Chapter 10
Performance analysis VI – Take-off and landing
(Lectures 32-34)

Keywords: Phases of take-off flight — take-off run, transition and climb; take-off distance; balanced field length; phase of landing flight; landing distance.

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Performance analysis VI – Take-off and landing –1

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10.1 Introduction

An airplane, by definition, is a fixed wing aircraft. Its wings can produce lift only when there is a relative velocity between the airplane and the air. In order to be airborne, the lift produced by the airplane must be at least equal to the weight of the airplane. This can happen when the velocity of the airplane is equal to or greater than its stalling speed. To achieve this velocity called ‘Take-off velocity ($V_{TO}$)’ the airplane accelerates along the runway. Thus, an airplane covers a certain distance before it can take-off. Similarly, when an airplane comes in to land, the lift produced must be nearly equal to the landing weight. Hence, the airplane has a velocity, called ‘Touch down speed ($V_{TD}$)’, when it touches the ground. It then covers a certain distance before coming to halt.

The estimation of take-off distance and landing distance are the topics covered in this chapter.
10.2. Definitions of take-off run and take off distance

The horizontal distance covered along the ground, from the start of take-off till the airplane is airborne is called the take-off run. However, to decide the length of the runway required for an airplane, it is important to ensure that the airplane is above a certain height before it leaves the airport environment. This height is called ‘Screen height’ and is equal to 15 m (sometimes 10 m), which is above the height of common obstacles like trees and electricity poles. The take-off distance is defined as the horizontal distance covered by an airplane from the start of the run till it climbs to a height equal to the screen height. It is assumed that the weight of the airplane during take-off is the gross weight for which it is designed and that the take-off takes place in still air.

10.3 Phases of take off flight

The take-off flight is generally divided into three phases namely (i) ground run (ii) transition (or flare) and (iii) climb (see Fig.10.1a).

10.3.1 Take-off ground run

During the ground run the airplane starts from rest and accelerates to the take-off speed \(V_{T0}\) or \(V_1\). The flaps and engine(s) are adjusted for their take-off...
settings. In the case of an airplane with tricycle type of landing gear, all the three wheels remain in contact with the ground till a speed of about 85% of the \( V_{T0} \) is reached. This speed is called ‘Nose wheel lift off speed’. At this speed the pilot pulls the stick back and increases the angle of attack of the airplane so as to attain a lift coefficient corresponding to take-off \( (C_{LT0}) \). At this stage, the nose wheel is off the ground (Fig.10.1b) and the speed of the airplane continues to increase. As the speed exceeds the take off speed the airplane gets airborne and the main landing gear wheels also leave the ground.

When the airplane has a tail wheel type of landing gear, the angle of attack is high at the beginning of the take-off run (Fig.10.1c). However, the tail wheel is lifted off the ground as soon as some speed is gained and the deflection of elevator can rotate the airplane about the main wheels (Fig.10.1d). This action reduces the angle of attack and consequently the drag of the airplane during most of the ground run. As the take-off speed is approached the tail wheel is lowered to get the incidence corresponding to \( C_{LT0} \). When \( V_{T0} \) is exceeded, the airplane gets airborne.

The point at which all the wheels have left the ground is called ‘Unstick point’ (Fig.10.1a).

![Fig.10.1b Nose wheel lift-off](image)
10.3.2 Transition and climb phases

During the transition phase the airplane moves along a curved path (Fig.10.1a) and the pilot tries to attain a steady climb. As soon as the airplane attains an altitude equal to the screen height, the take-off flight is complete. For airplanes with high thrust to weight ratio the screen height may be attained during the transition phase itself.
10.4 Estimation of take-off performance

From the point of view of performance analysis, the following two quantities are of interest.
(i) The take-off distance (s) (ii) The time (t) taken for it.

Since the equations of motion are different in the three phases of take-off flight, they (phases) are described separately in the subsequent subsections.

10.4.1 Distance covered and time taken during ground run

The forces acting on the airplane are shown in Fig.10.1a. It is observed that the ground reaction (R) and the rolling friction, \( \mu R \), are the two additional forces along with the lift, the drag, the weight and thrust ; \( \mu \) is the coefficient of rolling friction between the runway and the landing gear wheels. The equations of motion are :

\[
T - D - \mu R = \frac{W}{g} a \quad (10.1)
\]
\[
L + R - W = 0 \quad (10.2)
\]

Hence, \( R = W - L \) and
\[
a = \frac{T - D - \mu (W-L)}{W/g} \quad (10.3)
\]

Further,
\[
a = \frac{dV}{dt} = \frac{dV}{ds} \frac{ds}{dt} = V \frac{dV}{ds}
\]

Hence, ground run \( (s_1) \) is given by:
\[
s_1 = \int_0^V \frac{V}{a} \frac{dV}{a} = \frac{W}{g} \int_0^V \frac{dV}{T - D - \mu (W-L)} \quad (10.4)
\]

The time taken during ground run \( (t_1) \) is given by:
\[
t_1 = \int_0^V \frac{dV}{a} = \frac{W}{g} \int_0^V \frac{dV}{T - D - \mu (W-L)} \quad (10.5)
\]
Equations (10.4) and (10.5) can be integrated numerically, when the variations of \( T, D \) and \( L \) are prescribed and \( \mu \) is known. The value of \( \mu \) depends on the type of surface. Typical values are given in Table 10.1.

