Neutron Spectrum & Cross sections

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This lecture will deal with the regions of neutron energy spectrum and the reaction cross sections of some isotopes as a function of neutron energy.

At the end of this lecture, the learners will be able to

(i) identify energy range associated with low-energy, resonance and continuum regions
(ii) understand the influence of neutron energy on fission, absorption and capture cross sections

1 Neutron spectrum
One of the essential studies in neutronics of a fast reactor is the study of neutron energy spectrum. The neutron capture and fission occurs over a wide range of neutron energies in a fast reactor compared to that in a thermal reactor.

Lethargy (u) is defined as the natural logarithm of ratio of maximum energy that a neutron might have in a nuclear reactor (E₀) to the neutron energy (E). Hence, lethargy is a measure of neutron energy. The maximum neutron energy (E₀) is taken as 10 MeV.

\[
Lethargy = u = \ln \left( \frac{E_0}{E} \right)
\]  

(1)

Differentiating Eq. (1), we may relate the change in energy and change in lethargy as:

\[
du = -\frac{dE}{E}
\]

(2)

However, small changes in lethargy (Δu) may be written as

\[
\Delta u = u_1 - u_2 = \ln \left( \frac{E_2}{E_1} \right)
\]

(3)

The neutron flux Vs neutron energy for typical fast reactor with metal oxide, metal carbide and metal as fuels is shown in Figure 1. The Y-axis is relative flux per unit lethargy. One may observe from Figure 1 that the relative flux increases with neutron energy till about 3 keV. The flux decreases slightly around 3 keV, which is attributed to the higher scattering by sodium at this neutron energy level. After this small depression, the relative flux increases with neutron energy and reaches maximum around 100 keV for the oxide fuel. Decrease in relative flux is rapid beyond the neutron energies of 2 MeV.
One may observe from Figure 1, that at lower neutron energies (< 20-50 keV), the relative neutron flux per unit lethargy is highest for oxide fuel, followed by that for carbide and metal fuels. Above 100 keV, the metallic fuel has the highest relative flux among these fuels.

The neutron flux as a function of neutron energy is given by:

\[ \phi(E) = C \sqrt{\frac{E}{(kT_m)^3}} \exp \left( \frac{-E}{kT_m} \right) \]  

(4)

‘kT_m’ is taken as 1.4 MeV for Pu-239. ‘kT_m’ changes with the isotope.
The variation of neutron flux with neutron energy influences the fraction of fission that occurs at different energy levels. By dividing neutron energies in to different groups with known energy range for each group, it is possible to calculate the fraction of fission that can occur over different neutron energy ranges. The results of such a study for an oxide-fueled, 1200 MWe fast reactor is available in the literature (Table 4.5, Page no. 74 of Ref #1). Few highlights from that table are:

(i) the fraction of fission over a energy range of 1.35-3.7 MeV is the largest (~0.18)
(ii) the neutron energy range over which a large fraction of fission occurs is 15 keV to 3.7 MeV, with the cumulative fission fraction in this range being 0.73
(iii) the median fission energy is ~ 150 eV. Median fission energy is the neutron energy above which the cumulative fission fraction is 0.5
(iv) Nearly 20% of fission occurs with neutron energies lower than 10 keV for the oxide-fueled reactor. The fission fraction in this energy range for metal–fuelled reactor is very low. The larger Doppler coefficient in oxide-fueled fast reactor is attributed to relatively higher fission fraction (compared to metal-fueled reactors) at neutron energies lower than 10 keV.

In a nuclear reactor, the neutron energies vary from MeV (fast neutron) to 0.025 eV (thermal neutron). The neutron energy spectrum can be divided into three regions: (i) low-energy region (< 1 eV); (ii) resonance region (1 eV <\(E_n\)<1 MeV) and (iii) continuum region (0.01<\(E_n\)<25 MeV).

The cross sections for various neutron reactions are dependent on neutron energy levels. Let us discuss the variation of cross section with neutron energy for few important isotopes: U-235, Pu-239 and U-238.

### 1.1 Cross sections for U-235

In the low energy region, the total cross section and the fission cross section for U-235 decrease with neutron energy, except over a small neutron energy range near 0.25-0.30 eV. For instance, at 0.025 eV, the fission cross section for U-235 is 580 b while at 1 eV the fission cross section falls to 65 b. This explains the reason for thermalization of fast neutron to energies around 0.025 eV in a thermal reactor.

Looking at the continuum region in which fast reactors operate, the fission cross section decreases with neutron energy. For U-235, the fission cross section at 0.01 MeV is about 3.3 b while the same at 0.5 MeV is about 1.2 b. Between 0.5 MeV and 3 MeV, the fission cross section is about 1.2 to 1.3 b. Beyond 6 MeV, the fission cross section increases and reaches about 3 b at about 20 MeV. Table 1 shows the cross sections for various neutron reactions for U-235 for different energy intervals of relevance to fast reactor.
### Table 1: Cross sections for U-235

