Homework

Lecture 9

Homework

1. Verify the property given in Equation (2.6) for rotation matrix.

2. Using Equation (2.6), show that

\[ a_{ik} a_{jk} = \delta_{ij} = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j) \end{cases} \]

where the term \( \delta_{ij} \), called Kronecker delta, has the value 1 on the diagonal and 0 on the off diagonal, that is, it represents an identity matrix when represented in matrix form.

3. Using relation for strain components (given in Equation (2.21)) write the expanded form of all strain components and understand the physical significance of all strain components. (The normal strain components denote the stretching of a line element, etc.)

4. Derive the principles of minimum of total potential and total complementary potential energy.

5. Derive the principle of virtual work.

Lecture 10

Homework:

1. Starting with hyperelastic material, first take \( x_2-x_3 \) plane as plane of material symmetry and obtain the stiffness matrix. Is this matrix the same as in Equation (3.20)? Justify your answer.

2. Starting with the stiffness matrix obtained in the above problem, take \( x_1-x_3 \) as an additional plane of symmetry and obtain the stiffness matrix. Is this matrix the same as in Equation (3.26)? Justify your answer.

Lecture 11

Homework:

1. Starting with the stiffness matrix for transverse isotropic material, take the transformations about \( x_1 \) and \( x_2 \) and show that you get the stiffness matrix as given in Equation (3.41).
Lecture 12

Homework:

1. Write the number of independent elastic constants for 3D hyperelastic, monoclinic, orthotropic, transversely isotropic and isotropic materials.

2. Are the Poisson’s ratio $\psi_{ij}$ and $\nu_{ji}$ independent of each other for an orthotropic unidirectional lamina?

3. Take the form of stiffness matrix for an orthotropic material as given in Equation (3.26). Using any symbolic calculation software like Maple or Mathematica, obtain the inverse of this matrix and confirm that the form of compliance matrix written in Equation (3.42) is correct. Further, confirm that this matrix is symmetric. (One should be able to do this using the concepts of linear algebra alone.)

4. Extend the Problem 3 to get the stiffness matrix given in Equation (3.51).

Lecture 13

Homework:

1. Verify the result given in Equation (3.74).

2. Using the invariance property of strain energy density function, show that

$$
\left[T_1^T\right]^{-1} = \left[T_2^T\right]^{-1} \quad \text{and} \quad \left[T_2^T\right]^{-1} = \left[T_1^T\right]^{-1}
$$

3. Obtain the individual terms of transformed stiffness and compliance matrices using Equation (3.73) and Equation (3.80), respectively and verify it with Equation (3.76) and Equation (3.82), respectively.

4. Obtain the strain transformation matrix $\left[T_2^T\right]$ using tensorial shear strains. Further, using this transformation matrix obtain the transformed stiffness and compliance matrix in the form similar to Equation (3.75) and Equation (3.81). Compare the new matrices and comment on the observations with justifications.

5. Calculate the stiffness and compliance coefficients for following transversely isotropic materials given in Table 3.1.

<table>
<thead>
<tr>
<th>Property/Material</th>
<th>T300/BSL914C Epoxy</th>
<th>E-glass/LY556/HT907/DY063 Epoxy</th>
<th>S-glass/MY750/HY917/DY063 Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
<td>138</td>
<td>53.48</td>
<td>45.6</td>
</tr>
<tr>
<td>$E_\perp$ (GPa)</td>
<td>11</td>
<td>17.7</td>
<td>16.2</td>
</tr>
<tr>
<td>$G_{12} = G_{13}$ (GPa)</td>
<td>5.5</td>
<td>5.83</td>
<td>5.83</td>
</tr>
</tbody>
</table>
6. The stiffness matrix for an orthotropic material is given as

\[
[C] = \begin{bmatrix}
141.3602 & 3.6453 & 3.6453 & 0 & 0 & 0 \\
3.6453 & 10.2763 & 4.1029 & 0 & 0 & 0 \\
3.6453 & 4.1029 & 10.2763 & 0 & 0 & 0 \\
0 & 0 & 0 & 2.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 4.082 & 0 \\
0 & 0 & 0 & 0 & 0 & 4.082
\end{bmatrix}
\]

Find:

a. The compliance matrix
b. The engineering constants \( E_1, E_2, E_3, G_{12}, G_{23}, G_{31}, \nu_{12}, \nu_{31}, \nu_{23} \).

