Assignment for Module–7

Due on: Sunday, Mar 6\textsuperscript{th}

INSTRUCTIONS

- Submit your work on the course website (https://onlinecourses.nptel.ac.in/noc16_ch01/)
- Please submit your results before the deadline.
- You do not need to upload your MATLAB files.

1. RK-2 RALSTON’S METHOD (2 points)

The RK-2 Ralston’s method is similar to the Heun’s method discussed in the lectures. The Ralston’s method is given by:

\[
y_{i+1} = y_i + h \left( \frac{k_1}{3} + \frac{2k_2}{3} \right)
\]

\[
k_1 = f(t_i, y_i), \quad k_2 = f \left(t_i + \frac{3h}{4}, y_i + \frac{3hk_1}{4} \right)
\]

Solve the following ODE using RK-2 Ralston’s method:

\[V - IR - L \frac{dI}{dt} = 0\]

for inductance \(L = 2\), resistance \(R = 2.5\) and voltage \(V = 5\). The initial value for this ODE is given by current \(I(0) = 0\).

Using step-size \(h = 0.25\), obtain the value of current \(I\) at time 2.

Repeat the problem for double the step size. With \(h = 0.5\), obtain the value of current, \(I(2)\).

1–2. Report the value of \(I(2)\) using Ralston’s method with \(h = 0.25\) and \(h = 0.5\).

2. STABILITY OF RK-2 RALSTON’S METHOD (1 point)

In Lecture 7.1, we saw that increasing the step-size to \(h = 1\) made Euler’s method unstable. We will repeat the same procedure to study the stability of Ralston’s method:

- Repeat the above problem for \(h = 0.25\) but with end-time of \(t_{end} = 20\).
- Double the value to \(h = 0.5\) and repeat again with \(t_{end} = 20\)
- Keep doubling the value of \(h\), and repeat Ralston’s method for \(h = 0.25, 0.5, 1.0, 2.0\) and \(4.0\). At certain \(h\), the numerical method may become unstable.

Does the Ralston’s method become unstable? If yes, at what value of \(h\)?

3. Choose the lowest value of \(h\) where Ralston’s method becomes unstable.
3. RL CIRCUIT (2 points)

Let us now use MATLAB solver \texttt{ode45} for the RL circuit model (with some changes):

\[
V - IR - L \frac{dI}{dt} = 0, \quad I(0) = 0
\]

**Part-1:** Solve the above ODE with \( L = 2, R = 2.5 \), but when voltage decreases with time as:

\[
V = 5e^{-0.2t}
\]

**Part-2:** Solve the above ODE for the case where \( R \) also depends on current \( I \). Thus, inductance is constant at \( L = 2 \) and:

\[
V = 5e^{-0.2t} \quad \text{and} \quad R = 2.5 + 0.5 \left( \frac{I}{2} \right)^{0.25}
\]

In both the cases, initial value is \( I(0) = 0 \). Use \texttt{ode45} to obtain \( I(5) \).

4. Report value of \( I \) using \texttt{ode45} at time 5 for Part-1 \((V = 5e^{-0.2t})\)

5. Report value of \( I \) using \texttt{ode45} at time 5 for \( V = 5e^{-0.2t}, R = 2.5 + 0.5 \left( \frac{I}{2} \right)^{0.25} \)

4. ODE USING HIGHER ORDER RK SOLVER (3 points)

One of the ODE solvers in MATLAB uses Bogacki-Shampine (B-S method), given by:

\[
y_{i+1} = y_i + \frac{7}{24}hk_i + \frac{1}{4}hk_2 + \frac{3}{8}hk_3 + \frac{1}{8}hk_4, \quad \text{where}
\]

\[
k_1 = f(t_i,y_i), \quad k_2 = f\left(t_i + \frac{h}{2},y_i + \frac{hk_1}{2}\right), \quad k_3 = f\left(t_i + \frac{3h}{4},y_i + \frac{3hk_2}{4}\right)
\]

\[
\text{and} \quad k_4 = f\left(t_i + h,y_i + \frac{2hk_1}{3} + \frac{hk_2}{3} + \frac{2hk_3}{3}\right)
\]

**Step-1:** Use the above method to solve the following ODE:

\[
\frac{dy}{dt} = -y^2 \sin(y), \quad y(0) = 1
\]

and obtain \( y(2) \) using step-size of \( h = 0.1 \). [Let’s call this solution \( y_{BS} \)]

**Step-2:** Using MATLAB solver \texttt{ode45}, compute the solution for the above ODE. The result \( y(2) \) will be named as \( y_{ode45} \). Hence, compute the error of B-S method as: \( e = |y_{ode45} - y_{BS}| \)

**Step-3:** Re-do the above problem to compute \( y(2) \) for \( h = 0.01 \). Compute the error as before.

**Step-4:** Compare errors in Step-2 and Step-3. When the step-size decreased by a factor of 10, how much did the error decrease. Based on this, what is the order of accuracy of this method?

6. Report the solution \( y \) at time 2 using B-S method for step-size of \( h = 0.1 \)

7-8. Report the errors in \( y(2) \) using \( h = 0.1 \) and \( h = 0.01 \).

9. What is the order of accuracy of the B-S method (multiple-choice)?

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