Introduction to Aerospace Propulsion

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Lecture No - 30
Propeller theories

Blade element theory

- The blade elements are assumed to be made up of airfoil shapes of known lift, $C_l$ and drag, $C_d$ characteristics.
- In practice a large number of different airfoils are used to make up one propeller blade.
- Each of these elements shall have its own lift, $C_l$ and drag, $C_d$ coefficient characteristics.
The thrust, $dT$ created by an element of elemental radial length $dr$ is created with contributions from the airfoil with lift, $dL$ and drag, $dD$. 
Using the blade elemental lift and drag characteristics the working capacity of the blade element may be found as:

**Thrust produced,**
\[
dT = dL \cdot \cos \phi - dD \cdot \sin \phi
= \frac{1}{2} \rho V_R^2 c \cdot dr \cdot (C_l \cos \phi - C_d \sin \phi)
\]

**Torque to be supplied,**
\[
dQ = (dL \cdot \sin \phi + dD \cdot \cos \phi) \cdot r
= \frac{1}{2} \rho V_R^2 c \cdot dr \cdot (C_l \sin \phi + C_d \cos \phi)
\]
Substituting for Resultant inflow velocity Incident and aligned to the blade element,

\[ V_R = \frac{V_\infty}{\sin \phi} \]

and for Incoming flow Dynamic head based on forward velocity of the element

\[ q = \frac{1}{2} \rho V_\infty^2 \]
The elemental thrust is:

\[ dT = \frac{q \cdot c \cdot dr}{\sin^2 \phi} \left( C_1 \cos \phi - C_d \sin \phi \right) \]

and

The elemental torque is:

\[ dQ = \frac{q \cdot c \cdot r \cdot dr}{\sin^2 \phi} \left( C_1 \sin \phi + C_d \cos \phi \right) \]
Propeller thrust and torque are now computed by integrating from the root to the tip of the blade and for number of blades, B

\[
T = qB \int_{0}^{R} \frac{c.dr}{\sin^2 \phi} (C_1 \cos \phi - C_d \sin \phi)
\]

\[
Q = qB \int_{0}^{R} \frac{c.r.dr}{\sin^2 \phi} (C_1 \sin \phi + C_d \cos \phi)
\]
• Thus, the net thrust and the torque are seen to be directly proportional to the number of blades, B and the chord, c.
• *This is not quite true in practice*, as more is the number of blades and wider the blade chord - it shall result in more surface area, more flow blockage and higher consequent aerodynamic losses.
• The optimum number of blades need to be found separately and not from the blade element theory.
The blade element efficiency,

\[ \eta_{el} = \frac{\text{Thrust power produced}}{\text{Torque power supplied}} \]

In terms of elemental airfoil characteristics \( C_l \) and \( C_d \), blade efficiency is:

\[ \eta_{el} = \frac{v.dT}{2\pi n.dQ} = \frac{V}{2\pi nr} \cdot \frac{C_l \cos \phi - C_d \sin \phi}{C_l \sin \phi + C_d \cos \phi} = \frac{C_l \cos \phi - C_d \sin \phi}{C_l \sin \phi + C_d \cos \phi} \cdot \tan \phi \]
Applying maxima condition it can be shown that maximum efficiency, $\eta_{el-max}$ occurs at

$$\phi = \frac{\pi}{4} - \frac{C_d}{2.C_l}$$

for a blade element airfoil characterized by its $C_d$ & $C_l$

The estimations from blade element theory is within 10% of the actually obtained results.
If the elemental performances are plotted in the form of $\frac{dC_T}{dx}$ and $\frac{dC_Q}{dx}$ variation in $X$, the span-wise direction of a blade (root to tip).
Low speed aircraft propeller Characteristics

Cruise Point

Courtesy: Theodorsen, 1948
High speed aircraft propeller characteristics

Courtesy: Dommasch et al., 1967
Efficiency of Propeller

Variable Pitch Propeller

Cruise Point selection

Thrust Co-efficient of Propeller

Courtesy: NACA, USA
Typical Propeller Thrust-Power Characteristics using NACA airfoils

Cruise Point selection

Variable Pitch Propeller

Courtesy: NACA, USA
$C_s$, the speed power coefficient, defined by,

$$C_s = (\frac{\rho \cdot V^5}{P \cdot n^2})^{1/5}$$

Is often used for design / selection of propeller

If coeff of power, $C_p$ as a function of $J$, is known, $C_s$ can be obtained from

$$C_s = \frac{J}{C_p^{1/5}}$$

The usefulness of $C_s$ is in the process of defining it -- diameter was eliminated. Thus the propeller design or selection related flow parameters may be estimated even before the propeller size is fixed.
Propeller Cs Characteristics
Transonic Swept bladed propellers
(a) Tractor ; (b) counter-rotating Pusher
In an aircraft application:

Propeller Power, $P_{prop} = P_{Engine} \cdot \eta_{shaft} \cdot \eta_{prop}$

Propeller Torque, $Q_{prop} = Q_{engine}$

Typically, at Take off,

$Q_{prop}$ is low, $\beta$ is low, $P_E$ is High, rpm is high

at Cruise,

$Q_{prop}$ is high, $\beta$ is high, $P_E$ is low, rpm is low
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Propeller Tutorial