Outline

- Principle of Missile Guidance

- Optimal Missile Guidance using MPSP
  - Ballistic Missile Guidance
  - Tactical Missile Guidance with Impact Angle Constraint

- A quick glimpse of other problems solved
Fundamental Problem of Strategic Missile Guidance

Guidance phase

Fundamental Problem of Tactical Missile Guidance

PN Guidance: $a_M = N V_M \dot{\lambda}$
Missile Guidance Laws

- Many classical missile guidance laws are inspired from “observing nature” (e.g. Proportional Navigation (PN) guidance law is based on ensuring “collision triangle”)
- Control theoretic based guidance laws are usually based on “kinematics” and/or “linearized dynamics”. Hence, they are usually not very effective!
- Nonlinear optimal control theory is a “natural tool” to obtain effective missile guidance laws

Point-mass Missile Model with Flat and Non-Rotating Earth

\[
\begin{align*}
\dot{x} &= V \cos \gamma \\
\dot{h} &= V \sin \gamma \\
\dot{V} &= \frac{1}{m} \left( T \cos \alpha - D - mg \sin \gamma \right) \\
\dot{\gamma} &= \frac{1}{mV} \left( T \sin \alpha + L - mg \cos \gamma \right)
\end{align*}
\]
Point-mass Missile Model with Spherical and Non-Rotating Earth

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\theta} &= \frac{V \cos \gamma}{r} \\
\dot{V} &= \frac{1}{m} \left( T \cos \alpha - D - mg \sin \gamma \right) \\
\dot{\gamma} &= \frac{1}{mV} \left( T \sin \alpha + L - mg \cos \gamma + \frac{mV^2}{R} \right), \quad R = \frac{r}{\cos \gamma}
\end{align*}
\]

MPSP for Ballistic Missile Guidance
(with solid motors)

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Introduction:
Liquid Engines vs. Solid Motors

<table>
<thead>
<tr>
<th>Liquid Engines</th>
<th>Solid Motors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quick firing is not possible</td>
<td>Quick firing is possible!</td>
</tr>
<tr>
<td>Sloshing and TWD effect</td>
<td>No sloshing and TWD effects</td>
</tr>
<tr>
<td>Higher cost</td>
<td>Lower cost</td>
</tr>
<tr>
<td>Thrust cut-off facility</td>
<td>No thrust cut-off facility</td>
</tr>
<tr>
<td>Burnout time is certain</td>
<td>Burnout time is uncertain</td>
</tr>
<tr>
<td>Manipulative T-t curve</td>
<td>Non-manipulative T-t curve</td>
</tr>
<tr>
<td>Guidance is easier</td>
<td>Guidance is difficult..!</td>
</tr>
</tbody>
</table>

System Dynamics

\[
\begin{align*}
\dot{r} & = V \sin \gamma \\
\dot{V} & = \frac{T}{M} \cos \delta - g \sin \gamma \\
\dot{\gamma} & = -\frac{T}{MV} \sin \delta + \left(\frac{V}{r} - \frac{g}{V}\right) \cos \gamma \\
\dot{\phi} & = -\frac{V \cos \gamma}{r} \\
\end{align*}
\]

- \( r \) : Local radius
- \( V \) : Velocity
- \( \gamma \) : Flt. path angle
- \( \phi \) : Range angle
- \( \delta \) : Shear angle (guidance parameter)
Free-Flight and Hit Equation

**FF Equation**

\[
\frac{r_{bo}}{r} = \frac{1 - \cos\theta}{\lambda_{ho} \cos^2\gamma_{ho}} + \frac{\cos(\theta + \gamma_{ho})}{\cos\gamma_{ho}}
\]

**Hit Equation**

\[
\frac{r_{bo}}{r_f} = \frac{1 - \cos\phi_{ho}}{\lambda_{ho} \cos^2\gamma_{ho}} + \frac{\cos(\phi_{ho} + \gamma_{ho})}{\cos\gamma_{ho}}
\]

\[
\lambda_{ho} = \frac{r_{bo} V_{bo}^2}{GM}
\]

