OPTICAL SENSORS

Practice Assignment 1
7) An electromagnetic wave is given by

\[ \vec{E} = E_0 \cos \left( \omega t - \frac{\sqrt{3}}{2} kx - \frac{1}{2} ky \right) \]

What will be the direction of propagation of electromagnetic wave?

- 30 degree from X-axis
- 30 degree from Y-axis
- Z-axis
- Y-axis

\[ \vec{B} = \frac{\vec{E} \times \vec{v}}{c} \]

\[ |\vec{E}| = E_0 \cos (\omega t - \frac{\vec{E} \cdot \vec{r}}{c}) \]

\[ k_x = \frac{\sqrt{3}}{2}, \quad k_y = \frac{1}{2} \]

\[ \tan \theta = \frac{k_y}{k_x} \]

\[ \theta = \tan^{-1} \left( \frac{k_y}{k_x} \right) \]

\[ \vec{E} \rightarrow \text{Electric field vector} \]

\[ \vec{B} \rightarrow \text{Magnetic field} \]

E0 magnitude of electric field

\[ \vec{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k} \]

\[ \vec{r} = (k_x \hat{i} + k_y \hat{j} + k_z \hat{k}) \]

\[ |\vec{r}| = \frac{2\pi}{\lambda} \]
\[ k_y = \frac{1}{2} \]

\[ k_x = \sqrt{3} \frac{1}{2} \]

\[ \tan \theta = \frac{k_y}{k_x} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \]

\[ \theta = \arctan \left( \frac{1}{\sqrt{3}} \right) = 30^\circ \]
2) If an unpolarised light passes through two polarizers, then for what value of $\theta$ intensity of the polarized light after the second polarizer will be one-tenth of the unpolarized light. $\theta$ is the angle made by the pass axis of second polarizer with the pass axis of first polarizer? (Following options are in degree)

- A $\bigcirc 60$
- B $\bigcirc 70$
- C $\bigcirc 63.4$
- D $\bigcirc 66$

\[
I_1 = \frac{I_0}{2} \quad I_2 = \frac{I_1 \cos^2 \theta}{2} = \frac{I_0 \cos^4 \theta}{2}
\]

\[
\cos \theta = \frac{1}{\sqrt{5}} \quad \theta = \cos^{-1} \left( \frac{1}{\sqrt{5}} \right)
\]
\[
2\cos^2 \theta = 1 + \cos 2\theta.
\]

\[
I_1 = \frac{I_0}{2}
\]

\[
\int_0^{2\pi} \cos^2 \theta \, d\theta = \frac{1}{2} \int_0^{2\pi} (1 + \cos 2\theta) \, d\theta = \frac{1}{2} \left[ \theta + \frac{\sin 2\theta}{2} \right]_0^{2\pi} = \pi
\]

\[
I_1 = I_0 \cos^2 \theta \rightarrow \int_0^{2\pi} \cos^2 \theta \, d\theta = \text{average}
\]

\[
= \frac{1}{2} (2\pi) = \pi
\]

\[
0.5 \text{cm} \quad 0.7 \text{cm} \quad 0.9 \text{cm} \rightarrow \text{average}
\]

\[
= \frac{1}{2}
\]
The following electromagnetic wave is:
\[
\vec{E}(x, y, z, t) = \left( A \hat{i} \cos(\omega t - \vec{k} \cdot \vec{z}) - \hat{j} \sin(\omega t - \vec{k} \cdot \vec{z}) \right)
\]

- Linearly polarized
- Elliptically polarized
- Left circularly polarized  \(\checkmark\)
- Right circularly polarized

\[
E_x^2 + E_y^2 = A_x^2 \cos^2(\omega t - \vec{k} \cdot \vec{z}) + A_y^2 \sin^2(\omega t - \vec{k} \cdot \vec{z})
\]

\[
E_x^2 + E_y^2 = A^2 \left( \cos^2(\omega t - \vec{k} \cdot \vec{z}) + \sin^2(\omega t - \vec{k} \cdot \vec{z}) \right)
\]

\[
E_x^2 + E_y^2 = A^2
\]
\[ E_x = A \cos(\omega t - kz) \]
\[ E_y = -A \sin(\omega t - kz) \]

\[ \omega t - kz = \theta \]

\[ E_x = A \cos \theta \]
\[ E_y = -A \sin \theta \]

\[ E_x^2 = A \cos \theta \]
\[ E_y = A \sin(\pi + \theta) \]
\[ E_y = -A \sin \theta \]

\[ t = 0 \]
\[ \theta = -kz \]
\[ \theta = 0 \]
\[ E_x = A \]
\[ E_y = 0 \]
\[ \theta = \pi \]
\[ E_x = -A \]
\[ E_y = 0 \]

\[ \theta \text{ increases} \]

\[ \theta = \frac{\pi}{2} \]
\[ E_x = 0 \]
\[ E_y = -A \]
transverse = perpendicular

4) For transverse electric (TE) polarization, which of the following is true?

- E is parallel to H (Crossed)
- E is parallel to plane of incidence (Crossed)
- E is transverse to plane of incidence (Corrected)
- H is transverse to plane of incidence (Crossed)

→ TM
\[ \vec{E} = a \cos(\omega t - kz) \hat{i} + b \cos(\omega t - kz + \delta) \hat{j} \]

\[ a = b = A \]

\[ \delta = \pm \pi/2 \]

\[ \cos(\pi/2 + \theta) = -\sin \theta \]

\[ \cos(-\pi/2 + \theta) = -\sin \theta \]

\[ E_x = a \cos(\omega t - kz) \]

\[ E_y = b \cos(\omega t - kz + \delta) \]

\[ E_n = A \cos(\omega t - kz) \]

\[ E^2_n + E^2_y = A^2 \]

\[ E_x + E_y = A \cos(\omega t - k(z + \pi/2)) + b \sin(\omega t - kz) \]

\[ = A^2 \left( \cos^2(\ldots) + \sin^2(\ldots) \right) \]
Extra Practice Problems
5) For normal incidence, if an electromagnetic wave is passing from one medium (refractive index, $n=1.78$) to another medium (refractive index, $n=1$), then what will be the reflectivity at that interface?

- 6%
- 10%
- 8%
- 11%
8) A sensor response in four different ranges of concentration (marked accordingly) is given in options where λ(nm) is the function of x. The sensitivity of the sensor is the largest in which range?

- $40x^2 + 2x$, for $x < 10\text{mM}$
- $5 + (1.2)^{4x+2}$, for $10 < x < 20\text{mM}$
- $300 + 2x^2$, for $25 > x > 20\text{mM}$
- $5x^3 + 2$, for $30 > x > 25\text{mM}$