Module 3: Lecture 8
Standard Terminologies in Missile Guidance

Keywords. Latax, Line-of-Sight (LOS), Miss-Distance, Time-to-Go, Fire-and-Forget, Glint Noise, Collision Triangle

4.3 Some Standard Terminologies in Missile Guidance

One often comes across a number of standard terms in the literature on missile guidance. In this section we will define many of these terms precisely.

Lateral Acceleration: This is also known as latax. This is the acceleration that the missile needs to apply in order to achieve a desired turn rate. It is called lateral acceleration since it is usually applied in a direction close to the normal to the missile longitudinal axis or the missile velocity vector. In fact, the guidance command generated by the guidance law is usually expressed as a lateral acceleration term. This is called the commanded latax and is given as an input to the lateral autopilots. Since the autopilots are essentially dynamical systems subject to time delay, the actual achieved latax is a time-varying quantity and is different from the commanded latax at any moment in time. This difference may also occur due to the saturation effect since the missile may not be able to pull a very high commanded latax.

Line-of-Sight (LOS) Rate: During a missile-target engagement the imaginary line joining the missile and the target at any given instant in time is called the instantaneous line-of-sight or the LOS. This line changes in length and orientation as the engagement proceeds. The change in its angular orientation is given by its angular velocity or rate of turn and is usually expressed in units of radians/sec. This is called the LOS rate.

Closing Velocity: This is the velocity with which the missile closes on to the target. Obviously this is given by the rate at which the length of the LOS or the LOS separation shrinks. Hence, it is the negative of the rate of change of the LOS separation or range.
Miss-Distance: It is also the doppler relative velocity of the target with respect to the missile along the line-of-sight. Note that the doppler relative velocity is positive when the target is approaching and negative when it is receding.

Miss Distance: This is the distance of closest approach of the missile to the target. When the missile directly hits the target, the miss distance is zero. But when the missile passes close to the target the miss distance is non-zero. In this case the proximity fuze detonates the warhead and the engagement comes to an end. Obviously, the primary objective of a guidance system is to minimize the miss-distance. Also note that the miss-distance is a non-negative quantity. Consider Figure 4.7 which shows the trajectories of a missile and a target, and also the miss-distance. It also shows the LOS at different instants in time, including the LOS at the instant of closest approach. The length of the LOS at this instant gives the miss-distance. Now, what distinguishes this LOS from all the other LOSs? It can be easily deduced that the closing velocity at this instant is zero. The closing velocity before this instant is positive and after this instant is negative.

Time-to-Go: The time-to-go is an important trajectory parameter which is used for the
implementation of many advanced guidance laws. Suppose we record the trajectory data of a missile-target engagement and find that the engagement ends with an interception at time $t_f$ (final time). Note that interception is assumed to have taken place when either the missile directly hits the target or at the time of closest approach. Now, at any given instant in time $t$ during the engagement the time-to-go is defined as the time remaining till interception and is given by $(t_f - t)$. It is usually denoted as $t_{go}$. This value is the actual time-to-go which is known only after the engagement is over. But, to implement the guidance law, we need to estimate the $t_{go}$ during the engagement. There are many ways by which this can be done. One of the ways, based on the available instantaneous information, is to use the formula,

$$\hat{t}_{go} = \frac{R}{(-R)}$$

(4.1)

where, $\hat{t}_{go}$ is the estimated time-to-go and $R$ is the LOS separation or the distance between the target and the missile at that instant in time. This is not a very accurate method of estimating the $t_{go}$, but for some limited cases it is satisfactory. For instance, it is accurate when we consider the collision triangle. There are other more accurate, but complicated, ways of finding the $t_{go}$ also.

**Blind Zone:** In a homing guidance system the seeker has to keep pointing towards the target in order to track it. However, during the last part of the terminal phase the missile could be pointing in such a direction that the seeker has to turn by a very large angle to keep the target within its field of view. However, seeker turn angle is subject to mechanical limitations. Hence, it may not be possible for the seeker to turn by such a large angle. In this case the seeker loses track of the target and cannot see it any more. This part of the missile trajectory is called the *blind zone* for the missile. This is shown in Figure 4.8. There is no information input from the seeker during this phase and the guidance system has to depend on previous information.

**LOBL - Lock On Before Launch:** In this mode the launch platform radar performs the search and acquisition functions and sends target position information to the missile seeker, directing it to lock on to the target before the missile is launched. However, this is normally an impractical procedure because of the following factors:
The missile seeker may suffer interference due to the signals emanating from the launch platform radar.

It is difficult for the missile guidance system to track the target when the missile is experiencing a high acceleration during the boost phase that occurs immediately after the launch.

The target may not be in the field-of-view of the missile seeker before launch.

**LOAL - Lock On After Launch:** This is a more complex procedure than LOBL. The missile has to be provided with the target information to enable the missile seeker to acquire the target. Even then the missile must go through the process of search and acquisition.

**Fire-and-Forget or Launch-and-Leave:** This refers to those missiles that have the capability to reach their targets after launch in the absence of any support from the launch platform or the operator.

**Glint Noise:** A target such as an aircraft has many radar reflector surfaces. The net return from these surfaces can be modelled as a movement of the apparent radar
position and is typically modelled as a Gaussian random variable with zero mean and some non-zero variance. This is called *glint noise*.

**Radome Error:** The missile seeker is used to track the target by processing the reflected signal from it. The radome that covers the missile seeker (in a homing guided missile) causes deterioration to the angle of the incoming signal due to refraction (in fact, due to this the radome has to be designed not only from the aerodynamic point of view but also from the viewpoint of electromagnetic considerations). Because of this the missile seeker actually tracks a *virtual target* whose position is shifted from the actual position of the target. This can be seen from Figure 4.9.