Reference:

Guidance Design Using MPSP:
Problem Specific Equations

Discretize System Dynamics:

\[
X_{k+1} = X_k + dt \begin{bmatrix}
V_k \sin\gamma_k \\
-\frac{F_{\gamma}}{M} \cos\theta_k - g \sin\gamma_k \\
-\frac{F_y}{M} \sin\gamma_k + \left(\frac{F_{z}}{M} - \frac{F_{\gamma}}{M}\right) \cos\gamma_k
\end{bmatrix}
\]

Discretize Output Equation:

\[
Y_N = \frac{r_N^2 V_{bo}^2 \cos^2\gamma_N}{GM(1 - \cos\phi_N) + r_N V_{bo}^2 \cos(\theta_N + \gamma_N) \cos(\gamma_N)}
\]
**Guidance Design:**
Some implementation issues

- Discretization of state equation for control computation: (Euler method: Faster computation)
- Discretization of state equation for Simulation studies: (RK-4 method: Higher accuracy)
- Step size (update interval): $\Delta t = 100 \text{ sec}$
- GLC: 1 sec after burnout of first stage
- All derivatives were carried out *symbolically*
- Iteration unfolding has been implemented

**Numerical Result:**
With Nominal Thrust

![Graphs showing shear angle and range over time for different ranges.](image)
Numerical Result:
With Uncertainties in Motor Performance

![Graph showing thrust variations](image)

![Graph showing range and height trajectories](image)
Control Computation Time

- PC Used
  - Pentium 4 (2.4 GHz), 512 MB RAM
- Programming platform used
  - MATLAB 7.0
- First control update: 1.755 sec
  - Further control updates require lesser time
- Code in low-level language will require much lesser computational time
  - Suitable for online implementation

Comparison Between MPSP and Nonlinear Programming

<table>
<thead>
<tr>
<th></th>
<th>MPSP</th>
<th>NLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td></td>
<td>1.6689</td>
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</tbody>
</table>

Cost function computation

$$J_{\text{NLP}} = 1.6843$$

$$J_{\text{MPSP}} = 1.6689$$
A Hybrid Design For Energy-Insensitive Guidance

- Step 1: Assume that the motor is guaranteed to burn up to a certain duration of predicted burnout time (say 90%). Design the MPSP Guidance.
- Step 2: Switch over to Dynamic inversion guidance, which assures that the free flight equation is satisfied for the remaining time continuously.
- Motivation: To eliminate VTP requirement.

Numerical Results:
MPSP + Dynamic Inversion

Height Error at the Target

![Graph showing height error at the target over time for different ranges.](image-url)
Computational Time

- Machine specification
  - Pentium 4 (3 GHz), 512 MB RAM (future on-board computers are expected to have better configurations)

- Language used
  - MATLAB 7.0 (a very high-level computationally inefficient language)

- First control update: 0.47 sec (averaged over ten simulations, which had very small variation)

- Code in ‘C’ language will require much lesser computational time (suitable for “real time” applications!)

Conclusions:
Ballistic Missile Guidance Problem

- The new MPSP guidance algorithm successfully works
  - Tested for various ranges (2000 km - 4500 km)
- It leads to high accuracy (±1 m height error)
  - With under/over performance of solid motors
- Computationally efficient algorithm
  - Can be implemented online in future
- Comparison with optimal control formulation (NLP solution) shows close match
Conclusions:
Strategic Missile Guidance Problem

- A new energy-insensitive guidance algorithm is designed by blending the MPSP technique with dynamic inversion.
- The composite guidance algorithm meets the specification and requirements of an energy insensitivity design.
- The composite guidance design can lead to elimination of Velocity Trimming package (VTP) requirement. However, it works only for “shallow trajectories”.
- The concept is equally valid for launch vehicle guidance.

MPSP for Tactical Missile Guidance with Impact Angle Constraint

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Why Impact Angle Constrained Guidance?