<table>
<thead>
<tr>
<th>Energy interval</th>
<th>Capture cross section (b)</th>
<th>Fission cross section (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 2.2 MeV</td>
<td>0.04</td>
<td>1.23</td>
</tr>
<tr>
<td>0.82 – 2.2 MeV</td>
<td>0.09</td>
<td>1.24</td>
</tr>
<tr>
<td>300 – 820 keV</td>
<td>0.18</td>
<td>1.18</td>
</tr>
<tr>
<td>110 – 300 keV</td>
<td>0.32</td>
<td>1.40</td>
</tr>
<tr>
<td>40 – 110 keV</td>
<td>0.53</td>
<td>1.74</td>
</tr>
<tr>
<td>15 – 40 keV</td>
<td>0.79</td>
<td>2.16</td>
</tr>
<tr>
<td>0.75 – 15 keV</td>
<td>1.71</td>
<td>4.36</td>
</tr>
</tbody>
</table>

### 1.2 Cross sections for Pu-239

It may recalled that the Pu-239 is not a naturally occurring fissile isotope; instead produced in nuclear reactors by neutron capture and transmutation in U-238. The fission and total cross sections decrease with neutron energy in the low-energy region, except in the energy range between 0.2-0.4 eV due to resonance. The fission cross section at 0.025 eV is about 752 b while the capture cross section is about 270 b, with the absorption and total cross sections being 1022 and 1028 b respectively. The total cross section falls to less than 10 b when the neutron energies reach 1 MeV. The average total cross section in the neutron energy range of 1 MeV to 4 MeV is 7.7 b, while the fission cross section in this energy range is around 1.8 b. The capture cross section decreases sharply with neutron energy in the region between 1 MeV to 4 MeV, with the values being 0.05 b (1 MeV) and 0.001 b (4 MeV).

Table 2 shows the cross sections for various neutron reactions for Pu-239 for different energy intervals of relevance to fast reactor.

### Table 2: Cross sections for Pu-239

<table>
<thead>
<tr>
<th>Energy interval</th>
<th>Capture cross section (b)</th>
<th>Fission cross section (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 2.2 MeV</td>
<td>0.01</td>
<td>1.85</td>
</tr>
<tr>
<td>0.82 – 2.2 MeV</td>
<td>0.03</td>
<td>1.82</td>
</tr>
<tr>
<td>300 – 820 keV</td>
<td>0.11</td>
<td>1.60</td>
</tr>
<tr>
<td>110 – 300 keV</td>
<td>0.20</td>
<td>1.51</td>
</tr>
<tr>
<td>40 – 110 keV</td>
<td>0.35</td>
<td>1.60</td>
</tr>
</tbody>
</table>
15 – 40 keV | 0.59 | 1.67
0.75 – 15 keV | 1.98 | 2.78

### 1.3 Cross sections for U-238

In the thermal region (<1 eV), the cross sections (total and fission) of U-235 and Pu-239 decrease with neutron energy as:

\[ \sigma \alpha \frac{1}{\sqrt{E}} \]  

The capture cross section for U-238 in the thermal region scales with neutron energy in accordance with Equation 5. The cross sections for various neutron reactions for U-238 for different energy intervals of relevance to fast reactor is shown in Table 3.

<table>
<thead>
<tr>
<th>Energy interval</th>
<th>Capture cross section (b)</th>
<th>Fission cross section (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 2.2 MeV</td>
<td>0.01</td>
<td>0.58</td>
</tr>
<tr>
<td>0.82 – 2.2 MeV</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>300 – 820 keV</td>
<td>0.11</td>
<td>-</td>
</tr>
<tr>
<td>110 – 300 keV</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td>40 – 110 keV</td>
<td>0.26</td>
<td>-</td>
</tr>
<tr>
<td>15 – 40 keV</td>
<td>0.47</td>
<td>-</td>
</tr>
<tr>
<td>0.75 – 15 keV</td>
<td>0.84</td>
<td>-</td>
</tr>
</tbody>
</table>

From the above discussion, it is clear that the cross section and neutron flux vary with the neutron energies. Hence the average cross section for a neutron reaction (absorption or fission) must account for the variation of cross section and neutron flux with neutron energy, as shown in Equation (6).

\[ \sigma = \frac{\int \sigma(E)\phi(E)dE}{\int \phi(E)dE} \]  

The following example illustrates the use of Equation (6) for the calculation of average fission cross section for Pu-239.
In the above table, Eq. (4) is re-written as:

$$\phi(E) = C \exp\left(\frac{-E}{kT_m}\right) = C^* (\sqrt{E}) \exp (-E)$$

(7)

where $C^*$ is a lumped parameter which is constant for a given isotope

$$C^* = \frac{C}{(kT_m)^3} \exp \left(\frac{1}{kT_m}\right)$$

(8)

The average fission cross section is

$$\sigma = \frac{\int \sigma(E) \phi(E) dE}{\int \phi(E) dE} = \frac{\sum \sigma(E) \phi(E) dE}{\sum \phi(E) dE} = 1.77 \text{ b}$$

Note: We do not need the value of ‘C’ or ‘$kT_m$’ for calculation of average cross section

2 References/Additional Reading
1. A.E. Waltar, D.R. Todd, P.V. Tsvetkov (Eds.), “Fast Spectrum Reactors”, Springer, 2012 (Chapter 1)
3. http://www.matpack.de/Info/Nuclear/Nuclids/P/Pu239.html