliability Analysis Methods**
Reliability Analysis Methods

1. Performance Function and Reliability Index
2. Direct Integration Method
3. Mean-Value First-Order Second-Moment (MFOSM) Method
4. Advanced First-Order Second-Moment (AFOSM) Method
   a) First-order approximation of performance function at design point.
   b) Algorithms of AFOSM for independent normal parameters.
   c) Treatment of correlated normal random variables.
   d) Treatment of non-normal random variables.
   e) AFOSM reliability analysis for non-normal, correlated random variables.
5. Monte Carlo Simulation Methods
1. Performance Function and Reliability Index

- To enable a quantitative analysis of the reliability of a structure, every failure mode has to be cast in a mathematical form.
- A limit state function is given by:
  \[ W(z, x) = R(z, x) - S(z, x) \]

where,

- \( z \): Vector of design variables;
- \( x \): Vector of random input variables;
- \( R \): Resistance of the structure;
- \( S \): Load on the structure.

The value of the limit state function for given values of \( x \) and \( z \) denotes the margin.
The reliability index is defined as the ratio of the mean to the standard deviation of the performance function $W(z, x)$

$$\beta = \frac{\mu_w}{\sigma_w}$$

where $\mu_w$ and $\sigma_w$ are the mean and standard deviation of the performance function.

The boundary that separates the safe set and failure set is the failure surface, defined by the function $W(z, x) = 0$, called the limit state function.
1. Performance Function and Reliability Index

System states defined by performance function

\[ W(z, x) < 0 \]
Failure Region

\[ W(z, x) > 0 \]
Safe Region

\[ W(z, x) = 0 \]
Limit-State Surface
2. Direct Integration Method

The reliability can be computed in terms of the joint PDF of the load and resistance as

\[
p_s = \int_{r_1}^{r_2} \left[ \int_{l_1}^{r} f_{R,L}(r, l) dl \right] dr = \int_{l_1}^{l_2} \left[ \int_{l}^{r_2} f_{R,L}(r, l) dr \right] dl
\]

where \( f_{R,L}(r, l) \) : joint PDF of random load, L, and resistance, R;

\( r, l \) : dummy arguments for the resistance and load, respectively;

\( (r_1, r_2), (l_1, l_2) \) : lower and upper bounds for the resistance and load, respectively.
3. Mean-Value First-Order Second-Moment (MFOSM) Method

- Here, the performance function $W(z, x)$, defined on the basis of the load and resistance functions, $S(z, x)$ and $R(z, x)$, are expanded in a Taylor series at a selected reference point.
- The second and higher order terms in the series expansion are truncated, resulting in an approximation that requires the first two statistical moments of the random variables.

The simplification of Taylor series greatly enhances the practicality of the first order methods because in many situations, it is difficult to find the PDF of the variables while it is relatively simple to estimate the first two statistical moments.
Here, it mitigates the deficiencies associated with the MFOSM method, while keeping the simplicity of the first–order approximation.

The difference between the AFOSM and MFOSM methods is that the expansion point in the first–order Taylor series expansion in the AFOSM method is located on the failure surface defined by the limit state equation, $W(x) = 0$. 
5. Monte Carlo Simulation Methods

- It is a general purpose method to estimate the statistical properties of a random variable that is related to a number of random variables which may or may not be correlated.

- The values of stochastic parameters are generated according to their distributional properties and are used to compute the value of performance function.

- Reliability of the structure can be estimated by computing the ratio of the number of realizations with \( W \geq 0 \) to the total number of simulated realizations.

- Disadvantage ➔ computational intensiveness.
Return period/recurrence interval: The reciprocal of annual exceedance probability

First order, Stationary Markov Process states that observed value of a random event at any time \( t+1 \), is a function of a stationary component, correlated component and a purely random component. Here, the random component is assumed to be normally distributed with zero mean and \( \sigma_t^2 \) variance.
Reliability is defined as the probability of non-failure, $p_s$, at which the resistance of the system exceeds the load.

- Failure occurs when the resistance of the system is exceeded by the load (floods, storms etc.)

Measurements of reliability are recurrence interval, safety margin and Safety factor

Techniques for Uncertainty Analysis:

- Analytical technique
- Approximate technique
- Reliability analysis methods
Highlights in the Module

- Reliability Analysis Methods:
  1. Performance Function and Reliability Index
  2. Direct Integration Method
  3. Mean-Value First-Order Second-Moment (MFOSM) Method
  4. Advanced First-Order Second-Moment (AFOSM) Method
  5. Monte-Carlo Simulation method