- Enhancement of warhead lethality (e.g. front-attack)
- Terminal trajectory shaping for attacking weak locations of targets (e.g. top attacks for tanks)
- Mission demand (e.g. bunker buster mission, terrorist hideouts in urban areas, water reservoir attack etc.)
- Coordinated attack by multiple munitions
- Countermeasure by enhanced stealthiness
- Range enhancement (indirectly)
- Increase in observability of the target

Existing Methods of 3D Guidance

Literature:

- 3D PN and Augmented PN Laws
- Nonlinear 3D guidance for guaranteed capture
- Sliding Mode Control based 3D impact angular guidance

Observations:

- Fewer references for 3D impact angle constraints
- Existing solutions render laws which are either non-optimal or based on kinematics only
- Latest “guaranteed 3D capture guidance” does not consider impact angular constraints.
**Motivations**

- PN guidance laws are usually adequate to assure small miss distance, but are silent on impact angle.
- Optimal control theory based techniques are available, but they rely on “linearized kinematics”…need to do better than that!
- The aim here is to develop a nonlinear optimal guidance law with 3-D impact angle constraints, using **nonlinear point-mass dynamic models**.

**Challenges**

- Strong (nonlinear) coupling between elevation angle and azimuth angle dynamics should be accounted for.
- Zero/Near-zero miss distance requirement cannot be compromised.
- Impact angle constraints in 3D (i.e. two angle constraints at the same time) must be ensured.
- Latax demand has to be as minimum as possible.
Problem Objectives

To design a 3-D optimal guidance law for maneuvering, moving and stationary targets,

(i) with thrust force (with autopilot delay)
(ii) without thrust force (with autopilot delay)

Aim: To obtain negligible miss distance as well as the desired azimuth and elevation impact angles simultaneously!
### Missile and Target Dynamics

**Missile Model**

\[
\begin{align*}
\dot{V}_m &= \frac{[T_m - D_m]}{m_m} - g \sin(\gamma_m) \\
\dot{\gamma}_m &= -\frac{a_z - g \cos(\gamma_m)}{V_m} \\
\psi_m &= \frac{a_y}{V_m \cos(\gamma_m)} \\
\dot{x}_m &= V_m \cos(\gamma_m) \cos(\psi_m) \\
\dot{y}_m &= V_m \cos(\gamma_m) \sin(\psi_m) \\
\dot{z}_m &= V_m \sin(\gamma_m)
\end{align*}
\]

**Target Model**

\[
\begin{align*}
\psi_i &= \frac{a_z}{m_m} \\
\dot{x}_i &= V_i \cos(\psi_i) \\
\dot{y}_i &= V_i \sin(\psi_i)
\end{align*}
\]

**State**

\[
X = [V_m \quad \gamma_m \quad \psi_m \quad x_m \quad y_m \quad z_m]^T
\]

**Control**

\[
U = [a_z \quad a_y]^T
\]

### Assumptions about target

- Speed \( V_i \) is constant
- Target moves with a latax command \( a_{yt} \) normal to its velocity \( V_t \), which is known.
- Coordinates of target in inertial frame \((x, y, z)\) are available
- \( a_{yt} \) can be:
  - Zero (straight line movement)
  - Constant (constant \( g \) maneuvers)
  - Sinusoidal (periodic maneuvers)
Normalized Dynamics

- **Missile Model**
  \[
  V_{n} = \frac{\left[ F_{n} - D_{n} \right]}{m_{n}V_{n}} - g \sin(\psi_{n}^{*}y_{n}^{*}) \\
  \dot{V}_{n} = -\alpha_{n}g - g \cos(\psi_{n}^{*}y_{n}^{*})V_{n}V_{n}'y_{n}^{*} \\
  \dot{\psi}_{n} = \frac{\alpha_{n}g}{V_{n}V_{n}'\cos(\psi_{n}^{*}y_{n}^{*})} \\
  \dot{x}_{n} = \frac{V_{n}V_{n}'\cos(\psi_{n}^{*}y_{n}^{*})\cos(\psi_{n}^{*}y_{n}^{*})}{x_{n}^{*}} \\
  \dot{y}_{n} = \frac{V_{n}V_{n}'\cos(\psi_{n}^{*}y_{n}^{*})\sin(\psi_{n}^{*}y_{n}^{*})}{y_{n}^{*}} \\
  \dot{z}_{n} = \frac{V_{n}V_{n}'\sin(\psi_{n}^{*}y_{n}^{*})}{z_{n}^{*}}
  \]

