Unit 11 - Week 11: ODE-IVP (Part-2)

Assignment 11

Due on: 2019-10-16, 23:59 IST

Course outline

How to access the part?
Course Pre-requisites and Information
Week 1 - Computation and Error Analysis
Week 2 - Linear Systems - 2
Week 3 - Linear Equations - 2
Week 4 - Nonlinear Equations in Single Variables
Week 5 - Nonlinear equations in Single and Multiple Variables
Week 6 - Regression Game Fitting
Week 7 - Interpolation
Week 8 - Numerical Differentiation
Week 9 - Numerical Integration
Week 10 - Ordinary Differential Equations - Initial Value Problems (ODE-IVP)
Week 11: ODE-IVP (Part-II)
Introduction to Predictor- Corrector Methods
Stability Analysis of Euler's Methods
Extension to Multiple Variables
Least-squares Regression
Romberg's extrapolation
BVP Solver of ODE's
Integrate
Adaptive Step-Sizing
Adaptive extrapolating and Enveloping Methods
Boundary Conditions and Extrapolation using Runge-Kutta
Accuracy and Error Analysis (Weeks 14 and 15)

Data: Assignment 11

My Assignment
Numerical Methods for Engineers: Week 11
Feedback Form
Solutions to Assignment 11

Video Download, Live Broadcast and Other Information

Info about our Final Exams

Assignment 11

The due date for submitting the assignment has passed.

As per our records you have not submitted this assignment.

For the first three problems of the assignment, consider the NC outlet model is given by

\[
\frac{dL}{dt} = \frac{L - \frac{L}{2}}{L}
\]

for induction \( L = 1 \) and \( L = 2 \), resistance \( R = 2 \), and voltage \( V \). The initial value of current \( I(t) \) is \( 0 \). Note that the analytical solution (true solution for current at any time) is given by

\[
I(t) = \left(1 - e^{-\frac{t}{R}}\right)
\]

Problem 1: RK-2 Method:

In the previous assignment, you used RK-1 + RK-1 Method for solving a certain Differential Equation (DE) - IVP. The RK-1 + RK-2 method is given as:

\[
y_k = y_{n-1} + \frac{h}{2} f(t, y_{n-1})\]

Recall that the initial value of \( y(0) \) is \( 0 \). In this problem, we will compute the solution \( S(0.5) \) with \( h = 0.5 \) and \( h = 0.25 \).

1. Report the solution using the RK-2 method with \( h = 0.5 \). Cost that solution\( S(0.5) \).

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( y(0.5) \approx 0.125 \)

2. Report the error between true solution and your RK-2 method with \( h = 0.5 \).

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( \text{Error} \approx 0.0125 \)

3. Report the solution with two steps of \( h = 0.25 \). Please round that you are still computing \( 0.5 \) in this question. Cost that solution \( S_1 \).

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( S_1 = 0.125 \)

4. Report the error between true solution and your RK-2 method with \( h = 0.25 \).

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( \text{Error} \approx 0.0039 \)

Problem 2: Richardson's extrapolation:

In this problem, we will use Richardson's extrapolation to improve the solution obtained using the midpoint method.

1. Set up the system of equations \( A = 0 \) or \( 0 \), and the solution \( y = 0 \) or \( 0 \) as in Problem 1. Use these two systems to compute the better approximation using Richardson's extrapolation method. Note that \( \alpha = 2 \).

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( \text{Solution} \approx 0.1 \)

2. Report the error between true solution and your solution using Richardson's extrapolation.

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( \text{Error} \approx 0.1 \)

Problem 3: Stability Analysis of Euler's Method

Let's verify the stability of Euler's method. Euler's explicit method is stable for small values of \( h \), but it becomes unstable if the value of \( h \) increases beyond a certain threshold. Based on the assumptions in the lecture, the value of \( h \leq 0.25 \). (Note: \( h = 0.25 \) in this example).

1. With \( h = 1 \), use four iterations of Euler's explicit method and report \( y_1 \).

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( y_1 \approx 1.25 \)

2. With \( h = 0.1 \), use four iterations of Euler's explicit method and report \( y_1 \).

Yes, the answer is correct.

Answer:

Accepted Answer:

(A) \( y_1 \approx 1.025 \)

3. With \( h = 0.25 \), use four iterations of Euler's explicit method and report \( y_1 \).

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( y_1 \approx 1.25 \)

4. Based on the above result, what is the threshold value of \( h \) beyond which Euler's explicit method becomes unstable.

No, the answer is incorrect.

Answer:

Accepted Answer:

(A) \( h > 0.25 \)

0.00 points

0.00 points

0.00 points

0.00 points

0.00 points

0.00 points

0.00 points

0.00 points

0.00 points