Hydrologic Analysis
(Contd.)

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Objective of this module is to learn linear-kinematic wave models and overland flow models
Topics to be covered

- Kinematic wave modeling
  - Continuity equation
  - Momentum equation
  - Saint Venant equation
- Kinematic overland flow modeling
- Kinematic channel modeling
Module 4

Lecture 1: Kinematic wave method
Kinematic wave method

- This method assumes that the weight or gravity force of flowing water is simply balanced by the resistive forces of bed friction.

- This method can be used to derive overland flow hydrographs, which can be added to produce collector or channel hydrographs and eventually, as stream or channel hydrograph.

- This method is the combination of continuity equation and a simplified form of St. Venant equations.

(Note:- The complete description of St. Venant equations is provided in Module-6)
Kinematic modeling methods

Continuity Equation

\[ \frac{Q}{2g} + \frac{\partial Q}{\partial x} \, dx \]

Energy line

Datum

\[ h \]

\[ Z \]

\[ \theta \]

\[ S_0 \]

\[ F_f \]

\[ F_g \]

\[ F_H \]
The general equation of continuity,

\[
\text{Inflow-Outflow} = \text{rate of change of storage}
\]

**Inflow**

\[
\text{Inflow} = \left( Q - \frac{\partial Q}{\partial x} \cdot \frac{\Delta x}{2} \right) \cdot \Delta t + q \Delta x \Delta t
\]

**Outflow**

\[
\text{Outflow} = \left( Q - \frac{\partial Q}{\partial x} \cdot \frac{\Delta x}{2} \right) \cdot \Delta t
\]

where,

- \( q \) = rate of lateral inflow per unit length of channel
- \( A \) = cross-sectional area

**Storage change**

\[
\text{Storage change} = \frac{\partial A}{\partial t} \Delta x \Delta t
\]
The equation of continuity becomes, after dividing by $\Delta x$ and $\Delta t$, 

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q$$

- For unit width $b$ of channel with $v =$ average velocity, the continuity equation can be written as 

$$y \frac{\partial v}{\partial x} + v \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = \frac{q}{b}$$
Kinematic modeling methods

Momentum equation

It is based on Newton’s second law and that is, Net force = rate of change of momentum

The following are the three main external forces acting on area A:

Hydrostatic: \[ F_H = -\gamma \frac{\partial(yA)}{\partial x} \Delta x \]

Gravitational: \[ F_g = -\gamma AS_o \Delta x \]

Frictional: \[ F_f = -\gamma AS_f \Delta x \]

where:
- \( \gamma \) = specific weight of water \((\rho g)\)
- \( y \) = distance from the water surface to the centroid of the pressure prism
- \( S_f \) = friction slope, obtained by solving for the slope in a uniform flow equation, (manning’s equation)
- \( S_o \) = Bed slope
The rate of change of momentum is expressed from Newton’s second law as

\[ F = \frac{d}{dt}(mv) \]  

---------4.1

where the total derivative of \( v \) W.R.T \( t \) can be expressed

\[ \frac{dv}{dt} = \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \]  

---------4.2
Kinematic modeling methods

Momentum Equation

Contd...

• Equating Eq. 4.1 to the sum of the three external forces results in

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{g}{A} \frac{\partial (yA)}{\partial x} + \frac{vq}{A} = g(S_o - S_f) \quad \ldots \ldots 4.3
\]

• For negligible lateral inflow and a wide channel, the Eq. 4.3 can be rearranged to yield

\[
S_f = S_o - \frac{\partial y}{\partial x} - \frac{v \partial v}{g \partial x} - \frac{1 \partial v}{g \partial t} \quad \ldots \ldots 4.4
\]
Kinematic modeling methods

Assumptions of Saint Venant equations

- In developing the general unsteady flow equation it is assumed that the flow is one-dimensional (variation of flow depth and velocity are considered to vary only in the longitudinal X- direction of the channel)
- The velocity is constant and the water surface is horizontal across any section perpendicular to the longitudinal flow axis
- All flows are gradually varied with hydrostatic pressure such that all the vertical accelerations within the water column can be neglected
- The longitudinal axis of the flow channel can be approximated by a straight line, therefore, no lateral secondary circulations occur
The slope of the channel bottom is small (less than 1:10)

The channel boundaries may be treated as fixed non-eroding and non-aggravating

Resistance to flow may be described by empirical resistance equations such as the manning or Chezy equations

The flow is incompressible and homogeneous in density
## Kinematic modeling methods

### Forms of momentum Equation

<table>
<thead>
<tr>
<th>Type of flow</th>
<th>Momentum equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic wave (study uniform)</td>
<td>$S_f = S_o$</td>
</tr>
<tr>
<td>Diffusion (non inertia) model</td>
<td>$S_f = S_o - \frac{\partial y}{\partial x}$</td>
</tr>
<tr>
<td>Steady no-uniform</td>
<td>$S_f = S_o - \frac{\partial y}{\partial x} - \left( \frac{v}{g} \right) \frac{\partial v}{\partial x}$</td>
</tr>
<tr>
<td>Unsteady non-uniform</td>
<td>$S_f = S_o - \frac{\partial y}{\partial x} - \left( \frac{v}{g} \right) \frac{\partial v}{\partial x} - \left( \frac{1}{g} \right) \frac{\partial v}{\partial t}$</td>
</tr>
</tbody>
</table>

Dynamic wave

Module 4
Kinematic modeling methods

Possible types of open channel flow

- Uniform
- Gradually varied
- Rapidly varied

Steady Flow

Gradually varied
Rapidly varied
Unsteady flow

E.L. = Energy line
W.S. = water surface
## Kinematic Modelling Methods

### Difference between Kinematic and Dynamic Wave

<table>
<thead>
<tr>
<th>Kinematic Wave</th>
<th>Dynamic Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is defined as the study of motion exclusive of the influences of mass and force</td>
<td>In this the influences of mass and force are included</td>
</tr>
<tr>
<td>When the inertial and pressure forces are not important to the movement of wave then the kinematic waves governs the flow</td>
<td>When inertial and pressure forces are important then dynamic waves govern the moment of long waves in shallow water (large flood wave in a wide river)</td>
</tr>
<tr>
<td>Force of this nature will remain approximately uniform all along the channel (Steady and uniform flow)</td>
<td>Flows of this nature will be unsteady and non-uniform along the length of the channel</td>
</tr>
<tr>
<td>Froude No. &lt; 2</td>
<td>Froude No. &gt; 2</td>
</tr>
</tbody>
</table>
Froude number

Fr = \frac{V}{\sqrt{gd}}

Where
V = velocity of flow
\( g \) = acceleration due to gravity
\( d \) = hydraulic depth of water

Wave celerity (C)
\[ c = \sqrt{gd} \]

1. Flows with Froude numbers greater than one are classified as supercritical flows
2. Froude number greater than two tend to be unstable, that are classified as dynamic wave
3. Froude number less then 2 are classified as kinematic wave
Kinematic modelling methods

Visualization of dynamic and kinematic waves

A dynamic wave appears as gradually varied, unsteady flow; streamlines and water surface profiles are not parallel.

A kinematic wave appears as uniform, unsteady flow; water surfaces and bed are parallel to each other and to the energy grade line.