In this lecture...

- Axial flow compressors
  - Basic operation of axial compressors
  - Velocity triangles
  - Work and compression
  - Design parameters
    - Flow coefficient
    - Loading coefficient
    - Degree of reaction
    - Diffusion factor
Basic operation of axial compressors

• Axial flow compressors usually consists of a series of stages.
• Each stage comprises of a row of rotor blades followed by a row of stator blades.
• The working fluid is initially accelerated by the rotor blades and then decelerated in the stator passages.
• In the stator, the kinetic energy transferred in the rotor is converted to static pressure.
• This process is repeated in several stages to yield the necessary overall pressure ratio.
Basic operation of axial compressors

• The compression process consists of a series of diffusions.
• This occurs both in the rotor as well as the stator.
• Due to motion of the rotor blades, two distinct velocity components: absolute and relative velocities in the rotor.
• The absolute velocity of the fluid is increased in the rotor, whereas the relative velocity is decreased, leading to diffusion.
• Per stage pressure ratio is limited because a compressor operates in an adverse pressure gradient environment.
Basic operation of axial compressors

- Turbines on the other hand operate under favourable pressure gradients.
- Several stages of an axial compressor can be driven by a single turbine stage.
- Careful design of the compressor blading is essential to minimize losses as well as to ensure stable operation.
- Some compressors also have inlet Guide Vanes (IGV) that permit the flow entering the first stage to vary under off-design conditions.
Velocity triangles

- Elementary analysis of axial compressors begins with velocity triangles.
- The analysis will be carried out at the mean height of the blade, where the peripheral velocity or the blade speed is, $U$.
- The absolute component of velocity will be denoted by, $C$ and the relative component by, $V$.
- The axial velocity (absolute) will be denoted by $C_a$ and the tangential components will be denoted by subscript $w$ (for eg, $C_w$ or $V_w$).
- $\alpha$ denotes the angle between the absolute velocity with the axial direction and $\beta$ the corresponding angle for the relative velocity.
Velocity triangles

\[ \vec{C} = \vec{U} + \vec{V} \]
Velocity triangles

\[ \Delta C_w \]

\[ C_1 \]

\[ C_2 \]

\[ V_1 \]

\[ V_2 \]

\[ \alpha_1 \]

\[ \beta_1 \]

\[ \alpha_2 \]

\[ \beta_2 \]

\[ C_w^2 \]

\[ C_w^1 \]

\[ V_{w1} \]

\[ V_{w2} \]

\[ U \]

\[ C_a \]
Property changes across a stage

- **Total enthalpy**
  - $h_{01}$
  - $h_{02}$
  - $h_{03}$

- **Absolute velocity**
  - $C_1$
  - $C_2$
  - $C_3$

- **Static pressure**
  - $P_1$
  - $P_2$
  - $P_3$

**Rotor**

**Stator**
Work and compression

• Assuming $C_a = C_{a1} = C_{a2}$, from the velocity triangles, we can see that

$$\frac{U}{C_a} = \tan \alpha_1 + \tan \beta_1 \quad \text{and} \quad \frac{U}{C_a} = \tan \alpha_2 + \tan \beta_2$$

• By considering the change in angular momentum of the air passing through the rotor, work done per unit mass flow is

$$w = U(C_{w2} - C_{w1}), \text{ where } C_{w1} \text{ and } C_{w2} \text{ are the tangential components of the fluid velocity before and after the rotor, respectively.}$$
Work and compression

The above equation can also be written as,

\[ w = UC_a (\tan \alpha_2 - \tan \alpha_1) \]

Since, \( (\tan \alpha_2 - \tan \alpha_1) = (\tan \beta_1 - \tan \beta_2) \)

\[ \therefore w = UC_a (\tan \beta_1 - \tan \beta_2) \]

In other words, \( w = U \Delta C_w \)

- The input energy will reveal itself in the form of rise in stagnation temperature of the air.
- The work done as given above will also be equal to the change in stagnation enthalpy across the stage.
Work and compression

\[ h_{02} - h_{01} = U \Delta C_w \]

\[ T_{02} - T_{01} = \frac{U \Delta C_w}{c_p} \Rightarrow \frac{\Delta T_0}{T_{01}} = \frac{U \Delta C_w}{c_p T_{01}} \]

Since the flow is adiabatic and no work is done as the fluid passes through the stator, \( T_{03} = T_{02} \)

Let us define stage efficiency, \( \eta_{st} \), as

\[ \eta_{st} = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}} \]

This can be expressed as

\[ \frac{T_{03s}}{T_{01}} = 1 + \eta_{st} \frac{\Delta T_0}{T_{01}} \]
Work and compression

In the above equation, \( \Delta T_0 = T_{03} - T_{01} \)

In terms of pressure ratio,

\[
\frac{P_{03}}{P_{01}} = \left[ 1 + \eta_{st} \frac{\Delta T_0}{T_{01}} \right]^{\gamma/(\gamma-1)}
\]