7. Why are the thermal effects important in composite materials? Explain in detail.

8. Plot variation of \( \alpha_x, \alpha_y, \alpha_z \) and \( \alpha_{xy} \) between \(-90^\circ < \theta < 90^\circ\) for T300/BSL914C Epoxy and E-glass/LY556/HT907/DY063 Epoxy. See Table 3.1 for the required thermal properties.

9. Why are the hygral effects important in composite materials? Explain.

10. Search literature to get the coefficients of moisture absorption for at least two composite materials and plot its variation between \(-90^\circ < \theta < 90^\circ\).

11. Write a computer code to read the properties of a transversely isotropic material and calculate all the terms of stiffness and compliance matrix. Verify your results with the results given in Example 1. Then use this code to get the stiffness and compliance matrices of T300/BSL914C Epoxy and S-glass/MY750/HY917/DY063 Epoxy.

12. Add another module to the code written for above problem to calculate the transformed stiffness and compliance matrices. Plot all the coefficients between \(-90^\circ < \theta < 90^\circ\). Compare the corresponding terms of these materials and comment.
**Lecture 14**

**Homework:**

1. Verify the result given in Equation (4.36).
2. Using the invariance property of strain energy density function, show that:
   \[ [T_1]^{-1} = [T_2]^T \quad \text{and} \quad [T_2]^{-1} = [T_1]^T. \]
3. Using the relation between \( Q' \) and \( C' \) as given in Equation (4.24) and \( C' \) in terms of engineering constants, show that \( Q' \) are as given in Equation (4.29).
4. Write the compliance coefficients in Equation (4.45) in terms of engineering constants.
5. Using Equation (4.24) in Equation (4.42) obtain the individual terms of \( Q' \) in terms of \( C' \).
6. For fibre orientation \( \theta = 30^\circ \) and \( \theta = 45^\circ \) obtain \( Q' \) matrix for materials given in Table 3.1.
7. The \( Q' \) matrix for a composite with fibre orientation of \( \theta = 60^\circ \) is given as
   \[
   \begin{bmatrix}
   18409 & 10436 & 3193 \\
   10436 & 33524 & 9896 \\
   3193 & 9896 & 11635
   \end{bmatrix} \text{ MPa}
   \]
   Find all engineering constants for this material.
8. Write a computer code to calculate reduced transformed stiffness and compliance matrix for any angle of fibre orientation with respect to global coordinate system.
9. Extend the code written for the above problem to plot the variation of \( Q' \) terms for orientation of fibres between \(-90^\circ < \theta < 90^\circ\). Plot the variation for materials given in Table 3.1.
10. Write a computer code to plot the variation of thermal and hygroscopic expansion coefficients with fibre orientation between \(-90^\circ < \theta < 90^\circ\) for T300/5208 composite.

**Lecture 15**

**Homework:**

1. Obtain the lamina engineering constants for materials given in Table 3.1 for fibre orientation of \( \theta = 30^\circ, 60^\circ, 67.5^\circ \)
2. Write a computer code to plot the variation of all lamina engineering constants and coefficients of mutual influence against the fibre orientation from \(-90^\circ < \theta < 90^\circ\). Further, plot the variations for the materials given in Table 3.1 simultaneously and compare their behaviour and comment on key observations.
Lecture 16

Homework:

1. Write the key points in the designation of laminate sequence.
2. What are the assumptions in the classical laminate theory?
3. What are the assumptions in the classical laminate theory?
4. Why the stresses at the interface of two laminae are different according to the classical plate theory?
5. Derive the expressions for resultant inplane forces and bending moments for laminate

