

# Unit 14 - Week 12: ODE - Boundary Value Problems

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### Week 2 - Linear Systems and Equations

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### Week 12: ODE - Boundary Value Problems

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## Assignment 12

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

**Due on 2019-10-23, 23:59 IST.**

### Problem 1: ODE-IVP

Consider the following mass-spring-damper system:

$$m \frac{d^2 x}{dt^2} + a \frac{dx}{dt} + bx = 0, \quad x(0) = 1, \quad \frac{dx}{dt} \Big|_{t=0} = 0.5$$

In the above model,  $m = 1$ ,  $a = 5$  and  $b = 1.5$

We will first convert into a set of two ODEs and then solve using Euler's explicit method

- 1) Let us define  $y = \frac{dx}{dt}$ . With this definition, the second ODE becomes:

$$\frac{dy}{dt} = \alpha y + \beta x$$

Please report the value of  $\alpha$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -5.05,-4.95

0.1 points

- 2) Please report the value of  $\beta$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -1.55,-1.45

0.1 points

With the above changes, the ODE-IVP in two variables can be written as:

$$\frac{d}{dt} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ \alpha y + \beta x \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

We will use Euler's explicit method with step-size  $h = 0.25$  to solve this problem starting with the initial condition given above

- 3) Euler's method gives  $Y_{i+1} = Y_i + h f(Y_i, t_i)$ . Let us first compute  $f(Y_i, t_i)$ , which is a  $2 \times 1$  vector. Please report the first element of vector  $f(Y_i, t_i)$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) 0.49,0.51

0.1 points

- 4) Please report the second element of vector  $f(Y_i, t_i)$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -4.1,-3.9

0.1 points

- 5) Hence use Euler's method to calculate  $Y(0.25)$ . Please report the value of the first element, i.e., displacement  $x$  at  $t = 0.25$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) 1.1,1.15

0.1 points

- 6) Please report the value of the second element, i.e., velocity  $dx/dt$  at  $t = 0.25$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -0.51,-0.49

0.1 points

- 7) Repeat this for one more iteration. This will give you  $Y(0.5)$ . Please report the first element, i.e., displacement at  $t = 0.5$  using two steps of Euler's method.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) 0.99,1.01

0.1 points

- 8) Also report the second element, i.e., velocity at  $t = 0.5$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -0.304,-0.29

0.1 points

- 9) Please repeat this process for two more steps and obtain the results at  $t = 1$  using four iterations of Euler's explicit method. Hence, report the displacement at  $t = 1$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) 0.83,0.87

0.1 points

- 10) Also report the velocity at  $t = 1$  using four steps of Euler's method

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -0.29,-0.25

0.1 points

### Problem 2: ODE-BVP using Finite Difference

Consider the following ODE-BVP resulting from a convective-diffusive process with source:

$$\frac{1}{P} \frac{d^2 C}{dx^2} + \frac{dC}{dx} - D(C - 1) = 0$$

where  $P = 5$  and  $D = 2$ .

This can be rewritten as:

$$\frac{d^2 C}{dx^2} + 5 \frac{dC}{dx} - 10C + 10 = 0$$

Let the boundary conditions  $C(0) = 0.1$  and  $C'(1) = 0$ .

We will divide the domain into 10 equal intervals with  $h = 0.1$ . The conditions at the two ends of the system are given by:

$$C_0 = 0.1, \quad \frac{C_{11} - C_{10}}{h} = 0$$

For all the internal nodes, we will use central difference formula. For each of the internal nodes, the central difference formula leads to:

$$\alpha C_{i+1} + \beta C_i + \gamma C_{i-1} = \delta$$

Node 1 and Node 10 will make use of the boundary conditions. Rest of the nodes use the above formula. Thus, the overall model becomes:

$$\begin{bmatrix} \beta & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & \beta & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & \beta & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & \beta & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma & \beta & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & \beta & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma & \beta & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \beta & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & \beta & \alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 200 & -210 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \end{bmatrix} = \begin{bmatrix} -17.5 \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ \delta \\ -10 \end{bmatrix}$$

Please provide the values of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$  below.

- 11) Please report the value of  $\alpha$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) 124,126

0.25 points

- 12) Please report the value of  $\beta$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -215,-205

0.25 points

- 13) Please report the value of  $\gamma$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) 74,76

0.25 points

- 14) Please report the value of  $\delta$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
(Type: Range) -10.1,-9.9

0.25 points