**Heading Error:** The heading error is the difference in angle between the actual missile velocity vector and the angle required by the missile velocity vector to satisfy the collision geometry conditions. This parameter is an important performance measure for missiles that follow the mixed guidance scheme and have to transit from a command phase to a homing phase.

**Factors affecting miss-distance:** There are many factors that affect the performance of the missile and its guidance law in terms of miss-distance. The main among these are

- Target maneuver
These factors affect miss-distance differently depending on the altitude and range of the target. This is shown in Figure 4.10.

4.4 The Kinematic Equations

The kinematic equations for the missile-target engagement, assuming point mass models for the missile and the target, are given below with reference to Figure 4.11.

\[ V_R = \dot{R} = V_T \cos(\alpha_T - \theta) - V_M \cos(\alpha_M - \theta) \]  \hspace{1cm} (4.2)

\[ V_\theta = \dot{\theta} = V_T \sin(\alpha_T - \theta) - V_M \sin(\alpha_M - \theta) \]  \hspace{1cm} (4.3)
Here, $V_T$ and $V_M$ are the target and missile velocities, and $R$ is the distance from the missile to the target (LOS separation). Usually $V_M > V_T$. The quantities $V_R$ and $V_\theta$ are very important for us – they are the components of the relative velocity (of the target with respect to the missile) along the LOS and normal to the LOS, respectively. We shall have occasion to use these quantities quite often in the later part of our lectures. The missile employs a lateral acceleration $a_M$ to turn the missile in an appropriate direction. Note that $\dot{\theta}$ is the LOS rate and $\dot{R}$ is the rate of change of the LOS separation. Also, the closing velocity $V_c$ is given by,

$$V_c = -\dot{R}$$ (4.4)

Equations (4.2) and (4.3) do not form the complete set of kinematic equations. The complete set will consist of equations modelling the variations in $\alpha_M$, $\alpha_T$, $\theta$, $V_M$, and $V_T$. Integrating these equations with respect to time from some given initial conditions will give the complete trajectory of this system of equations.

4.5 The Collision Geometry

The collision geometry or the collision triangle is the most fundamental concept in guidance law design. In this section, we will take a closer look at it and try to pin down what the collision triangle means in terms of some basic guidance parameters. See Figure 4.12 which shows the collision triangle.
Essentially the collision triangle defines the trajectories of the missile and the target when they culminate in an intercept, while both the vehicles are moving with constant speeds and in straight-line trajectories. This implies that the LOS does not rotate in space and so in (4.3) we have $V_\theta = 0$. Also, since the missile and target speeds and directions of flight are constant, in (4.2) $V_R$ remains a constant negative quantity. These two together ensure that the engagement ends up in a successful interception of the target by the missile. So, for the conditions of the collision geometry to be satisfied we must have

$$V_\theta = 0, \quad V_R < 0$$  \hspace{1cm} (4.5)

Note also that $V_\theta = 0$ implies that the component of the target velocity perpendicular to the LOS is equal to the component of the missile velocity perpendicular to the LOS.

### 4.6 Capturability

Capturability or interceptability is another fundamental notion in the performance evaluation of guidance laws. The primary objective of any missile is to intercept the target, all other performance measures are secondary to it. So it stands to reason that we
would like to identify all the initial conditions in the state space from which a missile can capture the target using a given guidance law. As we shall see in later chapters, the performance of guidance laws are limited by many factors. Because of this, a particular guidance law may be able to capture from a given set of initial conditions while it may fail to do so from another. A schematic representation of this is given in Figure 4.13.

The region in the state space or initial condition space, from which the missile is able to capture the target with a given guidance law (that is, the collection of those points in the initial condition space which satisfy the requirements of capturability) is called the capture region. The capture regions of two guidance laws may intersect, may form a subset or superset of each other, or may be totally disjoint. But based on the shape and size of the capture region we can derive many important observations regarding the suitability of the guidance law.

In our subsequent analysis we will try to express the capture region in the initial condition space of the relative velocities $V_R$ and $V_\theta$. We shall also show that this is the most natural representation of capturability, from the viewpoint of kinematics. I should also point out that the capture region we will obtain are those which will be based on purely theoretical considerations. Although they help us to draw many important and useful conclusions the actual capture region will be modified further (and will
be perhaps smaller) if we take into account other realistic factors like lateral saturation, constraints on fuel, and atmospheric conditions.

4.7 Concluding Remarks

In this chapter I have tried to introduce some of the fundamental concepts and terminologies used in the guidance literature. These will help us to go deeper into the design and actual working of several guidance laws and help us to understand them better.

Questions

In all the following questions, sketch a figure, if it helps you to explain the concept better.

1. Define commanded lateral acceleration and achieved lateral acceleration. Why are they different?
2. Define LOS angular rate. Write down the expression.
3. Define closing velocity. Write down the expression. When is the closing velocity positive and when is it negative?
4. What is miss distance?
5. What is time-to-go? How is it usually computed? When is this computation accurate?
6. What is blind zone and when does it occur?
7. Define (a) LOBL (b) LOAL (c) Fire-and-forget (d) Glint noise.
8. What are radome error and heading error?
9. What are the factors that affect miss distance? What values of altitude and range make these factors relatively more important?
10. Sketch a missile-target engagement geometry and write down the kinematic equations that govern the evolution of this system in time.
11. What is the relative velocity space?

12. What is a collision triangle and what are its features?

13. Define capturability in general and capturability in the relative velocity space.

References