Lecture 17

Homework:

1. Write laminate constitutive equation and obtain its partially and fully inverted form.
2. What are the types of laminate?
3. Differentiate between symmetric and unsymmetric laminates.
4. What is an unsymmetric and antisymmetric laminate? Are they the same?
5. For antisymmetric laminates show that the terms $A_{16}, A_{26}, D_{16}, D_{26}$ are zero.
6. Show that for a symmetric laminate there is no coupling between extension and bending responses.
7. Classify the following laminates
   a. $[-60/30/-30/60]$  
   b. $[45/0/45]$  
   c. $[\pm 22.5/\mp 22.5]$  
   d. $[90/0]$  
   e. $[0/90/0/90]$  
   f. $[0/90/0/90/0]$  
8. Write an example for following laminates:
a. Antisymmetric laminate
b. Cross-ply
c. Cross-ply symmetric
d. Angle ply symmetric
e. Balanced angle ply
f. Quasi-isotropic
g. Specially orthotropic

9. For the composite material T300/5208, calculate the $[A]$, $[B]$, and $[D]$ for the following laminates. The thickness of each lamina is 0.1 mm.

a. $[(0/90)_2]$  

b. $[(0/90)_s]$  

c. $[\pm45]$  

d. $[\pm45]_s$  

e. $[0/-30/-30/0/30/0]$  

10. For the above laminate sequences calculate the compliance relation (for midplane strains and curvatures). Develop a computer code for this.

11. Using the code developed in exercise (10), verify the solutions given for Example 5.6 and Example 5.7.

12. Show that the T300/5208 $[0/\pm60]$ laminate is a quasi-isotropic laminate. Is it an isotropic laminate?
Lecture 18

Homework:

1. Derive the effective in-plane engineering constants for a laminate.

2. Derive the effective flexural engineering constants for a laminate.

3. For the composite material T300/5208, calculate the effective in-plane and engineering constants for the following laminates. The thickness of each lamina is 1 mm.

   a. \([0/(90)_2]_s\)

   b. \([\pm 45]_s\)

   c. \([\mp 45]_s\)

   d. \([0/45/0]_s\)

   e. \([0/45/90]_s\)

Lecture 19

Homework:

1. Derive the resultant in-plane forces and moments for a laminate with thermal effects.

2. Derive the resultant in-plane forces and moments for a laminate with hygral effects.

3. Derive an expression for laminate coefficient of thermal expansion under the uniform temperature condition.

4. Derive an expression for laminate coefficient of hygral expansion under the condition of uniform moisture absorption.

5. Derive the expressions for resultant in-plane forces and moments for a laminate with hygro-thermal effects.

6. Derive the governing differential equations for classical laminate theory.
7. Calculate the laminate coefficients of thermal expansion for the following laminates of AS4/3501-6 Epoxy from Soden et al [4]. Take thickness of each layer as 1 mm.
   a. $[\pm 45]_s$
   b. $[0/45/0]$
   c. $[0/\pm 45]_s$
   d. $[0/90/0]_s$

8. Calculate the thermal residual stresses for a temperature change of $-75^\circ C$ at the top and bottom of the following laminates of AS4/3501-6 Epoxy. (Write a computer code for this problem. Repeat the Example 5.12).
   a. $[90/0]$
   b. $[0/45]$

9. Calculate the laminate coefficients of hygral expansion for laminate sequences in exercise example 7 with T300/5208 material as in Example 5.11. Take thickness of each layer as 1 mm.

