Chapter 3

1. What is the difference between symmetric and asymmetric membrane?
   
   Ans: In symmetric membrane, the material is same with uniform porosity. In asymmetric membrane, a thin skin of lower porosity over a symmetric support acts as a membrane.

2. What is the relationship of osmotic pressure with concentration and molecular weights?
   
   Ans: Osmotic pressure decreases with molecular weight and increases with concentration.

3. What is the relationship between observed and real retention?
   
   Ans: Real retention > observed retention.

4. Consider filtration of 5 kg/m³ concentration of ultrafiltration is gel layer controlled with gel concentration of 150 kg/m³. Filtration occurs in a tube of diameter 25 mm and length 1m. The flow rate is 150 L/h, protein diffusivity is 3x10⁻¹¹ m²/s. Find the permeate flux?
   
   Ans: 
   
   \[
   J = \text{Permeate flux} = k \ln \frac{c_g}{c_0} \\
   = k \ln \frac{150}{5} = 3.4k
   \]

   Estimation of k:

   For tubes, 
   \[
   Sh = \frac{kd}{D} = 1.62 \left( \frac{ReSc}{d} \right) ^{\frac{1}{3}}
   \]

   \[
   k = 1.62 \left( \frac{D^3 u_0 d r}{r d^3 r D L} \right) ^{\frac{1}{3}}
   \]
\[ Q = 150 \text{L/h} = \frac{150 \times 10^{-3}}{3600} \text{m}^3/\text{s} = 4.17 \times 10^{-5} \text{m}^3/\text{s} \]

\[ u_0 = \frac{Q}{A} = \frac{4.17 \times 10^{-5}}{\pi (25 \times 10^{-3})^2 / 4} = 0.085 \text{m/s} \]

\[ k = 1.62 \left( \frac{0.085 \times 9 \times 10^{-22}}{25 \times 10^{-3} \times 1} \right)^{\frac{1}{3}} = 2.35 \times 10^{-7} \text{m/s} \]

\[ J = 2.35 \times 10^{-7} \times 3.4 = 8 \times 10^{-7} \text{m}^3/\text{m}^2.\text{s} \]

5. Consider reverse osmosis of salt solution at a concentration of 10 kg/m\(^3\), in a thin channel of length 2 m and equivalent diameter of 2 mm. The membrane permeability is 2 \times 10^{-12} \text{m}^3/(\text{N}\cdot\text{s}). The real retention is 0.95. \( \Delta P = 2500 \text{kPa} \) and \( u_0 = 1.0 \text{ m/s} \). The osmotic pressure of salt is \( \Pi = 85000c \) (\( c \) is in kg/m\(^3\)) and salt diffusivity is 1.5 \times 10^{-9} \text{m}^2/\text{s}. Find the permeate flux and concentration assuming a film theory?

Ans: \( c_0 = 10\text{kg/m}^3; \quad L_p = 2 \times 10^{-12} \text{m}^3/\text{N}\cdot\text{s}; \quad L = 2\text{m}; \quad d_e = 2\text{mm}; \quad R_r = 0.95; \quad \Pi = 85000\text{C}; \quad D = 1.5 \times 10^{-9} \text{m}^2/\text{s}; \quad u_0 = 1\text{m/s} \)

Mass transfer coefficient:

\[ Sh = \frac{k d_e}{D} = 1.85 \left( \text{Re} \cdot \text{Sc} \cdot \frac{d_e}{L} \right)^{\frac{1}{3}} \]

\[ \text{Re} = \frac{\rho u_0 d_e}{\mu} = 10^6 \times 1 \times 2 \times 10^{-3} = 2000 \]
So laminar flow

\[ k = 1.85 \left( \frac{u_0 D^2}{d_c L} \right)^{1/3} \]

\[ = 1.85 \left( \frac{1 \times 1.25 \times 10^{-18}}{2 \times 10^{-3} \times 2} \right)^{1/3} \]

\[ = 1.53 \times 10^{-5} \text{ m/s} \]

Film Theory:

\[ j = k \ln \left( \frac{c_m - c_p}{c_0 - c_p} \right) \]

\[ C_p = C_0 (1 - R_m) = 0.05 \times 10 = 0.5 \text{ kg/m}^3 \]

\[ J = 0.53 \times 10^{-5} \ln \left( \frac{C_m - 0.5}{9.5} \right) \]

Osmotic Pressure Model

\[ J = L_P (\Delta P - \Delta \pi) = L_P (\Delta P - aC_m R_m) \]

\[ \therefore J = 2 \times 10^{-12} (2500 \times 10^3 - 85000 \times 0.95 c_m) \]

\[ J = 2 \times 10^{-12} (2.5 \times 10^6 - 8 \times 10^4 c_m) \]

\[ \therefore J = 5 \times 10^{-6} (1 - 0.032 c_m) \ldots \ldots \ldots (2) \]

From equations (1) and (2),
\[ 1.53 \times 10^{-5} \ln \left( \frac{c_m - 0.5}{9.5} \right) = 5 \times 10^{-6} (1 - 0.032c_m) \]

\[ 3 \ln \left( \frac{c_m - 0.5}{9.5} \right) = (1 - 0.032c_m) \]

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<td>RHS</td>
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\[ \therefore C_m = 12.2 \text{ kg/m}^3 \]

\[ C_p = 0.05 \times 12.2 = 0.61 \text{ kg/m}^3 \]

\[ J = 3.05 \times 10^{-6} \text{ m}^3/\text{m}^2\cdot\text{s} \]