<table>
<thead>
<tr>
<th>Type of surface</th>
<th>Coefficient of rolling friction (( \mu ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete, wood or asphalt</td>
<td>0.02</td>
</tr>
<tr>
<td>Hard turf</td>
<td>0.04</td>
</tr>
<tr>
<td>Average field-short grass</td>
<td>0.05</td>
</tr>
<tr>
<td>Average field-long grass</td>
<td>0.1</td>
</tr>
<tr>
<td>Soft ground</td>
<td>0.1-0.3</td>
</tr>
</tbody>
</table>

Table 10.1 Coefficient of rolling friction

The thrust during take-off run can be approximated as \( T = A_1 - B_1 V^2 \). The angle of attack and hence, the lift coefficient \( (C_L) \) and the drag coefficient \( (C_D) \) can be assumed to remain constant during the take-off run. With these assumptions, the left-hand side of Eq.(10.1) becomes:

\[
T - D - \mu (W - L) = A_1 - B_1 V^2 - \mu W - \frac{1}{2} \rho V^2 S (C_D - \mu C_L)
\]

\[
= A - B V^2 \quad \text{where, } A = A_1 - \mu W \text{ and } B = B_1 + \frac{1}{2} \rho S (C_D - \mu C_L)
\]

Substituting in Eqs.(10.4) and (10.5) gives:

\[
s_1 = \frac{W}{g} \int_0^V \frac{V dV}{A - B V^2} = \frac{W}{2gB} \ln \left\{ \frac{1}{A/(A - B V^2)} \right\}
\]

(10.6)
and \( t_1 = \frac{W}{g} \int_{0}^{V_1} \frac{dV}{A-BV^2} = \frac{W}{2g\sqrt{AB}} \ln \left( \frac{\sqrt{A} + \sqrt{B}}{\sqrt{A} - \sqrt{B}} \right) \frac{V_1}{V_1} \) \hspace{1cm} (10.7)

Remarks:

i) The denominator in the integrands of Eqs.10.4 and 10.5, i.e. \([T - D - \mu (W - L)]\), is the accelerating force during the take-off run. A good approximation to \( s_1 \) and \( t_1 \) is obtained by taking an average value of the accelerating force \( (F_a) \) to be its value at \( V = 0.7 \ V_1 \) i.e.

\[
F_a = [T - D - \mu (W - L)]_{V = 0.7 \ V_1}
\]

Consequently,

\[
s_1 = \frac{W}{g} \int_{0}^{V_1} \frac{dV}{F_a} = \frac{W \ V_1^2}{2g F_a} \hspace{1cm} (10.8)
\]

and \( t_1 = \frac{W}{g} \int_{0}^{V_1} \frac{dV}{F_a} = \frac{WV_1}{g F_a} \)

(10.9)

ii) Generally the flaps are kept in take-off setting (partial flaps) right from the beginning of the take-off run. Hence, \( C_D \) during the take-off run should include the drag due to flaps and landing gear.

Reference 3.6, section 3.4.1 may be consulted for increase in \( C_{DO} \) due to the flap deflection and the landing gear. See also section 2.9 of Appendix A. The proximity of the ground reduces the induced drag. As a rough estimate, the induced drag with ground effect can be taken to be equal to 60% of that in free flight at the same \( C_L \).

(iii) The take-off speed \( (V_{TO} \text{ or } V_1) \) is \((1.1 \text{ to } 1.2) \ V_S \); where \( V_S \) is the stalling speed with \( W = W_{TO} \) and \( C_L = C_{LTO} \). As mentioned in subsection 3.7.4, \( C_{LTO} \) is 0.8 times \( C_{L\text{land}} \).
10.4.2 Various speeds during take-off run

In the subsection 10.3.1 the nose wheel lift-off speed and take-off speed have been explained. Section 6.7 of Ref.1.10 mentions additional flight speeds attained during the ground run. A brief description of the speeds, in the sequence of their occurrence, is as follows.

(a) Stalling speed ($V_S$): It is the speed in a steady level flight at $W = W_{TO}$ and $C_L = C_{LTO}$.

(b) Minimum control speed on ground ($V_{mcg}$): At this speed, the deflection of full rudder should be able to counteract the yawing moment due to failure of one engine of a multi-engined airplane when the airplane is on ground.

(c) Minimum control speed in air ($V_{mca}$): At this speed, the deflection of full rudder should be able to counteract the yawing moment, due to failure of one engine of a multi-engined airplane if the airplane was in air.

(d) Decision speed ($V_{decision}$): This speed is also applicable to a multi-engined airplane. In the event of the failure of one engine, the pilot has two options. (I) If the engine failure takes place during the initial stages of the ground run, the pilot applies brakes and stops the airplane. (II) If the engine failure takes place after the airplane has gained sufficient speed, the pilot continues to take-off with one engine inoperative.

If the engine failure takes place at decision speed ($V_{decision}$), then the distance required to stop the airplane is the same as that required to take-off with one engine inoperative. See subsection 10.4.8 for additional details.

(e) Take-off rotation speed ($V_R$): At this speed the elevator is powerful enough to rotate the airplane to attain the angle of attack corresponding to take-off.

(f) Lift-off speed ($V_{LO}$): This is the same as unstick speed mentioned in subsection 10.3.1. This speed is between (1.1 to 1.2) $V_S$. 
It is mentioned in Ref. 1.10, chapter 6, that $V_{mcg}$, $V_{mca}$, $V_{decision}$, $V_R$ lie between $V_S$ and $V_{LO}$. 