Unit 13 - Week 11: ODE-IVP (Part-2)

Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-16, 23:59 IST.

For the first three problems of this assignment, consider the RL circuit model is given by

$$\frac{dI}{dt} = \frac{V}{L} - \frac{IR}{L} \qquad (1)$$

for inductance $L = 2$, resistance $R = 2.5$, voltage $V = 5$. The initial value of current $I(0) = 0$. Note that the analytical solution (true solution) for current at any time is given by:

$$I = \frac{V}{R}\left(1 - exp\left(-\frac{Rt}{L}\right)\right) \qquad (2)$$

Problem 1: RK-2 Midpoint Method :

In the previous assignment, you used $RK-2$ Midpoint Method for solving a different set of $ODE-IVP$. The $RK-2$ Midpoint Method is given as:

$y_{i+1} = y_i + hk_2$, where $k_1 = f(y_i, t_i)$, $\quad k_2 = f\left(y_i + \frac{hk_1}{2}, t_i + \frac{h}{2}\right)$

Recall that the initial value is $I(0) = 0$. In this problem, we will compute the solution $I(0.5)$ with $h = 0.5$ and $h = 0.25$

1) Report the solution using Midpoint Method with $h = 0.5$. Call this solution I_1

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.844,0.875

0.25 points

2) Report the error between true solution and your Midpoint method (with $h = 0.5$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.069,0.071
(Type: Range) -0.071,-0.069

0.25 points

3) Report the solution with two steps of $h = 0.25$. Please note that you are still computing $I(0.5)$ in this question. Call this solution I_2

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.9,0.93

0.25 points

4) Report the error between true solution and your Midpoint method (with $h = 0.25$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0133,0.0143
(Type: Range) -0.0143,-0.0133

0.25 points

Problem 2: Richardson's extrapolation :

In this problem, we will use Richardson's extrapolation to improve the solution obtained using the Midpoint method

5) Let us call the solution using $h = 0.5$ as I_1 and the solution using $h = 0.25$ as I_2, as done in Problem-1. Use these two values to compute better approximation using the Richardson's extrapolation. Note that LTE is $\mathcal{O}(h^3)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.910,0.936

0.5 points

6) Report the error between true solution and value using Richardson's extrapolation

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.0058,0.0060
(Type: Range) 0.0047,0.0049
(Type: Range) 0.0046,0.0050
(Type: Range) 0.0056,0.0060
(Type: Range) -0.0050,-0.0046
(Type: Range) -0.0060,-0.0056

0.5 points

Problem 3: Stability Analysis of Euler's Method

Let's verify the stability of Euler's method. Euler's explicit method is stable for small values of h, but can become unstable if the value of h increases beyond a certain threshold. Based on the discussions in the lectures, this value is $h \leq 2/\lambda$ (what is λ in this example?).

7) With $h = 1$, use four iterations of Euler's explicit method and report y_4

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.94,2.04

0.25 points

8) With $h = 1.6$, use four iterations of Euler's explicit method and report y_4

Yes, the answer is correct.
Score: 0.25
Accepted Answers:
(Type: Range) -0.01,0.01

0.25 points

9) With $h = 2.5$, use four iterations of Euler's explicit method and report y_4

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -40.0,-38.0

0.25 points

10) Based on the above result, what is the threshold value of h beyond which Euler's explicit method becomes unstable.

Hint

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.58,1.62

0.25 points