MODULE 2: DIFFUSION

LECTURE NO. 3

2.3 Diffusion through variable cross-sectional area

2.3.1 Diffusion through a conduit of non-uniform cross-sectional area

Consider a component A is diffusing at steady state through a circular conduit which is tapered uniformly as shown in Figure 2.3. At point 1 the radius is \( r_1 \) and at point 2 it is \( r_2 \). At position Z in the conduit, A is diffusing through stagnant, non-diffusing B.

At position Z the flux can be written as

\[
N_A = \frac{\overline{N}_A}{\pi r^2} = -D_{AB} \frac{dP_A}{RT} \frac{dP_A}{1-P_A/P} dz
\]  

(2.23)

Using the geometry as shown, the variable radius \( r \) can be related to position \( z \) in the path as follows:
\[ r = \left( \frac{r_2 - r_1}{z_2 - z_1} \right) z + r_1 \]  

(2.24)

The Equation (2.24) is then substituted in the flux Equation to eliminate \( r \) and then the Equation is integrated and obtained as:

\[
\frac{N_A}{\pi} \int_{z_1}^{z_2} \left( \frac{r_2 - r_1}{z_2 - z_1} \right) z + r_1 \, dz = -\frac{D_{AB}}{RT} \frac{P_A}{P_{A1}} \int_{p_{A1}}^{P} \frac{dP}{1 - P/P}
\]

Or

\[
\frac{N_A}{4\pi} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{D_{AB} P}{RT} \ln \frac{P - P_{A2}}{P - P_{A1}}
\]

(2.25)

### 2.3.2 Evaporation of water from metal tube

Suppose water in the bottom of a narrow metal tube is held at constant temperature \( T \). The total pressure of air (assumed dry) is \( P \) and the temperature is \( T \). Water evaporates and diffuses through the air. At a given time \( t \), the level is \( Z \) meter from the top as shown in Figure 2.4. As diffusion proceeds, the level drops slowly. At any time \( t \), the steady state Equation holds, but the path length is \( Z \).

![Figure 2.4: Schematic of evaporation in metal tube](image-url)
Thus the steady state Equation becomes as follows where $N_A$ and $Z$ are variables:

$$N_A = \frac{D_{AB} P}{RT Z P_{BM}} (P_A - P_{A_0})$$  \hspace{1cm} (2.26)

Where

$$P_{BM} = \frac{P_{B_1} - P_{B_2}}{\ln(P_{B_1} / P_{B_2})} = \frac{P_A - P_{A_0}}{\ln[(P - P_{A_0})/(P - P_{A_0})]}$$  \hspace{1cm} (2.27)

Assuming a cross sectional area of 1 m$^2$, the level drops $dZ$ meter in $dt$ sec, and $\rho_A(dZ.1)/M_A$ is the kmol of A that has been left and diffused. Then

$$N_A.1 = \frac{\rho_A(dZ.1)}{M_A dt}$$  \hspace{1cm} (2.28)

Substituting the Equation (2.28) in Equation (2.26) and integrating, one gets

$$\frac{\rho_A}{M_A} \int_{Z_0}^{Z_f} Z dZ = \frac{D_{AB} P (P_A - P_{A_0})}{RT P_{BM}} \int_0^1 dt$$  \hspace{1cm} (2.29)

$$t_f = \frac{\rho_A (Z_F^2 - Z_0^2) RT P_{BM}}{2M_A D_{AB} P (P_A - P_{A_0})}$$  \hspace{1cm} (2.30)

The Equation (2.30) represents the time $t_f$ for the level to drop from a starting point of $Z_0$ meter at $t = 0$ to $Z_F$ at $t = t_f$.

### 2.3.3 Diffusion from a sphere

There are lots of examples where diffusion can take place through the spherical shape bodies. Some examples are:

- Evaporation of a drop of liquid
- The evaporation of a ball of naphthalene
- The diffusion of nutrients to a sphere-like microorganism in a liquid

Assume a constant number of moles $\tilde{N}_A$ of A from a sphere (area = $4\pi r^2$) through stagnant B as shown in Figure 2.5.
From the Fick's law of diffusion, the rate of diffusion can be expressed as:

\[ N_A \left( 1 - \frac{P_A}{P_{total}} \right) = - \frac{D_{AB}}{RT} \frac{dP_A}{dr} \]  

\[ (2.31) \]

where \( N_A = \frac{\bar{N}_A}{4\pi r^2} \)  

\[ (2.32) \]

\[ \frac{-RT \bar{N}_A}{4\pi D_{AB}} \frac{dr}{r^2} = P_{total} \frac{dP_A}{(P_{total} - P_A)} \]  

\[ (2.33) \]

Integrating with limits of \( P_A \) at \( r_2 \) and \( P_A \) at \( r_1 \) gives:

\[ \frac{-RT \bar{N}_A}{4\pi D_{AB}} P_{total} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) = \ln \left( \frac{P_{total} - P_{A2}}{P_{total} - P_{A1}} \right) \]  

\[ (2.34) \]

As \( r_1 \ll r_2 \), then \( 1/r_2 \approx 0 \):

\[ \frac{\bar{N}_A}{4\pi r_1^2} = \frac{D_{AB} P_{total} (P_{A1} - P_{A2})}{RTP_{BM} r_1} = N_{A1}, \text{ the flux at the surface} \]  

\[ (2.35) \]

This Equation can be simplified if \( P_{A1} \) is small compared to \( P \) (a dilute gas phase), \( P_{BM} \approx P \). Also setting \( 2r_1 = D_1 \), diameter, \( C_{A1} = P_{A1}/RT \)

\[ N_{A1} = \frac{D_{AB}}{D_1} (C_{A1} - C_{A2}) \]  

\[ (2.36) \]