MEASURE THEORY

PROF. INDRAVA
Department of Mathematics
IMSC

TYPE OF COURSE : New | Core | PG
COURSE DURATION : 12 weeks (20 Jul' 20 - 9 Oct' 20)
EXAM DATE : 17 Oct 2020

PRE-REQUISITES : Set theory and Basic real analysis

INTENDED AUDIENCE : 1st year M.Sc. onwards

COURSE OUTLINE :
One of the main goals of Lebesgue's measure theory is to develop a fundamental tool for carrying out integration which behaves well with taking limits, and admitting a vast class of functions for which Riemann's integration theory is not applicable. Even though the crux of measure theory was to produce a good integration theory, it turns out that it also gives new ways of thinking about “measuring” objects, which is very useful for many other areas of mathematics such as probability theory as well as more advanced topics like harmonic analysis, ergodic theory, etc. Real-world applications of measure theory can be found in physics, economics, finance etc. Measure theoretic techniques are thus a must-have for any mathematician.

ABOUT INSTRUCTOR :
Prof. Indrava Roy is an Assistant Professor of Mathematics at the Institute of Mathematical Sciences, Chennai.

COURSE PLAN :

Week 1: Introduction and Motivation of Measure theory, Jordan measurability and Jordan content

Week 2: Basic properties of Jordan content and connection with Riemann integrals, Motivation and definition of Lebesgue outer measure on R^n

Week 3: Properties of Lebesgue outer measure on R^n, Caratheodory extension theorem

Week 4: Lebesgue measurability, Vitali and Cantor sets, Boolean and sigma algebras

Week 5: Abstract measure spaces with examples: Borel and Radon measures, Metric outer measures, Lebesgue-Stieljes measures, Hausdorff measures and dimension* (extra content)

Week 6: Measurable functions and abstract Lebesgue integration, Monotone convergence theorem, Fatou's lemma, Tonnelli's theorem.

Week 7: Borel-Cantelli Lemma, Dominated convergence theorem, the space L^1

Week 8: Various modes of convergence and their inter-dependence

Week 9: Riesz representation theorem, examples of measures constructed via RRT

Week 10: Product measures and Fubini-Tonnelli theorem

Week 11: Hardy-Littlewood Maximal inequality and Lebesgue's differentiation theorem

Week 12: Lebesgue's differentiation theorem (continued)