**Lecture 20**

**Home Work:**

1. What is meant by failure and damage?
2. Why is the study of damage mechanisms and their mechanics important for fibrous composites?
3. What are the defects in a composite?
4. What are the sources of defects in a laminated composite?
5. What are the damage mechanisms in a fibrous laminated composite?
6. What are the causes of delamination?
7. What are the remedies to suppress the delamination?
Lecture 21

Home Work:

1. What are issues in failure theories for composites as compared to theories for homogeneous and isotropic materials?
2. Explain in detail the following failure theories.
   a. Maximum stress theory
   b. Maximum strain theory
   c. Tsai-Hill theory
3. What are the differences between maximum stress (or strain) and Tsai-Hill theory?
4. The strains $\{\varepsilon\}_{xy} = 10^{-2} \{0.2 \ 1.2 \ 0.1\}^T$ are acting on 45° ply. Check whether the ply of AS4/3501-6 Epoxy material will fail or not using a) Maximum stress b) maximum strain and c) Tsai-Hill theory.
5. For 45° ply the state of stress acting on it is $\{\sigma\}_{xy} = \{0 \ 0 \ \tau\}^T$. Find the value of $\tau > 0$ for which this ply of AS4/3501-6 Epoxy material will fail by a) Maximum stress b) maximum strain and c) Tsai-Hill theory.

Lecture 22

Home Work:

1. Explain in detail the following failure theories.
   a. Hoffman Theory
   b. Tsai-Wu Theory
2. What is the difference between Tsai-Hill and Hoffman theory?
3. What are the key features of the Tsai-Wu theory?
4. What are the different methods to find the strength parameter $F_{12}$ in Tsai-Wu theory?
5. Explain the methods to find the strength parameter $F_{16}$ in Tsai-Wu theory.
6. A ply of AS4/3506-1 material with 45° fibre orientation is in the planar state of stress. The strains are $\{\varepsilon\}_{xy} = 10^{-2} \{1.2 \ 0.2 \ 0.1\}^T$. Check that whether lamina will fail if a) Hoffman theory b) Tsai-Wu theory is used.
7. Find the maximum value of $P > 0$ if a state of stress of $\sigma_{xx} = 2P$, $\sigma_{yy} = -3P$, and $\tau_{xy} = 4P$, is applied to the lamina of AS4/3506-1 material using a) Hoffman theory b) Tsai-Wu theory.
Lecture 23

Home Work:

1. Explain Hashin's criteria for three dimensional and planar state of stress.
2. Explain connection of various failure theories with respect to tensor polynomial criterion.
3. Verify the strength parameters of all the theories studied according tensor polynomial criterion.

Lecture 24

Home Work:

1. What are the assumptions in a typical micromechanical analysis?
2. Write a short note on RVE/Unit Cell.
3. Define volume and mass fractions for fibre and matrix and derive expressions for them.
4. Derive an expression for density of a composite in terms of densities of its constituents.
5. Using strength of materials approach, derive expressions for effective axial modulus, Poisson's ratio and transverse modulus.

Lecture 25

Home Work:

1. Using strength of materials approach, derive the expression for effective transverse modulus with deformation constrains satisfied.
2. Derive an expression for effective axial shear modulus of the composite using strength of materials approach.
3. Using strength of materials approach, derive the expressions for effective coefficients of thermal and hygral expansions in axial and transverse directions.
4. For fibre volume fraction of 0.6, determine all the effective mechanical and thermal properties of the fibre and matrix materials given in Table 7.1 and Table 7.2 and compare them with the experimental effective properties as reported in Soden et al [5]. Calculate percentage difference for all properties with respect to experimental effective properties.

Lecture 26

Home Work:

1. What is statistical homogeneity?
2. Write a short note on volumetric averaging.
3. Write a short note on energy equivalence approach in averaging.
4. Explain in detail the standard mechanics approach.
5. For the one dimensional problem as shown in Figure 7.8, show that the effective Young's modulus determined using standard mechanics approach is same as given in Equation (7.85) (which is same as in Equation (7.88)).
Lecture 27

Home Work:

1. Explain in detail the Hill's concentration factors approach.
2. What are Reuss and Voigt approximations in connection with Hill's concentration factors approach?
3. For fibre volume fraction of 0.6, determine all the effective mechanical properties for the fibre and matrix materials given in Table 7.1 and Table 7.2 and compare them with the experimental effective properties as reported in Soden et al [7]. Calculate percentage difference for all properties. Use Voigt and Reuss approximation for this exercise.

