

Unit 13 - Week 11: ODE-IVP (Part-2)

Course outline

How to access the portal?

Course Pre-requisites and Introduction

Week 1 - Computation and Error Analysis

Week 2 - Linear Systems and Equations

Week 3 - Linear Equations - 2

Week 4: Nonlinear Equations in Single Variable

Week 5: Nonlinear equations in Single and Multiple Variables

Week 6: Regression (Curve Fitting)

Week 7: Interpolation

Week 8: Numerical Differentiation

Week 9: Numerical Integration

Week 10: Ordinary Differential Equations – Initial Value Problems (ODE-IVP)

Week 11: ODE-IVP (Part-2)

- Introduction to Predictor-Corrector Methods
- Stability of Implicit Methods: Overview
- Stability Analysis of Euler's Methods
- Extension to multiple variables
- Local vs. Global Truncation Errors
- Richardson's Extrapolation
- Stiff System of ODEs: Introduction
- Adaptive Step-sizing
- Adaptive step-sizing and Embedded Methods
- Bonus: Errors and Extrapolation using MS-Excel
- Summary and Recap (Weeks 10 and 11)
- Quiz : Assignment 11

Week 12: ODE - Boundary Value Problems

Video Download, Live Session and Other Information

Info about our Final Exam

Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-16, 23:59 IST.

For the first three problems of this assignment, consider the RL circuit model is given by

$$\frac{dI}{dt} = \frac{V}{L} - \frac{IR}{L} \quad (1)$$

for inductance $L = 2$, resistance $R = 2.5$, voltage $V = 5$. The initial value of current $I(0) = 0$. Note that the analytical solution (true solution) for current at any time is given by:

$$I = \frac{V}{R} \left(1 - \exp\left(-\frac{Rt}{L}\right) \right) \quad (2)$$

Problem 1: RK-2 Midpoint Method :

In the previous assignment, you used $RK - 2$ Midpoint Method for solving a different set of $ODE - IVP$. The $RK - 2$ Midpoint Method is given as:

$$y_{i+1} = y_i + hk_2, \text{ where } k_1 = f(y_i, t_i), \quad k_2 = f\left(y_i + \frac{hk_1}{2}, t_i + \frac{h}{2}\right)$$

Recall that the initial value is $I(0) = 0$. In this problem, we will compute the solution $I(0.5)$ with $h = 0.5$ and $h = 0.25$

- 1) Report the solution using Midpoint Method with $h = 0.5$. Call this solution I_1

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 0.844,0.875

0.25 points

- 2) Report the error between true solution and your Midpoint method (with $h = 0.5$)

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 0.069,0.071
(Type: Range) -0.071,-0.069

0.25 points

- 3) Report the solution with two steps of $h = 0.25$. Please note that you are still computing $I(0.5)$ in this question. Call this solution I_2

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 0.9,0.93

0.25 points

- 4) Report the error between true solution and your Midpoint method (with $h = 0.25$)

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 0.0133,0.0143
(Type: Range) -0.0143,-0.0133

0.25 points

Problem 2: Richardson's extrapolation :

In this problem, we will use Richardson's extrapolation to improve the solution obtained using the Midpoint method

- 5) Let us call the solution using $h = 0.5$ as I_1 and the solution using $h = 0.25$ as I_2 , as done in Problem-1. Use these two values to compute better approximation using the Richardson's extrapolation. Note that LTE is $\mathcal{O}(h^3)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 0.910,0.936

0.5 points

- 6) Report the error between true solution and value using Richardson's extrapolation

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 0.0058,0.0060
(Type: Range) 0.0047,0.0049
(Type: Range) 0.0046,0.0050
(Type: Range) 0.0056,0.0060
(Type: Range) -0.0050,-0.0046
(Type: Range) -0.0060,-0.0056

0.5 points

Problem 3: Stability Analysis of Euler's Method

Let's verify the stability of Euler's method. Euler's explicit method is stable for small values of h , but can become unstable if the value of h increases beyond a certain threshold. Based on the discussions in the lectures, this value is $h \leq 2/\lambda$ (what is λ in this example?).

- 7) With $h = 1$, use four iterations of Euler's explicit method and report y_4

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 1.94,2.04

0.25 points

- 8) With $h = 1.6$, use four iterations of Euler's explicit method and report y_4

Yes, the answer is correct.
Score: 0.25

Accepted Answers:
(Type: Range) -0.01,0.01

0.25 points

- 9) With $h = 2.5$, use four iterations of Euler's explicit method and report y_4

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) -40.0,-38.0

0.25 points

- 10) Based on the above result, what is the threshold value of h beyond which Euler's explicit method becomes unstable.

Hint

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 1.58,1.62

0.25 points