This can be combined with the earlier equation to give,

\[
\frac{P_{03}}{P_{01}} = \left[ 1 + \eta_{st} \frac{U \Delta C_w}{c_p T_{01}} \right]^{\gamma/(\gamma-1)}
\]
Work and compression

• From the above equation that relates the per stage temperature rise to the pressure ratio, it can be seen that to obtain a high temperature ratio for a given overall pressure ratio (for minimizing number of stages),
  – High blade speed: limited by blades stresses
  – High axial velocity, high fluid deflection $(\beta_1-\beta_2)$: Aerodynamic considerations and adverse pressure gradients limit the above.
Design parameters

• The following design parameters are often used in the parametric study of axial compressors:
  – Flow coefficient,
    \[ \phi = \frac{C_a}{U} \]
  – Stage loading,
    \[ \psi = \frac{\Delta h_0}{U^2} = \frac{\Delta C_w}{U} \]
  – Degree of reaction, \( R_x \)
  – Diffusion factor, \( D^* \)
Degree of reaction

- Diffusion takes place in both rotor and the stator.
- Static pressure rises in the rotor as well as the stator.
- Degree of reaction provides a measure of the extent to which the rotor contributes to the overall pressure rise in the stage.
Degree of reaction

\[ R_x = \frac{\text{Static enthalpy rise in the rotor}}{\text{Stagnation enthalpy rise in the stage}} \]

\[ = \frac{h_2 - h_1}{h_{03} - h_{01}} \approx \frac{h_2 - h_1}{h_{02} - h_{01}} \]

For a nearly incompressible flow,

\[ h_2 - h_1 \approx \frac{1}{\rho} (P_2 - P_1) \text{ for the rotor} \]

and for the stage, \[ h_{03} - h_{01} \approx \frac{1}{\rho} (P_{03} - P_{01}) \]

\[ \therefore R_x = \frac{h_2 - h_1}{h_{02} - h_{01}} \approx \frac{P_2 - P_1}{P_{02} - P_{01}} \]
Degree of reaction

From the steady flow energy equation,

\[ h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \]

\[ \therefore R_x = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{V_1^2 - V_2^2}{2U(C_{w2} - C_{w1})} \]

For constant axial velocity, \(V_1^2 - V_2^2 = V_{w1}^2 - V_{w2}^2\)

And, \(V_{w1} - V_{w2} = C_{w1} - C_{w2}\)

On simplification, \(R_x = \frac{1}{2} - \frac{C_a}{2U} (\tan \alpha_1 - \tan \beta_2)\)

or, \(R_x = \frac{C_a}{2U} (\tan \beta_1 + \tan \beta_2)\)
Degree of reaction

- Special cases of $R_x$
  - $R_x=0, \beta_2 = -\beta_1,$ There is no pressure rise in the rotor, the entire pressure rise is due to the stator, the rotor merely deflects the incoming flow: impulse blading
  - $R_x=0.5,$ gives $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1,$ the velocity triangles are symmetric, equal pressure rise in the rotor and the stator
  - $R_x=1.0, \alpha_2 = -\alpha_1,$ entire pressure rise takes place in the rotor while the stator has no contribution.
Degree of reaction

\[ \alpha_2 = -\alpha_1 \]

\[ \beta_2 = -\beta_1 \]

\[ \alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1 \]

\[ R_x = 0.0 \]

\[ R_x = 0.5 \]

\[ R_x = 1.0 \]
Diffusion factor

- Fluid deflection \((\beta_2 - \beta_1)\) is an important parameter that affects the stage pressure rise.
- Excessive deflection, which means high rate of diffusion, will lead to blade stall.
- Diffusion factor is a parameter that associates blade stall with deceleration on the suction surface of the airfoil section.
- Diffusion factor, \(D^*\), is defined as
  \[
  D^* = \frac{V_{\text{max}} - V_2}{V_1}
  \]
  Where, \(V_{\text{max}}\) is the ideal surface velocity at the minimum pressure point and \(V_2\) is the ideal velocity at the trailing edge and \(V_1\) is the velocity at the leading edge.
Diffusion factor

- Suction surface
- Pressure surface

Symbols:
- $V_1$
- $V_{\text{max}}$
- $V_2$

Axes:
- Velocity
- Percent chord

Indices:
- 0
- 50
- 100
Diffusion factor

- Lieblein (1953) proposed an empirical parameter for diffusion factor.
  - It is expressed entirely in terms of known or measured quantities.
  - It depends strongly upon solidity (C/s).
  - It has been proven to be a dependable indicator of approach to separation for a variety of blade shapes.
  - $D^*$ is usually kept around 0.5.

$$D^* = 1 - \frac{V_2}{V_1} + \frac{V_{w1} - V_{w2}}{2\left(\frac{C}{s}\right)V_1}$$

Where, $C$ is the chord of the blade and $s$ is the spacing between the blades.
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In the next lecture...

• Cascade analysis
  – Cascade nomenclature
  – Loss and blade performance estimation