Lecture 28

Home Work:

1. What are the lacunas in standard mechanics approach?
2. Explain the importance of applied boundary conditions on RVE in determining the effective properties.
3. Explain in detail the concept of homogenization.
4. Show that for the one dimensional case as shown in Figure 7.9 the effective Young's modulus determined using homogenization approach is same as given by the standard mechanics approach.

Lecture 29

Home Work:

1. What is a CCA model?
2. Give a brief description of the background for CCA model.

Lecture 30

Home Work

1. What are the deformations or load conditions to be imposed on the concentric cylinders to determine the effective axial modulus and Poisson's ratio?
2. What are the continuity conditions to be imposed on the concentric cylinders to determine the effective axial modulus and Poisson's ratio?
3. Outline the methodology with key points to determine the effective axial modulus and Poisson's ratio using CCA model.
Lecture 31

Home Work:

1. Write a short note on deformations/loads to be imposed on the concentric cylinders to obtain the effective plane strain bulk modulus.
2. Derive an expression for the effective plane strain bulk modulus using CCA model.

Lecture 32

Home Work:

1. Write a short note on the deformation or the loads to be imposed on the concentric cylinders to determine the effective axial shear modulus.
2. Derive the expression for the effective axial shear modulus of the composite using CCA model.

Lecture 33

Home Work:

1. What is a three-phase cylinder model? Why is it required?
2. Write a short note on determination of transverse shear modulus using three phase cylinder model.

Lecture 34

Home Work:

1. What is a three-phase cylinder model? Why is it required?
2. Write a short note on determination of transverse shear modulus using three phase cylinder model.

Lecture 35

Home Work:

1. Write a short note on societies of mechanical testing.
2. What are the objectives of the mechanical testing?
3. What are the effects of the anisotropy of composites in their mechanical testing?
4. What are the issues with the mechanical testing of the specimen?
Lecture 36

Home Work:

1. List the parameters needed to assess the three main properties – modulus, strength and ductility.
2. What are the ways of assessing the quality of the composite?
3. What the physical properties of composite that needs to be quantified?
4. Explain the methods in short to measure the physical properties of a composite.

Lecture 37

Home Work:

1. What are the methods for strain measuring? Explain in detail the rosette principle.
2. What are the key points in tensile testing?
3. Explain the method of measuring the tensile modulus using symmetric laminates with 0° and 90° laminae.
4. Explain the method of measuring the tensile modulus using off-axis laminae.
5. Explain the method to measure the modulus from stress-strain curve.
6. Write a short note on compression testing.
7. What are the ways of imposing a compression load on a specimen for compression testing?
8. The 60° strain rosette is mounted on a shaft as shown in Figure below. The following readings are obtained for each gage: \( \varepsilon_a = 200 \times 10^{-6}, \varepsilon_b = -450 \times 10^{-6}, \text{ and } \varepsilon_c = 250 \times 10^{-6} \). Determine the strains in global directions.

![Figure: Rosette on a shaft](image-url)
Lecture 38

Home Work:

1. Explain in brief the tension test on \( [\pm 45]_s \) and off-axis lamina to determine the shear properties.
2. What is Iosipescu shear test?
3. Write short note on measurement of shear modulus by torsion of a thin walled tube.

Lecture 39

Home Work:

1. Explain in brief: (a) Two and three rail shear test, (b) picture frame test and (c) three and four point bending test.
2. What are differences between two and three rail shear test? Give their relative advantages and disadvantages.
3. Differentiate between three and four point bending tests along with their pros can cons.

Lecture 40

Home Work:

1. What are the key points in the design considerations?
2. Explain in detail the development of interlaminar stresses in the free edge region.
3. What is positive and negative shear? Which one has detrimental effect?