FUNCTIONAL ANALYSIS

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Retired from The Institute of Mathematical Sciences, Chennai

TYPE OF COURSE : New | Core | PG
COURSE DURATION : 12 Weeks (18 Jan' 21 - 09 Apr' 21)
EXAM DATE : 25 Apr 2021

PRE-REQUISITES : BSc (Mathematics) Real Analysis, Topology, Linear Algebra, Measure Theory

INTENDED AUDIENCE : MSc (Mathematics) and above

COURSE OUTLINE :
Functional Analysis is a core course in any mathematics curriculum at the masters level. It has wide ranging applications in several areas of mathematics, especially in the modern approach to the study of partial differential equations. The proposed course will cover all the material usually dealt with in any basic course of Functional Analysis. Starting from normed linear spaces, we will study all the important theorems, with applications, in the theory of Banach and Hilbert spaces. One important feature of the proposed course is the detailed treatment of weak topologies. Prerequisites are familiarity with real analysis, topology and linear algebra. Knowledge of measure theory is desirable.

ABOUT INSTRUCTOR :
S. Kesavan retired as Professor from the Institute of Mathematical Sciences, Chennai. He obtained his doctoral degree from the Universite de Pierre et Marie Curie (Paris VI), France. His research interests are in Partial Differential Equations. He is the author of five books. He is a Fellow of the Indian Academy of Sciences and the National Academy of Sciences, India. He has served as Deputy Director of the Chennai Mathematical Institute (2007-2010) and two terms (2011-14, 2015-18) as Secretary (Grants Selection) of the Commission for Developing Countries of the International Mathematical Union.

COURSE PLAN :

Week 1: Normed linear spaces, examples. Continuous linear transformations, examples.

Week 2: Continuous linear transformations. Hahn-Banach theorem-extension form. Reflexivity.

Week 3: Hahn-Banach theorem-geometric form. Vector valued integration.

Week 4: Baire's theorem,.Principle of uniform boundedness. Application to Fourier series. Open mapping and closed graph theorems.

Week 5: Annihilators. Complemented subspaces. Unbounded operators, Adjoints.

Week 6: Weak topology. Weak-* topology. Banach-Alaoglu theorem. Reflexive spaces.

Week 7: Separable spaces, Uniformly convex spaces, applications to calculus of variations.

Week 8: L^p spaces. Duality, Riesz representation theorem.

Week 9: L^p spaces on Euclidean domains,.Convolutions. Riesz representation theorem.

Week 10: Hilbert spaces. Duality, Riesz representation theorem. Application to the calculus of variations. Lax-Milgram lemma. Orthonormal sets.

Week 11: Bessel's inequality, orthonormal bases, Parseval identity, abstract Fourier series. Spectrum of an operator.

Week 12: Compact operators, Riesz-Fredholm theory. Spectrum of a compact operator. Spectrum of a compact self-adjont operator.