- **Target Model**
  \[
  \psi_{0} = \frac{a_{n}g}{V_{n}V_{n}'\psi_{n}^{*}} \\
  \dot{\psi}_{0} = \frac{V_{n}V_{n}'\sin(\psi_{n}^{*}y_{n}^{*})}{y_{n}^{*}}
  \]

- **Subscript** \( n \):
  Normalized value

- **Superscript** \( * \):
  Normalizing value

MPSP Guidance: Discretization and Output Selection

\[
X_{n_{k+1}} = F_{k}(X_{n_{k}}, U_{n_{k}}) = X_{n_{k}} + \Delta t f_{k}(X_{n_{k}}, U_{n_{k}}) \\
\frac{\partial X_{n_{k+1}}}{\partial X_{n_{k}}} = \frac{\partial F_{k}}{\partial X_{n_{k}}} = I_{6 \times 6} + \Delta t \frac{\partial f_{k}}{\partial X_{n_{k}}} \\
Y_{N} = \begin{bmatrix} \gamma_{m_{N}} & \psi_{m_{N}} & x_{m_{N}} & y_{m_{N}} & z_{m_{N}} \end{bmatrix}^{T} \\
Y_{N}^{*} = \begin{bmatrix} \gamma_{m_{f}} & \psi_{m_{f}} & x_{i_{N}} & y_{i_{N}} & 0 \end{bmatrix}^{T} \\
Y_{N} \to Y_{N}^{*}
\]

Subscript \( f \): Terminal impact angles
Stationary Targets
MPSP Vs. APN: A Comparison

\[ \gamma_{m_f} = -90^\circ \]
\[ \gamma_{m_b} = 10^\circ \]
\[ \psi_{m_h} = 20^\circ \]

Constraint in single angle
Stationary Targets: Same initial conditions & Different Terminal Constraints

Missile Trajectories with terminal constraints in $\gamma_m$ & $\psi_m$

- $\gamma_m = 10^\circ$
- $\psi_m = 10^\circ$

Various constraints in both angles

Stationary Targets: Different initial conditions for Same Terminal Constraints

3D Trajectories with Different Initial Conditions

- $\gamma_m = -20^\circ$
- $\psi_m = 20^\circ$

Initial condition perturbation with same terminal constraint
Stationary Targets:
Perturbation in initial conditions (Angle Histories)

Zero Effort Miss (ZEM) Plot
(Sinusoidal Maneuver)
Missile for missile (air) defense: A typical engagement scenario

Challenges

- Very high speed targets
  - Very less engagement time
  - Very high line-of-sight rate
- Zero/Near-zero miss distance is desired
- Impact/Aspect angle constraint
- Directional warhead
- Latax saturation (due to less dynamic pressure) should be avoided
Philosophy of Partial Integrated Guidance & Control (PIGC)

Three Loop Conventional Design  Partial IGC Design

- Exploits the inherent time scale separation property between faster rotational and slower translational dynamics
- Operates in a Two Loop Structure:
  - Commanded body rates generated in outer loop – MPSP
  - Commanded deflections generated in inner loop – Dyn. Inv.

Conclusions

- MPSP technique is very promising for optimal missile guidance (trajectory optimization philosophy is brought into guidance design).
- Various challenging strategic/tactical missile guidance problems have been (and are being) solved.
- MPSP has also been successfully demonstrated for Re-entry guidance of a Re-usable Launch Vehicle.
- An important extension of the MPSP is the MPSC design with control parameterization. It has additional desirable characteristics like control smoothness, faster computation over MPSP etc.
- MPSP has found good world-wide acceptance.
References: Journal Publications


References: Journal Publications

Thanks for the Attention....!!

questions - ??