Choose the most appropriate answer for each question.

1. If we expand \( f(x_0 + 2\Delta x) \) about the point \( x_0 \) in Taylor series, then the third term in the expansion is given by
   \[ \text{(a)} \ 4 \left. \frac{d^2f}{dx^2} \right|_{x_0} \Delta x^2 \quad \text{(b)} \ -2/3 \left. \frac{d^3f}{dx^3} \right|_{x_0} \Delta x^3 \quad \text{(c)} \ -4/3 \left. \frac{d^3f}{dx^3} \right|_{x_0} \Delta x^3 \quad \text{(d)} \ 2 \left. \frac{d^2f}{dx^2} \right|_{x_0} \Delta x^2 \]

   **Expand and see for yourself!**

2. \( \frac{d^2f}{dx^2} = \frac{-f_{i-2} - 16f_{i+1} + 30f_i - 16f_{i-1} - f_{i-2}}{(12\Delta x^2)} \) is an approximation of what order of accuracy?
   \( \text{(a)} \ 2 \quad \text{(b)} \ 3 \quad \text{(c)} \ 4 \quad \text{(d)} \ \text{None of the above} \)

   **The coefficients on the RHS do not add up to zero!**

3. For what value of \( p \) is the following a second order accurate approximation:
   \[ \left. \frac{d^3f}{dx^3} \right|_i = \frac{-3f_{i+4} + 14f_{i+3} - 24f_{i+2} + p f_{i+1} - 5f_i}{(2\Delta x^3)} \]
   \( \text{(a)} \ 12 \quad \text{(b)} \ 1 \quad \text{(c)} \ 16 \quad \text{(d)} \ 18 \)

   **The coefficients on the RHS should add up to zero.**

4. If we expand \( f(x_0 - 3\Delta x) \) about the point \( x_0 \) in Taylor series, then the fourth term in the expansion is given by
   \[ \text{(a)} \ (3/2) \left. \frac{d^2f}{dx^2} \right|_{x_0} \Delta x^2 \quad \text{(b)} \ -27/3! \left. \frac{d^3f}{dx^3} \right|_{x_0} \Delta x^3 \quad \text{(c)} \ (9/2) \left. \frac{d^3f}{dx^3} \right|_{x_0} \Delta x^3 \quad \text{(d)} \ -(9/2) \left. \frac{d^2f}{dx^2} \right|_{x_0} \Delta x^2 \]

   **Check it out by expanding.**

5. \( \frac{df}{dx} = \frac{-25f_i + 48f_{i+1} - 36f_{i-1} + 16f_{i+3} - 3f_{i+4}}{(\Delta x)} \) is an approximation of what order of accuracy?
   \( \text{(a)} \ 4 \quad \text{(b)} \ 3 \quad \text{(c)} \ 2 \quad \text{(d)} \ 5 \)

   **First derivative, one-sided differencing, five successive points**

6. Consistency condition is about the equivalence between
   \( \text{(a)} \) the exact and the numerical solution of the partial differential equation
   \( \text{(b)} \) the exact solution and the numerical solution of the discretized equation
   \( \text{(c)} \) the exact partial differential equation and the discretized equation
   \( \text{(d)} \) None of the above.

   **From definition**

7. Stability condition is about the equivalence between
a) the exact and the numerical solution of the partial differential equation
(b) the exact solution and the numerical solution of the discretized equation
(c) the exact partial differential equation and the discretized equation
(d) None of the above.

**From definition**

8. Convergence condition is about the equivalence between
a) the exact and the numerical solution of the partial differential equation
(b) the exact solution and the numerical solution of the discretized equation
(c) the exact partial differential equation and the discretized equation
(d) None of the above.

**From definition**

9. Which of the statements is true about differencing schemes?
(a) Inconsistent schemes cannot be stable.
(b) For stable schemes, inconsistency does not matter.
(c) An inconsistent scheme will not yield the correct solution.
(d) Stable scheme will lead to the converged solution within truncation errors.

*An inconsistent but stable scheme will yield the correct solution of a modified pde, not the one we are seeking to solve.*

10. A grid-independent solution is expected to agree with
(a) the analytical solution of the pde.
(b) the exact solution of the discretized equation.
(c) Both of the above.
(d) Neither of the above.

**We still need convergence and stability**

11. Lax theorem on assurance of convergence is applicable only for
(a) linear problems, (b) well-posed problems (c) initial value problems (d) all of the above

12. For discretization of which of the following derivative(s) at (i, j) may we get one or more of the corner points A, B, C, D in Figure 1?

(a) \( \frac{\partial^2 u}{\partial x \partial y} \)  (b) \( \frac{\partial^2 u}{\partial y^2} \)  (c) \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \)  (d) \( \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} \)

![Figure 1](image)