Unit 14 - Week 12: ODE - Boundary Value Problems

Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-23, 23:59 IST.

Problem 1: ODE-IVP

Consider the following mass-spring-damper system:

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = 0,\ \ x(0) = 1,\ \ \frac{dx}{dt}\Big|_{t=0} = 0.5$$

In the above model, $m = 1, a = 5$ and $b = 1.5$

We will first convert into a set of two ODEs and then solve using Euler's explicit method

1) Let us define $y = \frac{dx}{dt}$. With this definition, the second ODE becomes:

$$\frac{dy}{dt} = \alpha y + \beta x$$

Please report the value of α

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -5.05,-4.95

0.1 points

2) Please report the value of β

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -1.55,-1.45

0.1 points

With the above changes, the ODE-IVP in two variables can be written as:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ \alpha y + \beta x \end{bmatrix},\ \ \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

We will use Euler's explicit method with step-size $h = 0.25$ to solve this problem starting with the initial condition given above

3) Euler's method gives $Y_{i+1} = Y_i + h\,\mathbf{f}(Y_i, t_i)$. Let us first compute $\mathbf{f}(Y_i, t_i)$, which is a 2×1 vector. Please report the first element of vector $\mathbf{f}(Y_i, t_i)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.49,0.51

0.1 points

4) Please report the second element of vector $f(Y_i, t_i)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -4.1,-3.9

0.1 points

5) Hence use Euler's method to calculate $Y(0.25)$. Please report the value of the first element, i.e., displacement x at $t = 0.25$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.1,1.15

0.1 points

6) Please report the value of the second element, i.e., velocity dx/dt at $t = 0.25$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -0.51,-0.49

0.1 points

7) Repeat this for one more iteration. This will give you $Y(0.5)$. Please report the first element, i.e., displacement at $t = 0.5$ using two steps of Euler's method.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.99,1.01

0.1 points

8) Also report the second element, i.e., velocity at $t = 0.5$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -0.304,-0.29

0.1 points

9) Please repeat this process for two more steps and obtain the results at $t = 1$ using four iterations of Euler's explicit method. Hence, report the displacement at $t = 1$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.83,0.87

0.1 points

10) Also report the velocity at $t = 1$ using four steps of Euler's method

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -0.29,-0.25

0.1 points

Problem 2: ODE-BVP using Finite Difference

Consider the following ODE-BVP resulting from a convective-diffusive process with source:

$$\frac{1}{P}\frac{d^2C}{dx^2} + \frac{dC}{dx} - D(C-1) = 0$$

where $P = 5$ and $D = 2$.

This can be rewritten as:

$$\frac{d^2C}{dx^2} + 5\frac{dC}{dx} - 10C + 10 = 0$$

Let the boundary conditions $C(0) = 0.1$ and $C'(1) = 0$.

We will divide the domain into 10 equal intervals with $h = 0.1$. The conditions at the two ends of the system are given by:

$C_0 = 0.1,\ \ \frac{C_{11} - C_9}{h} = 0$

For all the internal nodes, we will use central difference formula. For each of the internal nodes, the *central difference formula* leads to:

$$\alpha C_{i+1} + \beta C_i + \gamma C_{i-1} = \delta$$

Node 1 and Node 10 will make use of the boundary conditions. Rest of the nodes use the above formula. Thus, the overall model becomes:

$$\begin{bmatrix} \beta & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & \beta & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & \beta & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & \beta & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma & \beta & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & \beta & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma & \beta & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \beta & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \beta & \alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 200 & -210 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \end{bmatrix} = \begin{bmatrix} -17.5 \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ -10 \end{bmatrix}$$

Please provide the values of $\alpha, \beta, \gamma, \delta$ below.

11) Please report the value of α

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 124,126

0.25 points

12) Please report the value of β

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -215,-205

0.25 points

13) Please report the value of γ

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 74,76

0.25 points

14) Please report the value of δ

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -10.1,-9.9

0.25 points